PUPILS' SERIES of ARITHMETICS

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Laura Kerrigan

The Pupils' Series of Arithmetics

HIGHER BOOK

BY

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CONTENTS.

SUGGESTIONS.

The work throughout this book has been carefully prepared to meet the wants of students who have completed elementary arithmetic. The exercises are designed to cultivate and strengthen the reasoning powers, and to assist the teacher in stimulating independent thought.

Notation is presented through a comparison of the two methods. The foundation of the Arabic system is shown to consist in the power of position, rendered universally effective by the use of the nought. Gaining this knowledge, the pupil has the key to the system of decimal notation.

The four fundamental operations are briefly given. The principles are clearly set forth, and the problems are suited to the advanced grade of the pupil.

In Properties of Numbers the processes of factoring and the illustrative work in Divisors and Multiples will develop a clearer insight into number-relations. The subject should be thoroughly taught. The comprehensive review which follows will test the pupil's knowledge.

Besides the methods usually employed in teaching fractions, will be found processes which lessen the one difficulty of the subject, the difficulty always experienced when only the ordinary rules for division of fractions are learned. The pupil gains a clear comprehension of the distinction between the fractional unit and the integral unit; he acquires a knowledge of the law which endows the reciprocal with power to simplify any question whose solution depends upon the relation of the

fraction to the integral unit; he easily masters division of fractions.

It is urged: —

1. That the principles of fractions be persistently studied until they are learned.
2. That comparison of the fractional unit with the integral unit be often required.
3. That throughout the subject of fractions the reciprocal be employed.

It will be observed that upon these points this book teaches no new thing, but that it provides for the constant use of the reasoning powers which are commonly developed with less insistence and continuity.

The treatment of Compound Numbers affords work of a practical and disciplinary nature. The distinction between two units of different orders, begun in the first pages of the book, and continued throughout fractions, is not abandoned. The forms and processes will prove valuable means of illustration; especially to be noted are the forms which show the method of reduction, ascending and descending, in linear and square measures.

In Ratio and Proportion the pupil extends his knowledge of the relations of numbers. The subject is not treated at great length, as the work is but little more than the application of principles previously taught. For arranging the three known terms of a proportion, space is given to but one method, which is fully presented. The teacher who desires may readily vary the form, by making the third term, or any other term, the unknown term of the proportion.

Percentage is treated from the standpoint of ratio, or the relations of numbers.

Interest, with its many forms and analyses, furnishes much varied work.

The remaining subjects are treated concisely, and much space is devoted to valuable work in review.

HIGHER BOOK OF ARITHMETIC.

NOTATION AND NUMERATION.

1. The characters used for denoting numbers by the Roman method are seven capital letters:

M	D	C	L	X	V	I
One Thousand	Five Hundred	One Hundred	Fifty	Ten	Five	One

2. Repeating a letter adds its denoted value to the denoted value of the letter repeated.

The highest number that can be denoted by employing each letter once is One Thousand Six Hundred Sixty-six (MDCLXVI). To add one thousand to this number an additional M is prefixed; to add five hundred an M is substituted for the D; to add one hundred an additional C is written before the L; to add fifty a C is substituted for the L; to add ten an additional X is written before the V; to add five an X is substituted for the V; to add one an I is suffixed.

3. A letter denoting a smaller number than that denoted by the letter to its right signifies that the larger number is decreased by the smaller number.

Three repetitions of a letter are not commonly used. To denote four V is written with an I to its left, IV, meaning one less than five; IX denotes one less than ten; XL denotes ten less than fifty; XC denotes ten less than one hundred; CD denotes one hundred less than five hundred; CM denotes one hundred less than one thousand.

4. The characters used to denote numbers by the Arabic, or Decimal, method, are the ten symbols, or figures:

0	1	2	3	4	5	6	7	8	9
Nought	One	Two	Three	Four	Five	Six	Seven	Eight	Nine

5. Repeating a figure adds its value to ten times the value of the figure repeated.

In writing one thousand six hundred sixty-six, 1 is one; suffixing the figure 6 to the figure 1, the 1 becomes ten, and the 6 is added to the ten, making sixteen (16); again suffixing a 6, the 16 becomes one hundred sixty, and the 6 is added, making one hundred sixty-six (166); again suffixing a 6, the 166 becomes one thousand six hundred sixty, and the 6 is added, making one thousand six hundred sixty-six (1666).

6. In **Roman Notation** the value of a letter is not altered by any change of position.

In XXX each X denotes ten ones, or units; one X is no greater than another. In VIII each I means one unit; any I is equal to any other I.

7. In **Arabic,** or **Decimal, Notation** the value of a figure is altered by any change of position.

In 999 every 9 does not denote nine ones, or units; the 9 on the right denotes nine *units;* the second, or middle 9, denotes nine *tens*, and the third 9 denotes nine *hundreds*. In 5111 every 1 denotes one, but not one of the same value; the 1 on the right denotes one that has the value of *one;* the second 1 denotes one that has the value of *ten;* the third 1 denotes one that has the value of *one hundred;* the 5 denotes five, but each one of the five has the value of *one thousand.*

8. In **Roman Notation** there is no letter used to represent no number, or *none.*

As each letter has but one value, regardless of the position of the letter, there is no need for an expression denoting *none.* L denotes 50, which is five tens and no unit; X denotes 10, which is one ten and no unit.

9. In **Decimal Notation** a character to represent no number, or *none,* is necessary.

Fifty-one, written 51, means *one* unit and five tens; 50 means *no* unit and five tens; without the *nought* the 5 would change its position, and, occupying the first place, it would there denote 5 units.

Write in both Arabic and Roman Notation the following:

1. One thousand six hundred sixty-six.
2. Six hundred sixty-six.
3. Seventy-six.
4. Eighty-six.
5. Fifty-six.
6. Fifty-seven.
7. Sixty-six.
8. Fifty-eight.
9. Fifty-nine.
10. Forty-nine.
11. Fifty.

Write in Roman the following:

1. Thirty-five.
2. Thirty-eight.
3. One hundred.
4. Two hundred.
5. Two hundred thirty-five.
6. Two hundred thirty-eight.
7. Three hundred thirty-three.
8. Five hundred thirty-five.

Write in Arabic the following:

1. Seven hundred sixty-six.
2. Eight hundred sixty-eight.
3. Nine hundred sixty-nine.
4. Five hundred sixty-five.
5. One thousand.
6. One thousand five hundred.
7. One thousand nine hundred.
8. Two thousand nine hundred.

Write in Roman the following:

1. 20	4. 301	7. 500	10. 70	13. 90	16. 1009
2. 30	5. 40	8. 501	11. 700	14. 100	17. 1776
3. 300	6. 50	9. 60	12. 702	15. 1000	18. 1893

Write in Arabic the following:

1. MD	5. MDL	9. XI	13. CXI
2. MC	6. MDXL	10. XIX	14. CXIX
3. MCC	7. MCXL	11. XIV	15. CXIV
4. MDC	8. MCLX	12. XVI	16. CXVI

Write in Arabic and in Roman the following:

1. One thousand nine.
2. One thousand eight.
3. One thousand one.
4. Three thousand ten.
5. Three thousand eleven.
6. Five hundred eleven.
7. Five hundred forty-one.
8. Five hundred nineteen.
9. Two thousand nineteen.
10. Two thousand nine.
11. Two thousand ninety.
12. Two thousand one hundred.
13. Two thousand ten.
14. Two thousand forty-nine.
15. Two thousand two hundred.
16. Two thousand two.

10. In Arabic Notation there are three **Orders** of units: units of ones, units of tens, units of hundreds. The first order is that of units, the second that of tens, the third that of hundreds.

TABLE OF ORDERS.

10 units make 1 ten.
10 tens make 1 hundred.
10 hundreds make 1 thousand.

11. A **Period** is composed of three orders; three orders of units make the period of units; three orders of thousands make the period of thousands.

TABLE OF THE THOUSANDS' PERIOD.

10 hundreds make 1 thousand.
10 thousands make 1 ten-thousand.
10 ten-thousands make 1 hundred-thousand.

12. Higher Periods are Millions, Billions, Trillions, Quadrillions, etc.

TABLE OF PERIODS TO TRILLIONS.

1000 units make 1 thousand.
1000 thousands make 1 million.
1000 millions make 1 billion.
1000 billions make 1 trillion.

13. For convenience either commas or spaces may be used *to separate* periods.

Write in words the following expressions:

1. 1,000	**14.** 400,444	**27.** 516,016	**40.** 1,000,108
2. 1,000,000	**15.** 410,440	**28.** 606,106	**41.** 1,101,008
3. 10,000	**16.** 404,404	**29.** 707,707	**42.** 1,901,910
4. 100,000	**17.** 400,004	**30.** 170,170	**43.** 9,010,010
5. 200,000	**18.** 404,004	**31.** 17,017	**44.** 9,101,011
6. 300,000	**19.** 414,014	**32.** 7,007	**45.** 9,011,010
7. 330,000	**20.** 410,040	**33.** 171,070	**46.** 9,000,101
8. 333,000	**21.** 401,001	**34.** 107,107	**47.** 9,101,909
9. 333,300	**22.** 410,004	**35.** 810,018	**48.** 8,190,019
10. 333,330	**23.** 400,400	**36.** 801,801	**49.** 9,019,109
11. 333,333	**24.** 40,040	**37.** 108,008	**50.** 9,109,091
12. 303,333	**25.** 4,004	**38.** 819,000	**51.** 9,901,919
13. 300,333	**26.** 404,044	**39.** 910,008	**52.** 9,010,101

14. NUMBERS SMALLER THAN UNITS and written by the Arabic, or Decimal, notation are called Decimals, or Decimal Fractions.

In the number 111, the figure on the left denotes 1 ***hundred;*** the next figure denotes 1 *ten*, which is the tenth part of 1 hundred; the next figure denotes 1 *unit*, which is the tenth part of 1 ten.

15. The value of any figure in a number is the tenth of the value of a like figure one place to its left.

The value of a unit of the first order of decimals is one tenth of 1. In writing by the decimal notation numbers smaller than units, it becomes necessary to keep the smaller numbers, or decimals, separated from the whole numbers, or integers; if it be required to write one hundred eleven and one tenth, the one tenth must be prevented from throwing the integers one place to the left, hence:

16. The Decimal point is used to separate Decimals from Integers.

The first figure to the right of the decimal point denotes tenths, the second figure hundredths, the third figure thousandths, the numbers expressed decreasing to the right by tens, as in whole numbers.

Write in words:

1. 21.1	**6.** 21.11	**11.** 21.111	**16.** 21.019
2. 21.2	**7.** 21.12	**12.** 21.121	**17.** 21.0195
3. 21.3	**8.** 21.13	**13.** 21.01	**18.** 21.0105
4. 21.5	**9.** 21.15	**14.** 21.09	**19.** 21.1005
5. 21.9	**10.** 21.19	**15.** 21.001	**20.** 22.0001

Write in figures, using the decimal point:

1. Three and three tenths.
2. Three and three hundredths.
3. Three and three thousandths.
4. Three ten-thousandths.
5. Three hundred-thousandths.
6. Three and three millionths.
7. Four hundred-thousandths.
8. Four ten-thousandths.
9. Fifteen thousandths.
10. Sixteen hundredths.

NOTATION. — *Notation in Arithmetic is writing numbers.*

NUMERATION. — *Numeration is reading numbers.*

ADDITION.

DEFINITIONS.

17. **Addition** is the process of finding how many units are contained in two or more numbers.

18. The whole number of units in two or more numbers is called their **Sum.**

19. The **Sign** of addition (+) is read *plus,* meaning *more.*

20. Dollars are expressed by writing the dollar-sign ($) to the left of the figures; as, $ 500, denoting five hundred dollars.

Cents are expressed as decimal fractions, the decimal point being preceded by the dollar-sign; as, $ .08, denoting eight cents.

In an expression for *dollars and cents* the decimal point separates the two numbers, as $ 500.08, denoting five hundred dollars, eight cents.

PROBLEMS.

1. Find the sum of $ 1.10 and $ 2.35.

$$\begin{array}{r} \$1.10 \\ 2.35 \\ \hline \$3.45 \end{array}$$

2. Monday a miller sold 532 barrels of flour, Tuesday he sold 341 barrels, and Wednesday he sold 123 barrels; how many barrels of flour did he sell in the three days?

3. A man paid $ 6.15 for car fare, and $ 2.50 for a berth in the sleeper; how much for both?

4. What is the sum of $ 5,814.15, $ 1,015.10, and $ 130?

5. A man owns three farms in different states; in Arkansas one of 1,431 acres, in Louisiana one of 5,243 acres, and in Texas one of 2,315 acres; how many acres in the three farms?

6. What is the sum of 123, 231, 132, and 213?

7. Find the sum of one thousand ten, three thousand three hundred, and two thousand nine.

8. Cæsar was born in the year 100 B.C. On the title-page of an old Cæsar's Commentaries MDCXLIV is printed. How many years from the year of Cæsar's birth to the year in which the old book was printed? *Ans.* 1,743 yr.

Find the sum of $5,192, $3,418, $7,565, and $1,891.

EXPLANATION.

$5,192
3,418
7,165
1,891
$17,666

(i) In writing columns of dollars, the dollar-sign is necessary only at the head of the column and at the number which gives the total of the columns.

(ii) The sum of the column of units is found; 1, 6, 14, 16 units; 16 units are 1 ten, 6 units; the 6 is placed under the column of units.

(iii) Combining with the column of tens the 1 ten from the 16 units, the sum of the column of tens is found; 1, 10, 16, 17, 26; 26 tens are 2 hundreds, 6 tens; the 6 is placed under the column of tens.

(iv) Combining with the column of hundreds the 2 hundreds from the 26 tens, the sum of the column of hundreds is found; 2, 10, 11, 15, 16; 16 hundreds are 1 thousand, 6 hundreds; the 6 is placed under the column of hundreds.

(v) Combining with the column of thousands the 1 thousand from the 16 hundreds, the sum of the column of thousands is found; 1, 2, 9, 12, 17; 17 thousands are 1 ten-thousand and 7 thousands; the 7 is placed under the column of thousands, and the 1 in the ten-thousands' place.

Find the sums of the following:

9.	**10.**	**11.**	**12.**
358	3,558	58,535	92,134
798	7,988	79,887	79,398
562	5,622	56,925	65,417
341	6,499	34,113	35,209
276	2,766	27,662	197

Find the sums of the following:

13.	14.	15.	16.
12,369	298,640	$1,998.65	$105,603.25
39,684	385,969	954.72	751.05
72,798	797,878	3,877.50	1,987.62
65,665	876,665	6,986.75	8,004.90
78,978	987,787	5,550.00	207,316.00
96,567	765,776	998.95	98,119.95
68,875	576,587	870.05	6,005.00
79,397	987,897	693.55	17,000.00
87,974	659,589	1,729.37	19.15
76,594	998,889	3,985.40	182,314.48

21. *The Double-Column Method.*

There are tests of addition but there is no absolute proof of accuracy where long columns are added.

One method of testing addition work is made by adding two columns at a time.

Add and test the columns here given:

EXPLANATION.

Adding by single columns, the sum is found to be 369. Adding by double columns, the first number is 72, the next 40, the next 5, and so on, the figures of the two columns being mentally kept separate as tens and units, so that from the bottom to the top of the two columns the successive numbers are $72 + 40 + 5 + 60 + 4 + 10 + 7 + 90 + 8 + 70 + 3$.

73
98
17
64
45
72
369

TENS.	UNITS.		
thirty-six,	nine.		3
thirty-six,	six;	7	
twenty-nine,	six;		8
twenty-eight,	eight;	9	
nineteen,	eight;		7
nineteen,	one;	1	
eighteen,	one;		4
seventeen,	seven;	6	
eleven,	seven;		5
eleven,	two;	4	
seven,	two;		2
Seven;		7	

Mentally the tens and units are called as tens and units, a ten being added whenever the sum of the units exceeds 9, as shown in the diagram, where the operation proceeds step by step from bottom to top, thus: Seven; seven, two; eleven, two; eleven, seven; seventeen, seven; eighteen, one; nineteen, one; nineteen, eight; twenty-eight, eight; twenty-nine, six; thirty-six, six; thirty-six, nine. (369)

The double-column method is much used by accountants, as it secures greater rapidity and accuracy, after skill is acquired, than the single-column method.

Add and test the following:

17.	18.	19.	20.
4,130	3,223	2,762	3,179
3,121	4,731	3,551	4,236
1,532	1,924	4,627	8,717
2,714	1,353	3,834	4,150
1,326	4,627	5,170	3,129

21.	22.	23.	24.
[illegible]	[illegible]864	658,349	418,927
[illegible]	[illegible]	498,755	697,596
[illegible]	[illegible]	717,667	387,265
[illegible]	[illegible]	729,776	756,957
[illegible]	[illegible]	595,669	967,686
[illegible]	[illegible]	976,875	458,977
[illegible]	[illegible]	468,566	765,487
[illegible]	[illegible]	753,857	986,795
[illegible]	[illegible]	698,594	439,682
[illegible]	[illegible]	855,767	587,755
[illegible]	[illegible]	967,596	496,646

[illegible]	[illegible]	27.	28.
[illegible]	[illegible]	98,160,704	99,245,678
[illegible]	[illegible]	32,876,519	78,654,867
[illegible]	[illegible]	27,684,157	55,368,687
[illegible]	[illegible]	92,869,549	72,186,786
[illegible]	[illegible]	86,544,694	65,579,768
[illegible]	[illegible]	77,695,587	43,297,876
[illegible]	[illegible]	63,986,778	67,858,597
[illegible]	[illegible]	47,176,897	41,585,795
[illegible]	[illegible]	28,868,765	86,249,579
[illegible]	[illegible]654	91,667,856	98,794,975
[illegible]	[illegible]867	54,578,677	67,976,957
23,585,462	[illegible]54,938	77,698,768	78,967,795
89,548,796	87,562,093	81,987,986	89,749,698
56,785,657	86,214,959	37,279,895	97,894,869
47,629,987	97,399,875	69,589,759	58,776,986

29. Grain products, 1879, in bushels; find all the totals.

1879.	CORN.	WHEAT.	OATS.	RYE.	TOTALS.
Ala.	25,451,278	1,529,657	3,039,639	28,402	xx,xxx,xxx
Ark.	24,156,417	1,269,730	2,219,822	22,387	xx,xxx,xxx
Fla.	3,174,234	422	468,112	2,965	x,xxx,xxx
Ga.	23,202,918	3,159,771	5,548,743	101,716	xx,xxx,xxx
Ky.	72,852,263	11,356,113	4,580,738	668,050	xx,xxx,xxx
La.	9,906,189	5,034	229,840	1,013	x,xxx,xxx
Miss.	21,340,800	218,890	1,959,620	5,134	xx,xxx,xxx
Mo.	202,485,723	24,966,627	20,670,958	535,426	xxx,xxx,xxx
N. C.	28,019,839	3,397,393	3,838,068	285,100	xx,xxx,xxx
S. C.	11,767,099	962,358	2,715,505	27,049	xx,xxx,xxx
Tenn.	62,764,429	7,331,353	4,722,190	156,419	xx,xxx,xxx
Tex.	29,065,172	2,567,760	4,893,359	25,399	xx,xxx,xxx
Va.	29,106,661	7,822,504	5,333,181	324,431	xx,xxx,xxx
Total	xxx,xxx,xxx	xx,xxx,xxx	xx,xxx,xxx	x,xxx,xxx	670,283,900

30. Grain products, 1889, in bushels; find all the totals.

1889.	CORN.	WHEAT.	OATS.	RYE.	TOTALS.
Ala.	30,073,036	208,591	3,231,085	14,618	xx,xxx,xxx
Ark.	33,982,318	955,668	4,180,877	15,181	xx,xxx,xxx
Cal.	2,381,270	40,869,337	1,463,068	243,871	xx,xxx,xxx
Del.	3,096,164	1,501,050	382,900	6,625	x,xxx,xxx
Fla.	3,701,264	290	391,321	13,389	x,xxx,xxx
Ga.	29,261,422	1,096,312	4,767,456	87,021	xx,xxx,xxx
Ill.	289,629,705	37,371,081	137,602,804	2,627,949	xxx,xxx,xxx
Ind.	108,843,094	37,318,798	31,491,661	877,532	xxx,xxx,xxx
Iowa	313,130,782	8,249,786	146,679,289	1,445,283	xxx,xxx,xxx
Kan.	259,574,568	30,399,871	44,629,034	2,917,386	xxx,xxx,xxx
Ky.	78,434,847	10,707,462	8,775,814	423,847	xx,xxx,xxx
Mich.	28,785,579	24,771,171	36,961,193	2,101,713	xx,xxx,xxx
Minn.	24,696,446	52,300,247	49,958,791	1,252,663	xxx,xxx,xxx
Neb.	215,895,996	10,571,059	43,843,640	1,085,083	xxx,xxx,xxx
N. H.	988,806	35,192	3,668,909	6,664	x,xxx,xxx
O.	113,892,318	35,559,208	40,136,732	1,007,156	xxx,xxx,xxx
Ore.	238,203	9,298,224	5,948,594	63,206	xx,xxx,xxx
Utah	84,760	1,515,465	597,947	33,928	x,xxx,xxx
Wash.	156,413	6,345,426	2,273,182	243,871	x,xxx,xxx
Wis.	34,024,216	11,698,922	60,739,052	4,250,582	xxx,xxx,xxx
Total	x,xxx,xxx,xxx	xxx,xxx,xxx	xxx,xxx,xxx	xx,xxx,xxx	x,xxx,xxx,xxx

SUBTRACTION.

DEFINITIONS AND PRINCIPLES.

22. If the difference between two unequal numbers be added to the smaller number, the sum will equal the larger number.

23. If the difference between two unequal numbers be deducted (or subtracted) from the larger number, the remainder will equal the smaller number.

24. **Subtraction** is the process of finding the difference between two numbers:

Or, it is the process for finding what remains of one number out of which another number is taken.

(Subtraction may be performed with equal numbers: the difference between two equal numbers is 0, or zero.)

25. The number to be subtracted is called the **Subtrahend.**

26. The number from which the **Subtrahend** is to be subtracted is called the **Minuend.**

27. The result obtained is called the **Difference,** or the **Remainder.**

(I) In finding how much one number exceeds another number, the result is the difference.

(II) In lessening a number, *i.e.*, taking a part out of a whole, the result is the remainder.

28. The **Remainder** is equal to the **Minuend** less the **Subtrahend.**

29. The **Subtrahend** is equal to the **Minuend** less the **Remainder.**

30. The **Minuend** is equal to the **Subtrahend** plus the **Remainder.**

31. The **Sign** of subtraction (—) is *minus,* meaning *less.*

32. The **Sign** of equality (=) is read *equal* or *equals.*

PROBLEMS.

1. A farmer had 2,965 bushels of corn, and sold 1,723 bushels; how many bushels did he then have?

2,965	bu.	Minuend
1,723	"	Subtrahend
1,242	bu.	Remainder

2. The minuend is nine thousand nine hundred sixty-eight; the subtrahend is six thousand three hundred four; what is the remainder?

3. The remainder is 3,216; the minuend is 5,798; what is the subtrahend?

4. The subtrahend is 2,987; the remainder is 1,146; what is the minuend?

5. The minuend is 4,133; the subtrahend is 2,987; what is the remainder?

Explanation.

4,133	Min.
2,987	Sub.
1,146	Rem.

(i) From the 3 units of the minuend, the 7 units of the subtrahend cannot be subtracted; changing into units 1 of the 3 tens of the minuend, the units of the minuend become 13, and the tens of the minuend become 2. Subtracting the 7 units of the subtrahend from the 13 units of the minuend, the remainder is 6 units, which is placed below.

(ii) From the 2 tens of the minuend the 8 tens of the subtrahend cannot be subtracted; changing into tens the 1 hundred of the minuend, the tens of the minuend become 12, and the 1 hundred of the minuend becomes 0. Subtracting 8 tens from the 12 tens the remainder is 4 tens, which is placed below.

(iii) From the 0 hundred of the minuend the 9 hundreds of the subtrahend cannot be subtracted; changing into hundreds 1 of the 4 thousands of the minuend, the hundreds of the minuend become 10, and the thousands of the minuend become 3. Subtracting 9 hundreds from 10 hundreds the remainder is 1 hundred, which is placed below.

(iv) Subtracting 2 thousands from 3 thousands the remainder is 1 thousand, which is placed below.

Solve and prove the following (see **30**):

6. The area of the United States in 1790 was 827,844 square miles; by 1890 it had become 3,603,884 sq. mi.; what was the increase of area for the century?

7. The United States in 1883 produced 1,551,066,895 bushels of corn. In 1880 the production of corn was 1,717,434,543 bushels; by how many bushels did the U. S. corn crop of 1880 exceed that of 1883?

8. In 1886 the imports of the United States amounted in value to $ 635,436,136; the exports of the same year amounted to $ 665,964,529; find the excess of exports.

9. The minuend is 300,031; the subtrahend is 187,229; find the remainder.

EXPLANATION.

When many noughts occur consecutively in the minuend it may be found well to use either of the two following methods:

Double-Column Subtraction.

30 00 31 18 72 29 11 28 02	Subtract by double columns: 29 from 31, the remainder is 02; 72 from 100, the remainder is 28; 18 from 29, the remainder is 11.

Left-Hand Subtraction.

300031 187229 112802	(i) Begin at the left. Subtract 1 from 2, as one of the 3 hundred-thousands in the minuend must be changed into 10 ten-thousands, because of the 8 ten-thousands in the subtrahend; 1 from 2 leaves 1.

(ii) Subtract 8 from 9, as 1 of the 10 ten-thousands in the minuend must be changed into 10 thousands, because of the 7 thousands in the subtrahend; 8 from 9 leaves 1.

(iii) Subtract 7 from 9, as one of the 10 thousands in the minuend must be changed into 10 hundreds, because of the 2 hundreds in the subtrahend; 7 from 9 leaves 2.

(iv) Subtract 2 from 10: it will not be necessary to change 1 of the hundreds into tens; 2 from 10 leaves 8.

(v) Subtract 2 from 2, as 1 of the 3 tens in the minuend must be changed into units, because of the 9 units in the subtrahend; 2 from 2 leaves 0.

(vi) Subtract 9 units from 11 units; 9 from 11 leaves 2.

Solve each of the following by the different methods:

10. In 1848 Missouri gave 40,077 votes for Cass, and 32,671 for Taylor; find Cass's majority in Missouri.

11. By the census of 1880, the population of North Carolina was 1,400,047, of whom 1,396,368 persons were natives of the United States; how many were foreign-born?

12. In 1871 the United States produced 8,365,809 bushels of oats; in 1872 the yield was 9,000,769 bushels; find the increase over 1871.

13. In 1879 Kansas produced 300,273 bushels of barley, and Tennessee 30,019 bushels; find the difference.

14. Cotton retained in U. S. in 1882 for home consumption amounted to 1,012,103,200 pounds against 953,049,105 pounds in 1881; find the difference.

15. Value of U. S. imports of wines: 1890, $ 8,859,956; 1891, $ 10,007,060; find the increase.

16. From a treasury containing $ 1,900,095 a payment of $ 98,908 was made; how much remained in the treasury?

17. To make a payment of $ 500,000 a company has on hand $ 249,875; how much additional money must be obtained?

18. From a stock of 1,000,000 pounds of flour, 987,418 pounds were shipped; how much remained?

19. How much must be added to the number 999,864 to make the number 2,000,000?

20. In 1891 the taxable values of a county were $ 1,999,886; in 1892 its taxable values were $ 2,200,005; find the increase.

21. What number must be subtracted from 8,000,000 to leave the remainder 5,675,089?

22. The earth's diameter at the poles is estimated to be 7,898.88 miles, and at the equator 7,924.11 miles; find the difference.

23. How much must be added to 1,000,000 – 987,654 to make a number equal to 2,000,000 – 136,759?

24. The difference between two numbers is 5,998; the larger number is 19,859; find the smaller number.

25. How much must be added to one million one, minus five hundred thousand nine, to make a number equal to two million one thousand, minus two thousand eight?

26. Find the difference between 16,000,000 and 75,409.

27. The difference between two numbers is 5,998; the larger number is 9,859; find the smaller number.

28. The subtrahend is the sum of the numbers of the years 1801–1831, inclusive; the minuend is the sum of the numbers of the years 1841–1879, inclusive; what is the remainder?

Ans. 16,244.

29. Expressed in Roman characters, the remainder is XCIX; the subtrahend is DXLIV; express the minuend in Roman.

30. The minuend is 98,675 greater than the subtrahend; the subtrahend equals the remainder; the remainder is 98,675; find the minuend.

31. The remainder is $ 9,090.95; the minuend is the difference between $ 100,000.95 and $ 1,999.09; find the subtrahend.

32. The minuend was 1,000,000; the sum of the subtrahend and remainder was estimated to be 989,989; find the error.

Ans. 10,011 too small.

33. The minuend is 1,500,506; the subtrahend is 99,009 less than the minuend; find the sum of minuend, subtrahend, and remainder. (*Ans.* 3,001,012.) Find the subtrahend.

34. The minuend is 6,789,987; find the sum of minuend, subtrahend, and remainder.

35. The minuend is 54,300,001; the subtrahend is 53,999,889; how much must be added to the remainder to make it equal to the subtrahend?

36. The sum of the subtrahend and remainder is 9,875,649; find the sum of the minuend, subtrahend, and remainder.

37. The sum of the minuend and remainder is $ 1,000,000.00; the minuend is $ 801,001.15; find the subtrahend.

REVIEW WORK.

How many more bushels of corn than oats did the 20 states produce in 1889, according to problem 30, page 10?

MULTIPLICATION.

DEFINITIONS AND PRINCIPLES.

33. A **Multiple** of a number is one or more times the number.

Fifty is a multiple of 10 and of 5; 55 is a multiple of 5, but not of 10.

34. The **Product** of two numbers is as many times either as there are units in the other.

The product of 10 and 5 is 50; the product of 10 and 5½ is 55; the product of ½ and 11 is 5½.

35. **Multiplication** is a short process for finding a multiple or a product.

In finding a multiple to multiply is to find the sum of equal numbers.

36. A **Multiplicand** is a number to be multiplied.

37. A **Multiplier** is a number to multiply by.

38. A **Factor** of a multiple is any one of any set of numbers whose product is the multiple.

39. Numbers applied to objects or things are sometimes called concrete.

40. Pure number, as 4, 6, is **Abstract Number.**

41. The product is of the same nature as the multiplicand.

There may be a concrete multiplicand; there cannot be a concrete multiplier.

42. A **Power** of a number is the number itself or the product of factors each of which is the number itself.

The first power of 2 is 2; its second power, or square, is 2 times 2; its third power, or cube, is 2 times 2 times 2; its fourth power is the product of four factors, each of which is 2; and so on.

The power sign is a small figure called an exponent; thus 5^2 means the square of 5; 5^3 means the cube of 5; and so on.

43. The **Sign** of multiplication (×) is read *times* or *multiplied by;* 3 × $4 is read *Three times four dollars;* $4 × 3 is read *Four dollars multiplied by three.*

44. MULTIPLICATION TABLE.

Showing Squares.

1	2	3	4	5	6	7	8	9	10	11	12
2	**4**	6	8	10	12	14	16	18	20	22	24
3	6	**9**	12	15	18	21	24	27	30	33	36
4	8	12	**16**	20	24	28	32	36	40	44	48
5	10	15	20	**25**	30	35	40	45	50	55	60
6	12	18	24	30	**36**	42	48	54	60	66	72
7	14	21	28	35	42	**49**	56	63	70	77	84
8	16	24	32	40	48	56	**64**	72	80	88	96
9	18	27	36	45	54	63	72	**81**	90	99	108
10	20	30	40	50	60	70	80	90	**100**	110	120
11	22	33	44	55	66	77	88	99	110	**121**	132
12	24	36	48	60	72	84	96	108	120	132	**144**

PROBLEMS.

1. The multiplicand is 4,197; the multiplier is 9; what is the product?

EXPLANATION.

$$\begin{array}{r} 4197 \\ 9 \\ \hline 37773 \end{array}$$

(i) The product of the 7 units is found: 9 times 7 units = 63 units: 63 units = 6 tens and 3 units: the 3 is placed under the units; the 6 tens are to be added to the product of the tens.

(ii) The product of the 9 tens is found: 9 times 9 tens = 81 tens: 81 tens and the 6 tens from the product of the units are 87 tens: 87 tens = 8 hundreds and 7 tens: the 7 is placed under the tens; the 8 hundreds are to be added to the product of the hundreds.

(iii) The product of the 1 hundred is found: 9 times 1 hundred = 9 hundreds: 9 hundreds and the 8 hundreds from the product of the tens are 17 hundreds: 17 hundreds = 1 thousand and 7 hundreds: the 7 is placed under the hundreds; the 1 thousand is to be added to the product of the thousands.

(iv) The product of the 4 thousands is found: 9 times 4 thousands = 36 thousands: 36 thousands and the 1 thousand from the product of the hundreds are 37 thousands: the 7 is placed under the thousands; the 3 is written in the ten-thousands' place.

2. A dealer bought 8 horses at $175 per head; what did the 8 horses cost?

ANALYSIS.

If the cost of each horse was $175, the cost of 8 horses was 8 times $175. 8 × $175 = $1,400. The cost of 8 horses was $1,400.

3. For 7 years the average yearly production of corn in the U. S. was 1,551,066,895 bushels; how many bushels were produced in the 7 years? *Ans.* 10,857,468,265 bushels.

4. The revenues of the U. S. government in 1886 amounted to $336,439,727; find the total of 9 such amounts.

5. For 6 years the cotton exported yearly from the U. S. averaged 1,862,572,530 pounds; how much for the 6 years?

6. For 9 years, beginning with 1866, the U. S. yearly production of cotton averaged 1,438,266,355 pounds; and for the 8 succeeding years the yearly production averaged 2,407,731,616 pounds; find the total production in pounds for the 17 years. *Ans.* 32,206,250,123 pounds.

7. The average yearly production of wheat in the U. S. for the years 1874–1882, inclusive, was 389,964,867 bushels; for the years 1883–1891, inclusive, it was 458,077,791 bushels; find the total for the 18 years.

8. For the years 1875–1882 the average yearly production of corn in the U. S. was 1,426,618,835 bushels; for the succeeding 9 years it was 1,783,908,764 bushels; find the total production of corn in the U. S. for the years 1875–1891.

9. For the years 1876–1883 the internal revenue of the U. S. averaged yearly $126,245,762.38; for the 7 succeeding years it averaged $126,785,602.49; find the total internal revenue of the U. S. for the years 1876–1890.

10. For the years 1876–1883 the customs revenue of the U. S. averaged yearly $170,781,021.67; for the 7 succeeding years it averaged $208,474,835.08; find the total customs revenue of the U. S. for the years 1876–1890.

11. What will 10 sheep cost @ $5.75 per head? *Ans.* $57.50.

12. What will 10 beeves cost @ $29 per head? *Ans.* $290.

13. What will 10 calves cost @ $11.25 per head? *Ans.* $112.50.
14. What will 100 horses cost @ $85.00 per head?

45. To multiply by any power of 10:

For the product write the multiplicand with as many noughts annexed as there are noughts in the multiplier, pointing off in the product as many figures for decimals as there are decimal places in the multiplicand.

15. What will 1,000 acres of land cost @ $14 per acre?
16. What will 1,000 acres of land cost @ $8.50 per acre?
17. What will 10,000 pounds sugar cost @ $.05 per pound?

18. What will 400 acres of land cost @ $15 per acre?

EXPLANATION.

The multiplier, 400, may be separated into two factors, 4 and 100; the multiplicand, $15, may be first multiplied by 100 by annexing two noughts; then multiplying this product, $1,500, by 4, the result is the product of $15 by 400. In practice the multiplicand is multiplied by 100 by writing two noughts in the last two places of the product.

$$100 \times \$15 = \$1500$$
$$\begin{array}{r} \$1500 \\ 4 \\ \hline \$6000 \end{array}$$

$$\begin{array}{r} \$15 \\ 400 \\ \hline \$6000 \end{array}$$

19. What will 80 acres of land cost @ $25 per acre?
20. What will 800 acres cost @ $12.50 per acre?
21. What will 9,000 pounds of rice cost @ $.04 per pound?
22. What will 8,000 pounds of cotton cost @ $.07 per pound?
23. Find the cost of 4,000 head of sheep @ $3.75 per head.
24. Find the cost of 900 hogs @ $9.50 per head.

25. Multiply 1,726 by 5.

$$\begin{array}{rcr} 1726 & & \\ 5 & & \\ \hline 30 = & 5 \times & 6 \\ 100 = & 5 \times & 20 \\ 3500 = & 5 \times & 700 \\ 5000 = & 5 \times & 1000 \\ \hline 8630 = & 5 \times & 1726 \end{array}$$

26. Multiply 5 by 1,726.

$$\begin{array}{rrr} 5 & & \\ 1726 & & \\ \hline 30 = & 6 & \times 5 \\ 100 = & 20 & \times 5 \\ 3500 = & 700 & \times 5 \\ 5000 = & 1000 & \times 5 \\ \hline 8630 = & 1726 & \times 5 \end{array}$$

27. A planter bought a farm, containing 1,726 acres of land, at \$6 per acre; find the cost of the farm.

$$\begin{array}{r} 1726 \\ \$\,6 \\ \hline \$\,10356 \end{array}$$

For convenience the factors may be reversed; the numerical result is the numerical product. The product is always like the multiplicand.

28. A bale of cotton weighing 475 pounds was bought at \$.06 per pound; what was paid for the bale?

29. Find the value of 10 *B/C*, averaging 475 pounds, at \$.07 per pound. *Ans.* \$332.50.

30. Find the value of 10 car-loads, averaging 35,000 pounds, of cotton-ties at \$.08 per pound.

31. What product is composed of the three factors 5, 182, and 10,000?

32. The multiplicand is 1,894; the multiplier is 365; what is the product?

$$\begin{array}{rcl} 365 & = & 300 + 60 + 5 \\ \hline 5 \times 1894 & = & 9470 \\ 60 \times 1894 & = & 113640 \\ \underline{300} \times \underline{1894} & = & \underline{568200} \\ 365 \times 1894 & = & 691310 \end{array}$$

$$\begin{array}{rcl} 1894 & & \\ 365 & & \\ \hline 9470 & = & 5 \times 1894 \\ 11364\cancel{0} & = & 60 \times 1894 \\ \underline{5682\cancel{0}\cancel{0}} & = & \underline{300} \times \underline{1894} \\ 691310 & = & 365 \times 1894 \end{array}$$

33. The multiplicand is 2,894; the multiplier is 465; what is the product? *Ans.* 1,345,710.

34. The multiplicand is 5,695; the multiplier is 987; find the product.

35. The multiplicand is \$8,532; the multiplier is 789; find the product. *Ans.* \$——.

36. The multiplicand is 3,621; the multiplier is 708; what is the product?

$$\begin{array}{rcl} 708 & = & 700 + 8 \\ \hline 8 \times 3621 & = & 28968 \\ \underline{700} \times \underline{3621} & = & \underline{2534700} \\ 708 \times 3621 & = & 2563668 \end{array}$$

$$\begin{array}{rcl} 3621 & & \\ 708 & & \\ \hline 28968 & = & 8 \times 3621 \\ \underline{25347} & = & \underline{700} \times \underline{3621} \\ 2563668 & = & 708 \times 3621 \end{array}$$

37. The multiplier is 3,006; the multiplicand is \$.24; find the product.

38. The multiplicand is 909; the multiplier is 909; find the product.

39. The multiplicand is 3,006; the multiplier is 3,006; find the product.

40. In a square mile there are 640 acres; in an acre there are 4,840 square yards; how many square yards in a square mile?

41. In a square yard there are 9 square feet; how many square feet in an acre?

42. How many square feet are in a square mile?

43. In a square orchard there are 16 rows of trees, 16 trees to the row; how many trees in the orchard?

44. Find the second power, or square, of 25. *Ans.* 625.

45. Find the third power, or cube, of 9. *Ans.* 729.

46. Find the fourth power of 6. *Ans.* 1,296.

47. Find the fifth power of 5. *Ans.* 3,125.

48. Find the square of 341. *Ans.* ——.

49. Find the cube of 19. *Ans.* ——.

50. Find the fourth power of 8. *Ans.* ——.

51. Raise 6 to the fifth power. *Ans.* ——.

52. How many square miles are in a square county that is 30 miles to the side?

53. How many square miles in a rectangular territory 275 miles long and 225 miles wide?

54. Find the value of the corn crop of Washington Co. in 1879; 364,510 bushels at $ .30 per bushel. *Ans.* $ 109,353.

55. Find the value of the wheat crop of Franklin Co. in 1892; 55,652 bushels at $ .65 per bushel. *Ans.* $ 36,173.80.

56. Find the value of the cotton crop of Johnson Co. in 1893; 14,093 bales, averaging 484 pounds, at $ .07 per pound.

57. U. S. corn crop of 1884; 1,795,528,432 bushels; find the value of the crop at $ .35 per bushel.

58. U. S. cotton crop of 1866; 1,041,962,263 pounds; find the value of the crop at $.43 per pound. *Ans.* $ 448,043,773.09.

59. U. S. cotton crop of 1881; 3,199,822,682 pounds; find the value of the crop at $ .09 per pound. *Ans.* $ 287,984,041.38.

60. U. S. wheat exports, 1880; 152,252,795 bushels; find the total value at $ 1.25 per bushel. *Ans.* $ 190,315,993.75.

61. U. S. flour exports, 1885; 9,152,260 barrels; find the total value at $ 5.58 per barrel. *Ans.* ——.

62. U. S. cotton exports, 1865; 19,063 bales, averaging 465 pounds to the bale; find the total value at $ .77 per lb.

Ans. $ 6,825,507.15.

CONTRACTIONS.

63. Multiply 2,947 by 93.

EXPLANATION.

$$\begin{array}{rl} 2947 & \\ 93 & \\ \hline 8841 & = 3 \times 2947 \\ 26523 & = 30 \times 3 \times 2947 \\ \hline 274071 & = 93 \times 2947 \end{array}$$

The product of the multiplicand by the 3 units is first found; then it is seen that the 90 of the multiplier is 30 times the 3 already used; 30 times *the product of* 3 *and the multiplicand* is 90 times the multiplicand. 30 is 3×10; the first partial product is multiplied by 3, and it is multiplied by 10 at the same time by placing the product's first figure in the column of tens. The process consists, in practice, of merely multiplying by the 3 (which is easier than multiplying by the 9), and taking care to place the figures in their proper columns; three times *the product of the multiplicand and* 3 is 9 times the multiplicand.

64. Multiply 2,947 by 63; by 62; by 82.

65. Multiply 4,845 by 84; by 142; by 162.

66. Multiply 9,867 by 255; by 244; by 153.

67. Multiply 9,867 by 39.

EXPLANATION.

$$\begin{array}{r} 9867 \\ 39 \\ \hline 29601 \\ 88803 \\ \hline 384813 \end{array}$$

The order is reversed; the 3 of the multiplier is first used, but the first figure of the product is written in the tens' place; the 9 is 3 times the 3; the product by the 3 tens is multiplied by 3, but the first figure of the product is written in the units' place.

68. Multiply 4,242 by 26; by 36; by 28.

69. Multiply 2,886 by 24; by 216; by 214.

70. Multiply 3,982 by 327; by 428; by 648.

71. Multiply 6,749 by 636; by 749; by 756.

72. Multiply 97,869 by 99.

Explanation.

Annexing two noughts to the multiplicand multiplies it by 100; subtracting from the product the multiplicand itself, which is the product of the multiplicand and 1, the result is the product of the multiplicand and 99.	9786900 97869 9689031

73. Multiply 9,876,547 by 99.

74. Multiply 487,632,198 by 999. *Ans.* 487,144,565,802.

75. Multiply 7,721,643 by 999; by 499; by 699.

REVIEW PROBLEMS.

76. The minuend is 160; find the sum of the minuend, subtrahend, and remainder. *Ans.* 320.

77. A minuend is 3,107; the sum of the minuend, subtrahend, and remainder is the multiplicand, and 79 is the multiplier; find the product.

78. The subtrahend is the square of 8; the minuend is the cube of 4; what is the remainder?

79. The minuend is the cube of 16; the subtrahend is the square of 64; find the remainder.

80. The sum of two numbers is 1,001,001; the smaller number is 90,090; write in words the larger number.

81. The sum of two numbers is two million one; the smaller number is ninety thousand two; find the larger number.

82. The multiplicand is 33,000,001 – 29,809,801; the multiplier is the third power of 9; find the product.

83. The minuend is the product of the square of 16 and the fourth power of 5; find the sum of the minuend, subtrahend, and remainder. *Ans.* 320,000.

84. The multiplicand is $.07; the multiplier is 789; find the product.

85. Bought 16 pounds butter at $.30 per pound
20 " coffee at .35 " "
18 " rice at .06 " "

Find total cost. *Ans.* $12.88.

86. Find amount of bill:

Louisville, Ky., Mch. 4, 1893.

Mrs. R. K. Wilson,

Bought of M. L. SMITH & CO.

14	*yd.*	*ribbon*	@	*$.25*				
54	*yd.*	*domestic*	@	*$.09*				
48	*yd.*	*sheeting*	@	*$.22*				
6	*doz.*	*sp. cotton*	@	*$.50*				

Received payment,

M. L. Smith & Co.

Make receipted bills for the following. Supply names and dates.

87. Groceries:

19 lb. cheese @ $.22	27 lb. coffee @ $.32
44 lb. sugar @ $.06	14 lb. tea @ $.65
37 lb. bacon @ $.09	1 bbl. flour @ $6.00

Amt. $33.89.

88. Fruits:

17 bbl. apples @ $3.50	9 bxs. lemons @ $5.75
23 bxs. oranges @ $2.75	8 doz. pine-apples @ $1.25

Amt. $184.50.

89. Country produce:

27 lb. butter @ $.25	14 doz. eggs @ $.24
28 lb. honey @ $.12	26 chickens @ $.22
29 lb. dried fruit @ $.06	18 lb. feathers @ $.39
16 lb. bees-wax @ $.25	3 bu. chestnuts @ $6.75

Amt. $52.20.

90. Grain, etc.: 175 bu. oats @ $.32; 3 tons cotton-seed meal @ $22; 225 bu. corn @ $.56; 9 tons hay @ $7.75; 87 bu. wheat @ $.77.

91. Millers' products: 29 bbl. flour @ $4.75; 17 bu. grits @ $.95; 36 bu. meal @ $.65; 72 bu. bran @ $.28.

ADDITIONAL REVIEW WORK. — All of page 15

FOR ORAL WORK.

Review. — Induction.

1. What is the result in addition called?

2. What is the result in subtraction called?

3. What is the Subtrahend? What is the Minuend?

4. Give the distinction between the difference and the remainder.

5. What does the subtrahend equal? What does the minuend equal? What does the remainder or difference equal?

6. What is Multiplication? Give the names of the factors in multiplication. What is the result called?

7. What does the multiplier show? What does the multiplicand show?

8. What is a Multiple? Give three multiples of 6.

9. What is a Power? Give the third power of 1; the cube of 2.

10. Give the distinction between a concrete number and an abstract number.

11. Which factor in multiplication must be an abstract number? Which factor is of the same nature as the product?

Example: 3×4 dollars $= 12$ dollars.

12. Name the product in the example. Name the multiplier. Name the multiplicand. Which are concrete numbers?

13. To make the sum of 12 dollars by addition, how many times must 4 dollars be taken?

14. Can the sum of 12 dollars be found by adding the abstract factor 3? Why?

15. How many times does the product 12 dollars contain the multiplicand? Why?

16. Does the product contain the multiplier? Why?

17. How many times can 4 dollars be subtracted from 12 dollars?

18. Given the product 12 dollars and the factor 3, what is the other factor?

19. What is *one-third of* 12 *dollars?*

DIVISION.

DEFINITIONS AND PRINCIPLES.

46. When two **Factors** compose a concrete **Product**, one of the factors is a concrete number, and the other is an abstract number.

47. When two **Factors** compose an abstract **Product**, each of the factors is an abstract number.

48. A **Product** or **Multiple** and one **Factor** being given, the process for finding the other **Factor** is called **Division**.

49. The given **Factor** in **Division** is called the **Divisor**.

The divisor is the number to divide by.

50. The **Product** or **Multiple** given is called the **Dividend**.

The dividend is the number to be divided.

A dividend is the product of its two factors.

51. The **Factor** sought is called the **Quotient**.

The quotient is the result obtained by dividing.

52. When both **Divisor** and **Dividend** are abstract numbers the **Quotient** sought is an abstract number.

53. When both **Divisor** and **Dividend** are concrete numbers the **Quotient** sought is an abstract number.

54. When the division is such that the quotient sought is an abstract number, the division is called **Subtractional Division**.

The quotient is the number of times that the divisor can be subtracted from the dividend. The divisor represents part of the dividend. The dividend equals the divisor multiplied by the quotient.

55. When the **Divisor** is an abstract number, and the **Dividend** is a concrete number, the **Quotient** sought is a concrete number.

56. When the division is such that the quotient sought is a concrete number, the division is called **Partitive Division.**

The divisor is the number of times that the quotient is contained in the dividend: the quotient represents part of the concrete dividend, and in writing the quotient its concrete nature must be shown: a mere number, or numerical quotient, is not the answer to the problem.

57. There are three **Signs** of division in common use.

(i) The principal **Sign** (÷) is read *divided by.* It shows that the number before it is the dividend, and the number following it is the divisor. 4 ÷ 2 is read, *four divided by two.*

(ii) The operative **Signs** [ ) and ( ] show that the number facing the concavity of the curved line is the divisor; 4)12, 12(4, 4)12(, all show that 4 is the divisor and 12 the dividend.

(iii) The fractional **Sign**, which, instead of dots as in the principal sign, has a number above it and a number below it, also signifies division. $\frac{2}{3}$ is read, *two divided by three,* or, as a fraction, *two thirds.*

58. If from the dividend there be a remainder which will not contain the divisor, the division of this remainder is expressed by writing, as part of the quotient, the remainder over the divisor with the fractional sign between.

In the expression 3)5(1$\frac{2}{3}$, the quotient is read, *One and two thirds.*

(The multiplication table may be used as a division table by considering each multiple a dividend.)

PROBLEMS.

1. Divide 604 by 2, and prove the result.

EXPLANATION.

The 6 hundreds of the dividend contain the divisor three hundred times; the 3 is written under hundreds. The 0 ten of the dividend does not contain the divisor; 0 is written under tens. The 4 units of the dividend contain the divisor twice; 2 is written under units. For proof see **50.**

$$\begin{array}{r} 2\underline{)604} \\ 302 \\ \underline{2} \\ 604 \end{array}$$

2. A company paid $2,700 for 9 acres of land; find the average cost per acre.

ANALYSIS.	PROCESS.
Cost of 9 A. = $2700	
Cost of 1 A. = $\frac{1}{9}$ of $2700	9)$2700
$\frac{1}{9}$ × $2700 = $300	$300
Average cost per A. = $300	

3. In 9 years the exports of wheat from the United States amounted to 894,563,536 bushels; find the average for the 9 years. *Ans.* 99,395,948$\frac{4}{9}$ bushels.

4. For the same 9 years the exports of cotton amounted to 16,476,548,785 pounds; find the average for the 9 years.
Ans. 1,830,727,642$\frac{7}{9}$ pounds.

5. The wheat exported, as above, amounted in value to $1,013,722,281; find the average value of the wheat exports for the 9 years. *Ans.* $112,635,809.

6. The cotton exported, as above, amounted in value to $1,818,804,920; find the average value of the cotton exports for the 9 years. *Ans.* $202,089,435$\frac{5}{9}$.

7. A farmer paid $1,768 for land at $8 per acre; how many acres did he buy?

ANALYSIS.

$8 = the cost of 1 acre.
$1768 = the cost of as many times 1 acre as $1768 contains $8.
$1768 contains $8 221 times.
221 × 1 acre = 221 acres; hence,
$1768 = the cost of 221 acres.
The farmer bought 221 acres of land.

8. A bale of cotton was sold for $34.24; how many cents are in $34.24? The price paid was 8 cents per pound; what was the weight of the bale? *Ans.* 428 pounds.

9. The sugar produced in Louisiana for the inclusive years 1884–1890 amounted to 1,932,738,480 pounds; find the average yearly production. *Ans.* 276,105,497$\frac{1}{7}$ pounds.

10. Divide 17,854,296 by 12; by 11.

11. Divide 191,300,007 by 7; by 8; by 9; by 11.

12. Divide 44,164 by 21.

EXPLANATION.

(i) The ten-thousands of the dividend do not contain the divisor any number of ten-thousand times; nothing is written in the ten-thousands' place in the quotient. Changing the 4 ten-thousands of the dividend into thousands there are 44 thousands in the dividend; 44,000 will contain the divisor 2,000 times; the 2,000 is written as a partial quotient. From the dividend the product of the partial quotient and divisor is subtracted.

21	44164	2000
	42000	
21	2164	100
	2100	
21	64	3
	63	
	1	$2103\frac{1}{21}$

(ii) The thousands of the remaining dividend do not contain the divisor any number of thousand times; therefore they are changed into hundreds; the remaining dividend contains the divisor 100 times; 100 is written as a partial quotient. From the remaining dividend, the product of the last partial quotient and the divisor is subtracted.

(iii) The remaining dividend does not contain the divisor any number of ten times; it contains the divisor 3 times; the 3 is written as units for the last partial quotient. From the remaining dividend the product of the last partial quotient is subtracted.

(iv) The sum of the partial quotients is the entire quotient. In the example the last remainder divided by the divisor ($1 \div 21$) is a partial quotient, and is expressed in the quotient by using the fractional sign of division.

The entire quotient is read *Two thousand one hundred three and one twenty-first.*

IN PRACTICE.

(i) See how many times 44 contains 21. Write the 2 in the thousands' place in the quotient. Twice 21 is 42. Place 42 under 44 and subtract. Bring down the next figure of the dividend.

$$\begin{array}{r}
2103\frac{1}{21} \\
21\overline{)44164} \\
\underline{42} \\
21 \\
\underline{21} \\
64 \\
\underline{63} \\
1
\end{array}$$

(ii) See how many times 21 contains 21. Write the 1 in the hundreds' place in the quotient. Multiply 21 by 1. Place the product under 21 and subtract. Bring down the next figure of the dividend.

(iii) See how many times 6 contains 21. Write the 0 in the tens' place in the quotient. Bring down the next figure of the dividend.

(iv) See how many times 64 contains 21. Write 3 in the units' place in the quotient. Multiply. Subtract.

(v) Write the remainder and divisor, with the fractional sign between them, after the units' place in the quotient.

13. The weight of 49 barrels of flour is 9,604 pounds; find the weight per barrel.

14. The weight of 301 bushels of corn is 16,856 pounds; find the weight per bushel.

15. The weight of 300 bushels of corn in the ear is 21,000 pounds; find the weight per bushel.

EXPLANATION.

The first form shows that one three-hundredth ($\frac{1}{300}$) of 21,000 pounds is 70 pounds.

The second form shows that one-third of one hundredth ($\frac{1}{3}$ of $\frac{1}{100}$) of 21,000 pounds is 70 pounds.

Comparison of the two forms shows that, before the final division is performed, both dividend and divisor have been divided by 100, and that their quotient is not affected.

(1)

$$\begin{array}{r} 70 \text{ lb.} \\ 300\overline{)21000} \text{ lb.} \\ \underline{2100} \\ 0 \end{array}$$

(2)

$$\begin{array}{r} 3\cancel{0}\cancel{0})\underline{210\cancel{0}\cancel{0}} \text{ lb.} \\ 70 \text{ lb.} \end{array}$$

59. If both dividend and divisor be divided by the same number, their quotient will not be affected.

16. The dividend is 100×210; the divisor is 100×3; find the quotient.

60. If both dividend and divisor be multiplied by the same number, their quotient will not be affected.

17. The dividend is 100×210; the divisor is 99×3; find the quotient.

18. The dividend is 99×210; the divisor is 100×3; find the quotient.

61. If the **Dividend** alone, or the **Divisor** alone, be either increased or lessened, their quotient will be affected.

19. The weight of 1,000 bushels of oats is 32,000 pounds; find the weight per bushel.

20. The weight of 100 bushels of corn is 5,600 pounds; find the weight per bushel.

21. The cost of 100 barrels of apples was \$125; find the cost per barrel. *Ans.* \$1.25.

62. To divide by any power of 10:

Remove as many noughts from the right of the dividend as there are noughts in the divisor. If noughts do not occur in the right of the dividend, point off for decimals as many figures in the dividend as there are noughts in the divisor.

22. How many thousands are in ten millions?

23. A farm of 109 acres was sold for \$3,379; find the price paid per acre. *Ans.* \$31.

24. Required, the number of hundreds in 1,500,000,000.

25. An agent for a cotton mill paid \$17,952 for 528 *B/C*; find the value of the average bale. *Ans.* \$34.

26. The dividend is 5,000 ÷ 20; the divisor is 500 ÷ 2; find the quotient.

27. The dividend is $\frac{500}{2}$; the divisor is $\frac{5000}{200}$; find the quotient.

28. The wages paid to 534 men for 1 week was \$6,408; find the average wages paid a man per day. *Ans.* \$2.

29. The dividend is $\frac{600}{20}$ pounds; the divisor is $\frac{300}{20}$ pounds; find the quotient.

30. The dividend is $\frac{750}{25}$ pounds; the divisor is $\frac{250}{50}$ pounds; find the quotient.

31. At an average speed of 27 miles per hour, a train runs 459 miles; find the number of hours required. *Ans.* 17 hours.

32. The dividend is $\frac{\$450}{\$15}$; the divisor is $\frac{\$900}{\$450}$; find the quotient.

33. The dividend is $\frac{\$1250}{125}$; the divisor is $\frac{54}{18}$; find the quotient.

34. The dividend is $\frac{181500}{605}$; the divisor is $\frac{28880}{161}$; find the quotient. *Ans.* $1\frac{12}{18}$.

Find the quotients of the following:

35. $\frac{909000}{4545} \div \frac{9891}{3297}$. *Ans.* $66\frac{2}{3}$.

36. $\frac{27720}{45} \div \frac{4928}{616}$. *Ans.* 77.

37. $\frac{2025495}{6555} \div \frac{4944}{48}$. *Ans.* 3.

38. $\frac{67760}{880} \div \frac{5901}{843}$. *Ans.* 11.

39. $\frac{9504}{88} \div \frac{11664}{108}$. *Ans.* 1.

40. $\frac{858754}{709} \div \frac{128018}{506}$. *Ans.* 2.

41. $\frac{46256}{892} \div \frac{4720}{40}$. *Ans.* 1.

42. $\frac{299676}{39} \div \frac{357806}{93}$. *Ans.* 2.

43. $\frac{6 \times 8 \times 9}{8 \times 6}$. *Ans.* 9. **44.** $\frac{7 \times 9 \times 5 \times 4}{4 \times 5 \times 7}$. *Ans.* 9.

CANCELLATION.

63. A short process of solving such problems as 43 and 44 is called *Cancellation.* The work is performed by striking out equal factors from the dividend and divisor (see **59**).

45. A farmer sold 40 bushels of corn at 50 cents per bushel; with the money received he bought cloth at 80 cents per yard; how many yards of cloth did he buy?

Analysis.

Value of 1 bu. corn = 50 cents.
Value of 40 " " = 40 × 50¢
40 × 50¢ = 2000¢

As the value of 1 yd. cloth was 80¢, he bought as many times 1 yd. as 2000¢ contains 80¢. 2000¢ contains 80¢ 25 times. 25 times 1 yd. = 25 yd. He bought 25 yd. of cloth.

By Cancellation.

$$\frac{40 \times 50}{80}$$

The dividend is 2000 (cents) separated into two factors. The divisor is 80 (cents), but not separated into factors. Separate the divisor into factors.

$$\frac{\overset{1}{\cancel{40}} \times 50}{\underset{1}{\cancel{40}} \times 2}$$

Cancel: the result is $\frac{50}{2} = 25$.

Cancelling the factor 40, common to 40 and 80, of the 40 the factor 1 remains, and of the 80 the factor 2 remains; then cancelling the factor 2, common to 50 and 2, of the 50 the factor 25 remains, and of the 2 the factor 1 remains.

Shorter Process.

$$\frac{\overset{1}{\cancel{40}} \times \overset{25}{\cancel{50}}}{\underset{\underset{1}{\cancel{2}}}{\cancel{80}}} = \frac{25}{1} = 25.$$

46. The factors of the dividend are 4, 6, 12, and 8; the factors of the divisor are 2, 3, and 24; find the quotient.

47. The factors of the dividend are 7, 8, 9, 9; the factors of the divisor are 72 and 21; find the quotient.

48. The factors of the dividend are 9, 17, 21, and 25; the factors of the divisor are 18, 34, 7, and 5; find the quotient.

By cancellation find the quotients:

49. $\frac{32 \times 25 \times 56 \times 72}{5 \times 8 \times 7 \times 9}$.

50. $\frac{16 \times 40 \times 55 \times 72}{9 \times 11 \times 4 \times 8}$.

51. $\frac{30 \times 64 \times 45 \times 39}{8 \times 13 \times 3 \times 15}$.

52. $\frac{98 \times 102 \times 150 \times 165}{51 \times 25 \times 49 \times 53}$.

53. $\frac{27 \times 32 \times 45 \times 56}{64 \times 3 \times 7 \times 9}$.

54. $\frac{3}{8} \times \frac{4}{5} \times \frac{2}{9}$. *Ans.* $\frac{1}{15}$.

REVIEW PROBLEMS.

1. Required, the number of units in the following four numbers: seventy-seven million; eighty-eight thousand; three hundred one million; three hundred one thousand five hundred five.

2. In a county 30 miles square there are 56,000 acres improved, and 520,000 acres unimproved, land; in Mr. Holt's ranch there are 7,680 acres; how many square miles in the ranch? *Ans.* 12 square miles.

3. The remainder is 455; required, a minuend twice as great as the subtrahend.

4. From two fields, one of 80 acres, the other 102 acres, a farmer harvested 3,458 bushels of wheat; find the average yield per acre in pounds, 60 pounds to the bushel. *Ans.* 1,140 lb.

5. If the dividend were twice as great as it is, the quotient would be 856; what is the quotient?

6. How many bushels of oats, 32 pounds to the bushel, will equal in weight 149 bu. corn, 56 pounds to the bushel?

Ans. $260\frac{3}{4}$ bu. oats.

7. The subtrahend and remainder are equal; the minuend is composed of the factors 7, 8, 9; find the remainder.

8. Mr. Franklin's cistern has a capacity of 739,200 cubic inches; a gallon-measure's capacity is 231 cubic inches; how many gallons of water will the cistern hold? *Ans.* 3,200 gal.

9. The divisor is 693; the quotient is three times $16; find the dividend.

10. A farmer sold 6 *B/C* averaging 450 pounds for $216; find the price per pound.

11. If the divisor were three times as great as it is, the quotient would be 16; the dividend is 960; find the divisor.

12. Five men earned at equal wages $810 in 9 weeks; find the daily wages paid each man.

13. Half of the quotient is 214; the dividend is $152,368; find the divisor. *Ans.* $356.

14. How many bushels of sweet potatoes, 55 pounds to the bushel, will equal the weight of 330 bushels corn-meal, 48 pounds to the bushel? *Ans.* 288 bushels sweet potatoes.

15. The divisor is 693; the dividend is the product of the factors 12, 16, 161; find the quotient.

16. In 18 work-days a farmer planted a field of 360 acres cotton; during the 18 days the rain prevented work for 3 days; find the average number of acres planted per day.

17. The multiplier is 498; the product is $14,940; find the sum of the product and multiplicand.

18. The dividend is one million nine hundred ninety thousand three hundred one; the divisor is the difference of nine thousand nine and 8,970; find the quotient.

19. The dividend is $1,090,095; the quotient is four times one dollar twenty-five cents; find the divisor.

20. A foot is 12 inches; a yard is 3 feet; a furlong is 220 yards; a mile is 8 furlongs; how many inches in a mile?

21. The sum of two numbers is 364; the larger number is composed of the factors 3, 4, 5, 6; find the smaller number.

22. There are 4,800 sheets in a bale of paper; a bale is 5 bundles; a bundle is 2 reams; a ream is 20 quires; how many sheets in a quire?

23. The difference between two numbers is 320; the smaller number is the product of 64 and 8; what is the larger number?

24. Given: a bicycle, the small wheel 45 inches, the large wheel 135 inches in circumference.

Required, the number of miles travelled when the large wheel has made 1,408 revolutions. *Ans.* 3 miles.

Required, the number of revolutions the small wheel makes in 1 mile.

25. A dividend is 7,506 bushels of wheat; one factor is 417; find the other factor.

26. Find the quotient of the expression $\frac{9875439}{121919}$; prove the result by multiplication.

27. Find the product of the expression $9{,}097 \times 99{,}079$; prove the result by division.

28. Cancel $\dfrac{30 \times 45 \times 56 \times 72 \times 108 \times 144}{12 \times 18 \times 5 \times 15 \times 14 \times 24}$. *Ans.* 15,552.

29. Find the quotient: $\frac{381}{127} \div \frac{10008}{3336}$.

30. Find the product: $\frac{516}{172} \times \frac{3060}{9}$.

31. Find the difference: $\frac{711}{79} - \frac{1005005}{201001}$.

32. Find the sum: $\frac{75}{25} + \frac{30}{15} + \frac{1}{2}$. *Ans.* $5\frac{1}{2}$.

33. Find the product of the factors 6, 9, 11, 13.

34. Raise 200 to the sixth power.

35. What multiple of 7 days contains 52 weeks?

36. Given: the multiple 200,970 and the factor 2,233; find the other factor.

37. The sum of seven equal numbers is 1,000,006; find the sum of three of the numbers.

38. The minuend is 15; the difference between the subtrahend and remainder is 1; find the product of the minuend, subtrahend, and remainder. *Ans.* 840.

39. The dividend is nineteen million ninety-four thousand eight hundred sixty-seven; the divisor is four thousand seven hundred sixty-three. Write the quotient in Roman. Prove the work.

REVIEW WORK.—Page 23, Problems 76–84.

FOR ORAL WORK.

Review.—Induction.

1. What is a power?
2. Give the method for multiplying by a power of 10.
3. Give the method for dividing by a power of 10.
4. Give examples of abstract numbers; of concrete numbers.
5. What is a multiple?

6. Give the names of the multiple and factors in multiplication.

7. Which factor in multiplication must be an abstract number? Why? How can multiplication be tested?

8. Give the names of the multiple and factors in division.

Examples. (1) $\frac{\$12}{3} = \$4.$ (2) $\frac{\$12}{\$4} = 3.$

9. What sign of division is used in the examples?

10. In which example is the quotient concrete?

11. In which example is the divisor abstract?

12. In which example is the dividend a multiple of the quotient? How can division be tested?

13. In which example does the dividend contain the divisor?

14. In which example is the division partitive? Why?

15. How is the division of a remainder expressed?

16. In example 1, if the divisor be multiplied by 2, what will the quotient be?

17. In example 2, if the dividend be multiplied by 2, what will the quotient be?

18. In example 1, if the dividend be divided by 2, what will the quotient be?

19. In example 2, if both dividend and divisor be multiplied by 2, what will the quotient be? If both dividend and divisor be divided by 2, what will the quotient be?

Examples. (1) $24 = 4 \times 6$. (2) $24 = (2 \times 2) \times (2 \times 3)$. (3) $15 = 3 \times 5$.

20. In example 1, what are the factors of 24?

21. In example 2, what are the factors of 24?

22. In example 2, what are the factors of the factor 4? What are the factors of the factor 6?

23. In example 3, what are the factors of 15? Are there any factors of 3? Of 5? $3 \times 1 =$ what? $5 \times 1 =$ what?

24. Give the name for a factor composed of other factors, as 4 and 6, etc. *Ans. A composite factor.*

25. Give the name for a factor that has no factors, except itself and 1, as 3 and 5, etc. *Ans. A prime factor.*

PROPERTIES OF NUMBERS.

FACTORS—DIVISORS—MULTIPLES.

64. An **Integer** is a whole number.

The numbers 1, 5, 2, are integers; .15, $\frac{4}{5}$, $\frac{1}{2}$, are not integers.

An even number is a number that is exactly divisible by the integer 2.

The numbers 2, 4, 6, 8, 10 are even numbers; the unit figure of an expression for any even number must be 0, 2, 4, 6, or 8.

An odd number is one that is not exactly divisible by the integer 2.

The numbers 1, 3, 5, 7, 9, are odd numbers; the unit figure of an expression for any odd number must be 1, 3, 5, 7, or 9.

65. A **Prime Number** is one that has but two integral factors, itself and 1.

All prime numbers are odd except the number 2: 1, 2, 3, 5, 7, 11, are prime numbers; the unit figure of an expression for a greater prime number must be 1, 3, 7, or 9.

A prime number used as a factor is called a **Prime Factor.** To find the prime factors of a number is to find the prime numbers which, multiplied together, produce the number given, hence:

66. A number is equal to the product of all its prime factors. The prime factors of 24 are 2, 2, 2, 3; $2 \times 2 \times 2 \times 3 = 24$.

67. A **Composite Number** is a multiple of integral factors, neither of which is 1.

A composite number used as a factor is called a composite factor.

There may be many composite factors of a number.

The composite factors of 24 are 4, 6, 8, and 12;

$$4 \times 6 \times 8 \times 12 = 2304.$$

68. The product of the composite factors of a number is greater than the number.

69. A **Factor** of a number is a factor of any **Multiple** of the number.

2 is a factor of 6, and also of 3×6.

PROBLEMS.

1. Find the two prime factors of 77.
2. Find the three prime factors of 8.
3. Find the four prime factors of 16.
4. Find the three prime factors of 30.
5. Find the prime factors of 720.

PROCESS (1).

$$\begin{array}{r|r} 2 & 720 \\ \hline 2 & 360 \\ \hline 2 & 180 \\ \hline 2 & 90 \\ \hline 3 & 45 \\ \hline 3 & 15 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

The prime factors of 720 are

2, 2, 2, 2, 3, 3, 5.

Proof.

$2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 = 720.$

(See **66**.)

PROCESS (2).

$$720 = 8 \times 9 \times 10$$
$$8 = 2 \times 2 \times 2$$
$$9 = 3 \times 3$$
$$10 = 2 \times 5$$

The number may first be resolved into any of the composite factors of which it is the product; the composite factors may then be resolved into prime factors.

$$720 = 2 \times 2 \times 2 \times 3 \times 3 \times 2 \times 5.$$

To denote many equal factors, simpler expressions are obtained by the use of the power sign, or exponent (see **42**); thus, 2^4 denotes four factors, each of which is 2; 3^2 denotes two factors, each of which is 3, and so on.

6. Find the prime factors of each composite number from 20 to 60.

7. Find the prime factors of each composite number from 60 to 100.

8. Find the prime factors of each composite number from 100 to 200.

9. Find the prime factors of 380; of 864.
10. Find the prime factors of 900; of 625.
11. Find the prime factors of 1,000; of 1,500.
12. Find the prime factors of 2,000; of 2,500.

13. Find the prime factors of 1,440; of 3,003; of 4,090.

TESTS OF DIVISIBILITY.

70. *A knowledge of the following truths is helpful in factoring large numbers:*

(2) The prime number 2 is an exact divisor, or factor, of every even number.

(3) The prime number 3 is an exact divisor of every number, the sum of whose digits, or figures, is exactly divisible by 3, as 123; the sum of 1, 2, and 3 is 6; 6 exactly contains 3.

(4) The composite number 4 is an exact divisor of every number the sum of whose tens and units is 0 or is exactly divisible by 4, as 1180; the sum of 80 and 0 is 80; 80 exactly contains 4.

(5) The prime number 5 is an exact divisor of every number whose unit figure is 0 or 5.

(6) The composite number 6 is an exact divisor of every even number that is exactly divisible by 3.

(8) The composite number 8 is an exact divisor of every number whose units' period is exactly divisible by 8.

(9) The composite number 9 is an exact divisor of every number, the sum of whose digits is exactly divisible by 9.

(10) The composite number 10 is an exact divisor of every number whose units' figure is 0.

(25) The composite number 25 is an exact divisor of every number, the sum of whose tens and units is 0 or is exactly divisible by 25.

Find the prime factors of:

14.	1,050	**21.**	9,600	**28.**	896
15.	5,760	**22.**	802	**29.**	1,098
16.	810	**23.**	1,020	**30.**	825
17.	1,628	**24.**	981	**31.**	8,010
18.	435	**25.**	999	**32.**	3,822
19.	987	**26.**	4,950	**33.**	8,605
20.	3,200	**27.**	4,411	**34.**	546

35. Find the prime factors of 983.

THE SUCCESSIVE TRIALS.

I.	It is useless to try 2 . . .	983 is an odd number.	
II.	Try other prime numbers; 3 will not answer . . .	The sum of the digits of 983 is not divisible by 3.	
III.	It is useless to try 5 . . .	The units' figure of 983 is neither 0 nor 5.	
IV.	Try 7; it fails	$983 \div 7 = 140\frac{3}{7}$	(*Quotient factor more than* 20 *times Divisor factor.*)
V.	Try 11; it fails	$983 \div 11 = 89\frac{4}{11}$	(*Q.* 8 *times D.*)
VI.	Try 13; it fails	$983 \div 13 = 75\frac{8}{13}$	(*Factors nearer.*)
VII.	Try 17; it fails	$983 \div 17 = 57\frac{14}{17}$	(*Q.* 3 *times D.*)
VIII.	Try 19; it fails	$983 \div 19 = 51\frac{14}{19}$	(*Q. not* 3 *times D.*)
IX.	Try 23; it fails	$983 \div 23 = 42\frac{17}{23}$	(*Q. not twice D.*)
X.	Try 29; it fails	$983 \div 29 = 33\frac{26}{29}$	(*D. almost = Q.*)

One further trial will test 983: if it be a composite number, the trial of 31 will show it; for it is seen that each succeeding trial brings trial factors more nearly equal to each other. The next trial must be with factors so nearly equal that any further trial would be useless; for, so soon as the trial divisor exceeds the trial quotient, there is proof that, unless there be error, the number tested is prime. When the divisor factors exceed the quotient factors, the quotients begin to repeat the search the divisors have already made.

XI. Try 31; it fails . . . . $983 \div 31 = 31\frac{22}{31}$

The number 983 is prime: the next trial divisor would be 37, which is evidently too great. It can be seen from the result of the tenth trial that 37 is not contained in 983 so many as 30 times.

Find the prime factors of:

36. 7,180	**39.** 2,818	**42.** 2,998	**45.** 9,101
37. 8,161	**40.** 3,941	**43.** 7,101	**46.** 7,823
38. 4,937	**41.** 6,157	**44.** 5,632	**47.** 6,572

COMMON FACTORS.

71. Any factor of each of two or more numbers is a **Common Factor** of the numbers.

Of the numbers 12 and 9, 3 is a common factor.

PROBLEMS.

1. Find a prime factor common to 27 and 36.
2. Find a composite factor common to 28 and 36.
3. Find a prime factor common to 6, 9, and 51.
4. Find the prime factors common to 18, 30, and 66.
5. Find the greatest (composite) factor common to 45, 105, 135.

HIGHEST COMMON FACTORS.

First Method.

72. The highest factor of each of two or more numbers is the **Highest Common Factor** of the numbers.

73. The highest common factor of two or more numbers is the product of all the prime factors common to the numbers.

6. Find the highest common factor of 75, 275, and 550.

PROCESS.

Prime factors of 75 are 3, 5, 5.
Prime factors of 275 are 11, 5, 5.
Prime factors of 550 are 2, 11, 5, 5.

The factors 2 and 3 are each found in but one of the numbers; the factor 11 is found in but two of the numbers. The factors 5 and 5 are found in all the numbers.

The common prime factors are 5, 5.
The highest common factor is 25.

$5^2 = 25$

7. Find the H. C. F. of 63, 105, and 120.
8. Find the H. C. F. of 93, 217, and 341.
9. Find the H. C. F. of 96, 120, and 168.
10. Find the H. C. F. of 14, 56, and 84.
11. Find the H. C. F. of 60, 70, and 85.
12. Find the H. C. F. of 175, 225, and 1,000.
13. Find the H. C. F. of 39, 1,170, and 4,680.

GREATEST COMMON DIVISORS.

74. As a factor is an exact divisor of a number, it follows that the highest common factor of two or more numbers is their **Greatest Common Divisor.**

14. Find the Greatest Common Divisor of 72 and 180.

15. Find the G. C. D. of 102, 357, and 663.

16. Find the G. C. D. of 567 and 88,349. *Ans.* 1.

The prime factors of the smaller number are 3, 3, 3, 7; it is obvious that 88,349 does not contain the factor 3. On trial it is seen that the number 7 is not a factor of 88,349; it is useless to try further.

17. Find the G. C. D. of 7,007 and 9,135.

Second Method.

75. Any common factor of two numbers is a factor of their **Sum.**

11 is a factor common to 33 and 55 and 88.

76. Any common factor of two numbers is a factor of their **Difference.**

11 is a factor common to 88 and 55 and 33.

18. Find the G. C. D. of 2,268 and 1,136.

Explanation.

(i) As any common factor of two numbers is a factor of their difference, the G. C. D. of 2268 and 1136 is also the G. C. D. of the three numbers expressed as a minuend, a subtrahend, and a remainder.

2268 Min.
1136 Sub.
1132 Rem.

(ii) The G. C. D. of the minuend, subtrahend, and remainder is the G. C. D. of other three numbers expressed by the subtrahend and remainder and *their* difference, therefore:

1136 Sub.
1132 Rem.
4 Dif.

(iii) The G. C. D. of 2268 and 1136 is the G. C. D. of 1136 and 1132, and the G. C. D. of 1136 and 1132 is the G. C. D. of 1132 and 4. The G. C. D. of 1132 and 4 is readily found by the divisibility test of 4 (see **70**).

The tens and units of 1132 exactly contain 4; 4 is the G. C. D. of 1132 and 4.

The G. C. D. of 2268 and 1136 is 4.

19. Find the G. C. D. of 1,442 and 722; of 5,060 and 2,529.

20. Find the G. C. D. of 18,197 and 9,099; of 6,307 and 3,164.

21. Find the G. C. D. of 38,192 and 5,278.

EXPLANATION AND PROCESS.

As it is evident that there must be many subtractions where two numbers are so unequal, the process is much shortened by subtractional division.

THE SUCCESSIVE STEPS.

Step 5.	Step 4.	Step 3.	Step 2.	Step 1.	
5	4	4	4	7	Number of subtrahends.
14) 70	) 294	) 1246	) 5278	) 38192	Successive minuends.
70	280	1176	4984	36946	Multiple subtrahends.
	14	70	294	1246	Differences.

A factor of a number is a factor of any multiple of the number (see **69**). A factor of 38192 and 5278 is a factor of 38192 and of any multiple of 5278, and of the difference between the multiple and 38192.

STEP 1.

The greatest multiple of 5278 that 38192 contains is subtracted from 38192.

The G. C. D. sought is a factor of their difference . . . 1246

$$\begin{array}{r} 7 \\ 5278)38192 \\ \underline{36946} = 7 \times 5278 \\ 1246 \end{array}$$

STEP 2.

The greatest multiple of 1246 that 5278 contains is subtracted from 5278.

The G. C. D. sought is a factor of their difference . . . 294

$$\begin{array}{r} 4 \\ 1246)5278 \\ \underline{4984} = 4 \times 1246 \\ 294 \end{array}$$

STEP 3.

The greatest multiple of 294 that 1246 contains is subtracted from 1246.

The G. C. D. sought is a factor of their difference . . . 70

$$\begin{array}{r} 4 \\ 294)1246 \\ \underline{1176} = 4 \times 294 \\ 70 \end{array}$$

STEP 4.

The greatest multiple of 70 that 294 contains is subtracted from 294.

The G. C. D. sought is a factor of their difference . . . 14

$$\begin{array}{r} 4 \\ 70)294 \\ \underline{280} = 4 \times 70 \\ 14 \end{array}$$

STEP 5.

The greatest multiple of 14 that 70 contains is subtracted from 70. The difference is 0.

$$\begin{array}{r} 5 \\ 14)70 \\ \underline{70} = 5 \times 14 \\ 0 \end{array}$$

As 14 is the highest factor of 14, it is the G. C. D. of 14 and 70 and of all the minuends, subtrahends, and differences that occur in the entire process, and is, therefore, the G. C. D. of 38192 and 5278.

22. Find the G. C. D. of 96, 120, and 128.

The G. C. D. of two of the numbers may be found: the H. C. F. of *this G. C. D. and the third number* is the G. C. D. of the three numbers.

23. Find the G. C. D. of 160, 256, and 1,600.

24. Find the G. C. D. of 765, 855, and 1,035.

25. Find the G. C. D. of 1,216, 1,472, and 1,984.

26. Find the concrete G. C. D. of $250, $500, and $950.

COMMON MULTIPLES.

77. Any multiple of each of two or more numbers is a **Common Multiple** of the numbers.

$24 = 4 \times 6$: $24 = 3 \times 8$: $24 = 2 \times 12$.
24 is a common multiple of 4, 6, 3, 8, 2, and 12.
A multiple is exactly divisible by each of its factors.

THE LEAST COMMON MULTIPLE.

78. The least number that is a multiple of each of two or more numbers is called their **Least Common Multiple.**

24 is the L. C. M. of 3 and 8; but it is not the L. C. M. of 4 and 6, because $4 = 2 \times 2$, and $6 = 2 \times 3$: a number that contains 4, contains the factor 2 of the 6.

The least common multiple of two or more numbers is the least dividend that exactly contains every factor of the numbers.

Comparison by Prime Factors.

The G. C. D. of 24 and 36	and	The L. C. M. of 24 and 36.
Prime factors of 24 are 2, 2, 2, 3.		2, 2, 2, 3, are the prime factors of 24.
Prime factors of 36 are 2, 2, 3, 3.		2, 2, 3, 3, are prime factors of 36.
Both 24 and 36 are multiples of their G. C. D.		The L. C. M. is a multiple of both 24 and 36.
The H. C. F. is the greatest factor which both 36 and 24 contain.		The L. C. M. is the least multiple that contains both 24 and 36.
The greatest factor that both 24 and 36 contain is 12:		The least multiple that contains both 24 and 36 is 72:
$2 \times 2 \times 3 = 12$.		$2 \times 2 \times 2 \times 3 \times 3 = 72$.
Each common factor is used once; all other factors are rejected.		Each common factor is used once; all other factors are used.

79. The least common multiple of two or more numbers is at least as great as the greatest of the numbers. The L. C. M. of 27 and 54 must be at least as great as 54.

Unless the greatest of two or more numbers is their Least Common Multiple, their L. C. M. contains the greatest number at least twice, and the next greatest number at least three times.

The L. C. M. of 3 and 6 is 6: 6 contains 6, 1 time, and 3, 2 times.
The L. C. M. of 2 and 3 is 6: 6 contains 3, 2 times, and 2, 3 times.

1. Find the Least Common Multiple of 24 and 36.

Method by Inspection.

The multiple required is the least multiple that exactly contains both 24 and 36.	L. C. M. must contain 24 and 36.
It exactly contains 24 one or more times. . . .	Useless to try 1×24.
It exactly contains 36 one or more times	Useless to try 2×24.
Does the L. C. M. contain 36 one time?	1×36 is not 2×24.
Does the L. C. M. contain 36 two times? . . .	2×36 is 3×24.

$3 \times 24 = 72$: $2 \times 36 = 72$: 72 is the L. C. M. of 24 and 36.

2. Find the Least Common Multiple of 24 and 36.

Method by Factoring.

What are the prime factors of 24?	2, 2, 2, 3.
What are the prime factors of 36?	2, 2, 3, 3.
Does 24 contain a factor not in 36?	2.
Can the L. C. M. reject any factor?	2, 2, 2, 3, 3.

$$2 \times 2 \times 2 \times 3 \times 3 = 72.$$

3. Find the L. C. M. of 36 and 108.
4. Find the L. C. M. of 64 and 32.
5. Find the L. C. M. of 108 and 144.
6. Find the L. C. M. of 800 and 1,200.
7. Find the L. C. M. of 450 and 720.
8. Find the L. C. M. of 560 and 640.
9. Find the L. C. M. of 972 and 1,782.
10. Find the L. C. M. of 336 and 472.
11. Find the L. C. M. of 66 and 462.
12. Find the L. C. M. of 462 and 963.

13. Find the L. C. M. of 70, 84, and 735.

Method by Division.

THE SUCCESSIVE STEPS.

	735	84	70
STEP 1.			
The factors common to 70 and 84 are cancelled (or divided by themselves) from 70.	$3\times 5\times 7\times 7:$	$2\times 2\times 3\times 7:$	$2\times 5\times 7.$
			$\cancel{2}\times 5\times \cancel{7}.$
The result is:	$3\times 5\times 7\times 7:$	$2\times 2\times 3\times 7:$	$1\times 5\times 1.$
STEP 2.			
The factors common to 70 and 735 are cancelled from 70.			$1\times \cancel{5}\times 1.$
The result is:	$3\times 5\times 7\times 7:$	$2\times 2\times 3\times 7:$	$1\times 1\times 1.$
STEP 3.			
The factors common to 84 and 735 are cancelled from 84.		$2\times 2\times \cancel{3}\times \cancel{7}:$	
The result is:	$3\times 5\times 7\times 7:$	$2\times 2\times 1\times 1:$	$1\times 1\times 1.$

The L. C. M. is: $3\times 5\times 7\times 7\times 2\times 2 = 2940.$

The factor 1 does not affect the product.

In the following operation the final result is more readily obtained.

EXPLANATION.

OPERATION.

$$\begin{array}{r|rrr}
2 & 70 & 84 & 735 \\ \hline
3 & 35 & 42 & 735 \\ \hline
5 & 35 & 14 & 245 \\ \hline
7 & 7 & 14 & 49 \\ \hline
 & 1 & 2 & 7
\end{array}$$

The operation eliminates, one by one, all of the redundant factors.

The first factor eliminated is 2, from 70 *or* 84.

The second factor eliminated is 3, from 42 *or* 735.

The third factor eliminated is 5, from 35 *or* 245.

The last factor eliminated is 7, from any two of the three numbers 7, 14, 49.

Each factor *seems* to be eliminated from all of the numbers of which it is a factor; its first power, however, being preserved in the divisor's place, is one of the factors still.

(i) The divisors and final quotients are prime factors of 70, 84, 135.

(ii) They comprise all the prime factors of 70, 84, 735.

(iii) They include no redundant factor; therefore:

The product of 2, 3, 5, 7, 2, 7 and 1 is the L. C. M. sought.

The operation also shows that 7 is the G. C. D. of the numbers, as 7 is only number that exactly divides each of the three numbers.

14. Find the L. C. M. of 48, 50, and 75.

Find the L. C. M. of each of the following:

15. 120, 144, and 200.

16. 66, 121, and 220.

17. 53, 170, and 300.

18. 30, 90, 180, and 225.

19. 47, 94, 470, and 625.

20. 95, 25, 285, and 625.

21. Find the L. C. M. of 7, 11, 13, 770, and 1,001.

22. Find the L. C. M. of 27, 36, 54, 96, and 500.

23. Find the L. C. M. of 28, 39, 84, 117, and 400.

24. Find the L. C. M. of 18, 19, and 27.

Rapid Method with Large Numbers.

25. Find the L. C. M. of 40 and 41.

The numbers have no common factor. Their product is their L. C. M.

$$40 \times 41 = 1640.$$

26. Find the L. C. M. of 64 and 72.

As 8 is their H. C. F., their L. C. M. is $\frac{1}{8}$ of 64×72, or 8 times 9 eighths.

$$64 = 8 \text{ eighths } (2 \times 2 \times 2).$$
$$72 = 9 \text{ eighths } (2 \times 2 \times 2).$$

27. Find the L. C. M. of 132 and 156.

As 12 is their H. C. F., their L. C. M. is $\frac{1}{12}$ of 132×156, or 11 times 13 twelves.

$$132 = 11 \text{ twelves } (2 \times 2 \times 3).$$
$$156 = 13 \text{ twelves } (2 \times 2 \times 3).$$

28. Find the L. C. M. of 381 and 453.

The H. C. F., found either by factoring, or by method on page 46, is 3.

$$381 = 127 \text{ threes}.$$
$$453 = 151 \text{ threes}.$$

It is evident that the L. C. M. is less than the product of the two numbers; it is $\frac{1}{3}$ of the product of the two numbers.

$$\tfrac{1}{3} \text{ of } 381 \times 453 = \tfrac{381}{3} \times 453 = 127 \times 453 = 57531;$$

or,

$$\tfrac{1}{3} \text{ of } 453 \times 381 = \tfrac{453}{3} \times 381 = 151 \times 381 = 57531.$$

The L. C. M. of 381 and 453 is 57531.

From the foregoing is derived the method of finding the L. C. M. of large numbers by multiplying either of the large numbers by the quotient resulting from the division of the other number by their greatest common divisor.

29. Find the L. C. M. of 777 and 1,110.

30. Find the L. C. M. of 508 and 608.

31. Find the L. C. M. of 608 and 614.

32. Find the L. C. M. of 7,114 and 7,396.

33. Find the L. C. M. of 3,140, 4,160, and 5,350. (See page 44, problem 22.)

34. Find the L. C. M. of 576, 676, and 776.

35. Find the L. C. M. of 9,523 and 11,663.

36. Find the L. C. M. of 3,763 and 4,757.

37. Find the L. C. M. of 7,142 and 9,356.

38. Find the L. C. M. of 5,184 and 6,084.

39. Find the L. C. M. of 7,014 and 6,307.

40. Find the L. C. M. of 11,011 and 12,100.

41. Find the product of the prime factors of the G. C. D. of 72 and 108.

Find the product of the prime factors of the L. C. M. of 72 and 108.

Prime factors of 72:	*of* 108:	*of G. C. D.*	*of L. C. M.*
2, 2, 2, 3, 3.	2, 2, 3, 3, 3.	2, 2, 3, 3.	2, 2, 3, 3, and 2, 3.
(5 factors, 2^3, 3^2.)	(5 factors, 2^2, 3^3.)	(4 factors, 2^2, 3^2.)	(6 factors, 2^3, 3^3.)
$72 = 2^3 \times 3^2$.	$108 = 2^2 \times 3^3$.	G. C. D. $= 2^2 \times 3^2$.	L. C. M. $= 2^3 \times 3^3$.

$72 \times 108 = 2^3 \times 3^2 \times 2^2 \times 3^3 = 2^5 \times 3^5$. G. C. D. $\times$ L. C. M. $= 2^2 \times 3^2 \times 2^3 \times 3^3 = 2^5 \times 3^5$.

$72 \times 108 = 2^5 \times 3^5 = 7776$. G. C. D. $\times$ L. C. M. $= 2^5 \times 3^5 = 7776$.

$72 \times 108 = 7776$. G. C. D. $\times$ L. C. M. $= 7776$.

$72 \times 108 =$ G. C. D. $\times$ L. C. M.

80. The product of the Greatest Common Divisor and Least Common Multiple of two numbers is equal to the product of the two numbers.

REVIEW PROBLEMS.

1. The minuend is the L. C. M. of 99 and 270; the subtrahend is the G. C. D. of 99 and 270; find the remainder. *Ans.* 2,961.

2. The remainder is the L. C. M. of 80 and 90; the minuend is the product of the L. C. M. and G. C. D. of 80 and 90; find the subtrahend. *Ans.* 6,480.

3. The quotient is the H. C. F. of 55, 132, and 11,011; the dividend is the next composite number greater than 1,000; find the divisor. *Ans.* 91.

4. Given: 48 rows of fruit trees, 144 trees to the row, 108 trees to the acre.

Required: the number of acres in the orchard.

5. The dividend is three times as great as the greatest prime number that can be expressed with three figures; the divisor is 21; write in words the quotient.

6. Given: 1,350 lb. seed-cotton to the bale; 36 cotton pickers, each hand averaging 150 lb. per day.

Required: the number of days in which they will pick 12 bales.

7. The minuend is 13^2; the subtrahend is the highest composite factor of 36; find the remainder. *Ans.* 151.

8. Given: oats, 32 lb.; wheat, 60 lb.; corn, 56 lb. to the bu.
" 136 acres in the three kinds of grain.
" 40 acres corn yielding 27 bu. per acre.
" 60 acres oats yielding 46 bu. per acre.
" — acres wheat yielding 17 bu. per acre.

Required: (1) the total yield in bushels.
(2) the average yield per acre in pounds.

9. Find the error. Cancel: $\frac{9 \times 8 \times 16 \times 30 \times 45}{6 \times 12 \times 12 \times 12 \times 75 \times 2} = 0.$

10. Find the value of the grain products in problem 8 at 52 cents for corn, 32 cents for oats, and 72 cents for wheat per bushel. *Ans.* \$1885.44.

11. The quotient is \$17.50; the divisor is 6^3; what is the dividend?

12. Given: A fence with panels 8 feet long; 2,000 16-ft. planks in the fence; the fence 5 planks high.

Required: the number of panels in the fence.

13. The dividend is \$755; one factor is \$5; find and name the other factor.

14. A man paid \$7,510 for a farm, and sold it for \$6,400, losing \$2 per acre; how many acres were in the farm?

15. The divisor is 6; the quotient is the highest factor of 28,913; find the dividend.

16. Express in Roman the G. C. D. of 18, 30, and 96.

17. The dividend, 1,003, is the product of two numbers; the divisor is the G. C. D. of the two numbers, and the quotient is the L. C. M. of the two numbers; find the prime factors of the product of quotient and divisor.

18. Find the cost of 1,000,000 fence rails @ 50 cents per hundred.

19. Find the cost of 1,000,000 bricks @ $10 per thousand.

20. Find the cost of 700,000 feet of weather-boarding @ $15 per M.

21. If 15^3 were added to a number, the sum would be the L. C. M. of 78, 86, and 99; find the number. *Ans.* 107,307.

22. Find five numbers the L. C. M. of which is 108.

23. The G. C. D. of 336, 1,008, and a third number, is the third number divided by 125; find the third number.

24. A dealer sold to a creditor a herd of cattle @ $16 per head; after paying $500 on the debt, he received a draft for $4,284; find the number of cattle in the herd.

25. Find the greatest concrete number that is a common divisor of $125, $390, and $1,000.

26. In writing the dividend 69,318, a student reversed the order of the figures in the thousands' period; therefore his quotient was 3,000 too great; find the divisor.

27. Seed-cotton less the cotton seed is lint cotton.

Given: 1,350 lb. seed-cotton to the bale.
450 lb. lint cotton to the bale.
72 acres in a field of cotton.
Yield 1,200 lb. seed-cotton per acre.

Required: The number of bales produced on the field.
" The weight of the cotton seed produced on the field.

28. The minuend is 120 greater than the subtrahend; the subtrahend is 70 greater than the remainder; find the minuend.

29. The product is 27 times as great as the multiplicand; the multiplicand is the third power of 3; what is the product?

30. The dividend is 25 times as great as the divisor; the divisor is the third power of 5; find the dividend.

31. The dividend is 12 times as great as the divisor, and 12 times as great as the quotient; find the dividend.

32. Find the G. C. D. of 4^4, 6^5, and 8^6. *Ans.* 32.

33. Find the L. C. M. of 3, 7, 11, and 13. *Ans.* 3,003.

34. The figures of the dividend are, for the millions' period, 123, for the thousands' period, noughts, and for the units' period, 123; the divisor is 123; find the quotient.

35. Cancel: $\dfrac{12012 \times 26026 \times 66}{13 \times 12 \times 66066}$. *Ans.* ——.

36. Cancel: $\frac{4}{9} \times \frac{3}{8} \times \frac{6}{19} \times \frac{38}{2}$. *Ans.* ——.

37. Given: A rectangular lot, 385 feet to the long side, 273 feet to the short side; fence panels of equal length; posts standing an exact number of feet apart.

Required: the number of posts. *Ans.* 188 posts.

38. Given: (1) $\$125 \div 5 = \$25$. (2) $\$125 \div \$5 = 25$.

Divide the concrete quotient by the abstract divisor.

Ans. ——.

39. If 159 be subtracted 80 times from a certain number the result will be 5; find the number. *Ans.* 12,725.

40. Cancel: $\frac{1}{3} \times \frac{3}{4} \times \frac{1}{2} \times \frac{2}{3}$. *Ans.* $\frac{1}{12}$.

ADDITIONAL REVIEW WORK.—Pages 32, 33, problems 46–54; page 35, problems 26–39.

FOR ORAL WORK.

Review.—Induction.

1. What is a prime number? Give examples.

2. What is a composite number? Give examples.

3. Name all odd numbers below 20.

4. Name all the prime numbers below 53.

5. Name all the composite numbers between 53 and 73.

6. What is a common factor? a common multiple?

7. What is the H. C. F. of two numbers? Give an example.

8. How may the highest prime factor common to two numbers be found?

9. Of what numbers is 3 an exact divisor?

10. Of what numbers is 4 an exact divisor?

11. Of what numbers is 9 a factor?

12. What is the G. C. D. of two numbers?

13. Show that any common factor of two numbers is a factor of their sum, and a factor of their difference.

14. Show that a factor of a number is a factor of any multiple of the number.

15. Define Least Common Multiple.

16. Show the distinction between the G. C. D. and the L. C. M. of two numbers.

17. What is the product of 4 and 6? What is the product of the L. C. M. and the G. C. D. of 4 and 6?

18. In what two ways can the product of the L. C. M. and the G. C. D. of two numbers be found?

19. What is an integer? Give examples of numbers that are not integers.

20. Give the name for numbers that are lower than whole numbers, and are written by the Arabic method.

21. How many dimes in a dollar?

Examples: $\$1.00 = 10 \times \$.10$; $1 = 10 \times .10$.

22. How many tenths must be added together to make an integral unit?

23. What must one tenth be multiplied by to make an integral unit?

24. What must an integral unit be divided by to make a tenth?

Examples: $\frac{10}{1} \times \frac{1}{10} = ?$ $\frac{4}{1} \times \frac{1}{4} = ?$

25. What sign of division is used in the examples?

26. Ten divided by 1 equals what? 1 divided by $10 = ?$

27. Four times $\frac{1}{4}$ equals what? One fourth of four equals what?

28. In expressing division by the fractional sign, where is the dividend placed? Where is the divisor placed?

29. What are the factors of a multiple used as a dividend called?

30. As a dividend is a multiple, and a divisor is a factor, what is the quotient factor in the expression $\frac{1}{4}$?

31. Is the quotient of $1 \div 4$ so great as a single integral unit?

32. Is it as great as any unit? *It is a fractional unit called a fourth.*

FRACTIONS.

DEFINITIONS AND PRINCIPLES.

81. A **Unit** is a number or a quantity used as a standard of comparison.

In abstract numbers the unit, or standard of comparison, is 1.

In United States money the unit is the **Dollar**; with $1 all money values are compared.

82. A whole number, used as a standard of comparison, is an **Integral Unit**. (See **64.**)

The least integral unit is the unit 1.

1 thousand is the unit of the thousands' period in numeration.

83. Every whole number higher than the unit 1 is a multiple of 1.

9 is nine abstract units; each unit is 1. The character 9 is the expression used for denoting the product resulting from the multiplication of the unit 1 by the multiplier nine.

9 is not a nine; it is 9 ones; but, if it be required to find how many nines there are in 18, a nine is the unit, or standard, by which 18 is compared with 9.

84. Ten characters being used to express numbers, the unit of each order is ten times as great as the unit of the next lower order; hence, the numbers are said to be written by a scale of tens, or by the **Decimal Scale**.

The removal of 1 to the next higher place multiplies 1 by ten; 10 denotes 10×1; 20 denotes $2 \times 10 \times 1$, or 2 integral units, each 10 times as great as the unit 1.

There are units of tens, units of hundreds, units of thousands, etc.

85. Every unit of lower order than the unit 1 is a quotient resulting from the division of the unit 1 by a divisor greater than 1.

The unit resulting from the division of 1 by 10 is 1 tenth.

86. When the divisor of the unit 1 is 10, or any power of 10, the quotient is the unit of a decimal fraction. (See **16.**)

The removal of 1 to the next lower place divides 1 by 10; .1, which is read 1 tenth, denotes $1 \div 10$; .2 denotes $2 \times 1 \div 10$, or 2 lower units each one tenth as great as the unit 1.

There are units of tenths, units of hundredths, units of thousandths, etc., each of which has its order in the decimal scale.

87. When the divisor of the unit 1 is neither 10, nor any power of 10, the quotient has no order in the decimal scale.

$1 \div 3$ cannot be expressed decimally; it is expressed by writing two integers with the fractional sign of division between them; thus, $\frac{1}{3}$.

88. A **Fractional Unit** is one of the equal parts of an integral unit, and is a quotient resulting from the division of the unit 1 by a divisor greater than 1.

The quotient of $1 \div 2$ is $\frac{1}{2}$, which is a fractional unit half as great as the unit 1.

The quotient of $1 \div 10$ is $\frac{1}{10}$, or .1, which is a fractional unit a tenth as great as the unit 1. There is no difference between $\frac{1}{10}$ and .1 except a difference of expression.

89. A **Fraction** is one or more fractional units.

.1, $\frac{1}{3}$, .01, $\frac{1}{5}$ are fractions; each fraction consists of 1 fractional unit.

.2, $\frac{2}{3}$, .02, $\frac{2}{5}$ are fractions; each fraction consists of 2 fractional units.

90. A **Common Fraction** is a fraction whose numerical expression requires a dividend and a divisor separated by the fractional sign of division.

The common fraction $\frac{1}{3}$ is also a fractional unit, because it is but 1 third. The common fraction $\frac{2}{3}$ is 2 fractional units each of which is $\frac{1}{3}$.

The fraction $\frac{3}{10}$ is not a common fraction, because it may be written by the decimal scale; thus, .3.

91. The number expressed by the dividend is called the **Numerator.**

The numerator is written above the fractional sign of division; it expresses the number of fractional units in the fraction.

In $\frac{5}{6}$ the numerator, 5, shows that there are 5 fractional units in the fraction.

92. The number expressed by the divisor is called the **Denominator.**

The denominator is written below the fractional sign of division; it expresses the number of equal parts into which the unit 1 is divided.

In $\frac{5}{6}$ the denominator, 6, shows that the unit 1 is divided into 6 equal parts, each of which is a fractional unit called a sixth.

93. The numerator and the denominator are called the **Terms** of the fraction.

94. The **Value** of a fraction is the quotient resulting from the division of the numerator by the denominator.

The fraction $\frac{3}{5}$ is equal to 3 times $\frac{1}{5}$, or to $3 \div 5$. Let it be required to find $\frac{3}{5}$ of a dime; $\frac{1}{5}$ of 1 dime is 2 cents, and $\frac{3}{5}$ of 1 dime is 6 cents; again, 3 times 1 dime is 30 cents, $\frac{1}{5}$ of which is 6 cents.

95. A **Proper Fraction** is a fraction whose value is less than the unit 1.

The numerator of a proper fraction is less than the denominator; $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$ are proper fractions.

96. An **Improper Fraction** is a fraction whose value is not less than the unit 1.

The numerator of an improper fraction is not less than the denominator; $\frac{3}{3}$, $\frac{4}{3}$, and $\frac{6}{5}$ are improper fractions.

97. A **Mixed Number** is a number which consists of an integer and a fraction.

$2\frac{2}{3}$ is a mixed number.

98. A fraction of a fraction is called a **Compound Fraction.**

$\frac{1}{2}$ of $\frac{3}{4}$ is called a compound fraction.

99. An expression indicating a dividend and a divisor, one or each of which is a fraction, is sometimes called a **Complex Fraction.**

The complex fraction $\frac{\frac{1}{3}}{2}$ is read *one third divided by two.*

100. An integer is easily compared with the unit 1; the name of the integer declares the result of the comparison.

To compare the unit 1 with fractions, use is made of **Reciprocals.**

101. The **Reciprocal** of a number is the quotient resulting from the division of the unit 1 by the number.

The reciprocal of an integral number is a fractional unit whose denominator is equal to the number; the reciprocal of 8 is $\frac{1}{8}$; the unit 1 is $\frac{1}{8}$ of 8.

The reciprocal of a fractional unit is an integral number equal to the denominator of the fractional unit; the reciprocal of $\frac{1}{8}$ is 8; there are 8 eighths in the unit 1.

The reciprocal of a fraction that contains more than one fractional unit, is expressed by inverting the terms of the fraction. Let it be required to find the reciprocal of $\frac{2}{3}$. The reciprocal of $\frac{1}{3}$ is 3; in the unit 1 there are 3 thirds. The reciprocal of $\frac{2}{3}$ cannot be so great as 3, because the unit 1 does not contain $\frac{2}{3}$ as many times as it contains $\frac{1}{3}$; it contains $\frac{2}{3}$ half as many times as it contains $\frac{1}{3}$; therefore, the reciprocal of $\frac{2}{3}$ is half as great as the reciprocal of $\frac{1}{3}$; the reciprocal of $\frac{1}{3}$ is 3; the reciprocal of $\frac{2}{3}$ is the half of 3, which is 3 halves, or, expressed as a fraction, $\frac{3}{2}$, whose terms are the inverted terms of the fraction $\frac{2}{3}$.

The number of times the unit 1 contains $\frac{2}{3}$ is $\frac{3}{2}$.

The number of times the unit 1 contains $\frac{3}{2}$ is $\frac{2}{3}$.

The fractions $\frac{2}{3}$ and $\frac{3}{2}$ are reciprocals of each other.

EXERCISES.

1. How many fractional units are there in 3 fourths?
2. How many times does the unit 1 contain $\frac{1}{4}$?
3. What is the reciprocal of $\frac{1}{4}$? Of $\frac{1}{5}$? Of $\frac{1}{8}$?
4. How many fractional units are there in $\frac{1}{9}$? In $\frac{2}{9}$?
5. How many times does the unit 1 contain $\frac{1}{9}$?
6. What is the reciprocal of $\frac{1}{9}$? Of $\frac{1}{8}$? Of $\frac{2}{8}$?
7. How many fractional units are there in $\frac{6}{12}$? In $\frac{1}{12}$?
8. What is the reciprocal of $\frac{1}{12}$? Of $\frac{6}{12}$?
9. How many fractional units are in $\frac{5}{4}$?
10. What is the reciprocal of $\frac{5}{4}$? Of $\frac{4}{5}$?
11. What is the fractional unit of $\frac{7}{8}$?
12. What is the reciprocal of $\frac{7}{8}$?

102. As the **value** of a fraction is the **quotient** resulting from the division of the **numerator** by the **denominator,** the **principles** of fractions are the principles of division.

(1) *If the numerator of a fraction is multiplied by any number, the value of the fraction is multiplied by that number.*

If, in the fraction $\frac{2}{5}$, the numerator 2 is changed to 4, the value of the fraction is multiplied by 2 ; $\frac{4}{5}$ is twice as great as $\frac{2}{5}$; the fractional unit is not changed ; the number of fractional units is doubled.

(2) *If the numerator of a fraction is divided by any number, the value of the fraction is divided by that number.*

If, in the fraction $\frac{4}{9}$, the numerator 4 is changed to 2, the value of the fraction is divided by 2 ; $\frac{2}{9}$ is half as great as $\frac{4}{9}$; the fractional unit is not changed ; the number of fractional units is halved.

(3) *If the denominator of a fraction is multiplied by any number, the value of the fraction is divided by that number.*

If, in the fraction $\frac{2}{11}$, the denominator is changed to 22, the value of the fraction is divided by 2 ; $\frac{2}{22}$ is half of $\frac{2}{11}$; the number of fractional units is not changed, but the fractional unit is changed. A twenty-second is half as great as an eleventh.

(4) *If the denominator of a fraction is divided by any number, the value of the fraction is multiplied by that number.*

If, in the fraction $\frac{2}{42}$, the denominator is changed to 21, the value of the fraction is multiplied by 2 ; $\frac{2}{21}$ is twice as great as $\frac{2}{42}$; the number of fractional units is not changed, but the fractional unit is changed. A twenty-first is twice as great as a forty-second.

(5) *If both the numerator and the denominator of a fraction are multiplied by the same number, the value of the fraction is not changed.*

If, in the fraction $\frac{1}{2}$, the numerator is changed to 2, and the denominator to 4, the value of the fraction is not changed ; the number of fractional units is doubled ; the value of each fractional unit is halved.

(6) *If both the numerator and the denominator of a fraction are divided by the same number, the value of the fraction is not changed.*

If, in the fraction $\frac{4}{6}$, the numerator is changed to 2, and the denominator to 3, the value of the fraction is not changed ; the number of fractional units is halved ; the value of each fractional unit is doubled.

REDUCTION.

103. Any process for changing a number without changing the value of the number is called **Reduction.**

104. Integers and Mixed Numbers to Improper Fractions.

PROBLEMS.

1. Reduce 3 to a fraction whose unit is a fifth.

ANALYSIS.

1 integral unit = 5 fifths.
3 integral units = 3 × 5 fifths.
3×5 fifths $= \frac{15}{5}$.

COMPARISON.

The reciprocal of $\frac{1}{5}$ is 5. (See **101.**) Since in 1 integral unit there are 5 fractional units, in 3 integral units there are 15 fractional units. $3 = \frac{15}{5}$.

2. Reduce 7 to eighths; 7 to tenths.
3. Reduce 9 to twenty-firsts; 9 to twenty-seconds.
4. Reduce 6 to thirty-thirds; 6 to sixteenths.
5. Reduce $20 to half-dollars; $20 to quarter-dollars.
6. Reduce 108 to ninths; 108 to sixteenths.
7. Reduce $20\frac{3}{4}$ to fourths.

ANALYSIS.

$$1 = \frac{4}{4};\ 20 = 20 \times \frac{4}{4} = \frac{80}{4};\ \frac{80}{4} + \frac{3}{4} = \frac{83}{4}.$$

8. Given: the integral number 931, and a number which consists of 7 fractional units called twelfths.

Required: the number of twelfths in the two numbers.

9. The minuend is the mixed number $301\frac{2}{3}$; reduce to thirds the sum of the integral and fractional units in the subtrahend and remainder. *Ans.* $\frac{905}{3}$.

10. The subtrahend is 17; the remainder is expressed by the integer 40 and a fraction whose numerator is 9, and whose fractional unit is a sixteenth.

Reduce to sixteenths the integral and fractional units of the minuend. *Ans.* $\frac{921}{16}$.

11. A number is composed of 3 integral units, and 6 fractional units each of which is as great as one fifteenth of an integral unit. Express the number as an improper fraction.

12. If granulated sugar be bought at the rate of 4 pounds for a quarter of a dollar, how many pounds can be bought for $\$50\frac{3}{4}$? *Ans.* 812 pounds.

13. The smaller of two numbers is $51\frac{3}{8}$; the larger number is 8 times as great as the smaller number; find the larger number. *Ans.* 411.

105. Improper Fractions to Integers and Mixed Numbers.

14. Find the number of integral units equal to 21 thirds.

ANALYSIS.

3 thirds = 1 integral unit.
21 thirds = as many times 1 integral unit as 21 contains 3; 21 contains 3, 7 times.
7 times 1 unit = 7 units.
$\frac{21}{3} = 7.$

COMPARISON.

The reciprocal of $\frac{1}{3}$ is 3. Since 3 of the fractional units equal 1 of the integral units, 21 of the fractional units equal 7 of the integral units, because there are 7 threes in 21.
$\frac{21}{3} = 21 \div 3 = 7.$

15. Find the number of integral units equal to 64 eighths.

16. Find the number of integral units equal to 81 ninths.

17. Find the number of integral units equal to 108 twelfths.

18. Express as an integer the fraction $\frac{132}{11}$.

19. The denominator is 17; the numerator is 17^2; express the integral value of the fraction.

20. The numerator is 64; the denominator is 9; express as a mixed number the value of the fraction.

ANALYSIS.

$\frac{9}{9} = 1$ integral unit.
$\frac{64}{9}$ = as many times 1 integral unit as $\frac{64}{9}$ contains $\frac{9}{9}$.
$\frac{64}{9}$ contains $\frac{9}{9}$, $7\frac{1}{9}$ times.
$7\frac{1}{9} \times 1$ integral unit $= 7\frac{1}{9}$ integral units, or 7 integral units and 1 ninth.

COMPARISON.

The reciprocal of $\frac{1}{9}$ is 9. Since 9 of the fractional units equal 1 of the integral units, 64 of the fractional units equal 7 of the integral units and 1 of the fractional units, because there are 7 nines in 64, and 1 over, which is 1 ninth.
$\frac{64}{9} = 64 \div 9 = 7\frac{1}{9}.$

21. The fractional unit is a thirteenth; the number of the fractional units is 143; reduce the number to integral units.

Ans. 11.

22. The dividend is 600; the divisor is 9; express the quotient.

23. The numerator is the highest composite factor of 24; the denominator is the highest common factor of 2, 4, and 8; express the fraction in integral units.

24. The numerator is the L. C. M. of 27 and 54; the denominator is the H. C. F. of 27 and 54; express the value of the fraction in integral units.

25. The dividend is the difference of 3^5 and 3^4; the divisor is the difference between 13^2 and 174; express the quotient in integral and fractional units.

26. How many twenty-fifths are in the sum of the numbers 3,968, 5,729, and 4,007$\frac{21}{25}$?

27. The minuend is 303$\frac{4}{7}$; express as an improper fraction the sum of the subtrahend and the remainder.

28. The product is 180,018; one factor is 17; express the other factor as an improper fraction.

106. Fractions to Lowest Terms.

107. Numbers are prime to each other when they have no common factor. A fraction is in its lowest terms when the numerator and denominator are prime to each other.

29. Reduce $\frac{18}{36}$ to a fraction whose terms are prime to each other.

ANALYSIS.

If 18 and 36 are not prime to each other, they have some H. C. F.

H. C. F. of 18 and 36 is 18. $\frac{\overset{1}{\cancel{18}}}{\underset{2}{\cancel{36}}} = \frac{1}{2}$. (See **102, 6.**)

30. Reduce $\frac{24}{36}$ to its lowest terms.

31. Resolve into their prime factors both the numerator 75 and the denominator 125; cancel the factors common to both, and show the result.

Cancel the common factors and show the lowest terms of each of the following fractions.

32. $\frac{28}{42}$; $\frac{36}{54}$; $\frac{63}{81}$; $\frac{126}{248}$; $\frac{75}{125}$; $\frac{64}{96}$; $\frac{78}{117}$.

33. $\frac{5}{100}$; $\frac{10}{100}$; $\frac{20}{100}$; $\frac{25}{100}$; $\frac{96}{100}$; $\frac{82}{100}$; $\frac{55}{100}$; $\frac{85}{100}$.

34. $\frac{119}{187}$; $\frac{209}{247}$; $\frac{299}{529}$; $\frac{1643}{1961}$; $\frac{1643}{52576}$; $\frac{217}{3906}$; $\frac{387}{6063}$.

35. $\frac{40}{12}$; $\frac{65}{15}$; $\frac{186}{16}$; $\frac{1375}{121}$; $\frac{2868}{112}$; $\frac{485}{45}$; $\frac{6447}{252}$.

36. A dividend is 49; the divisor is 77; express the quotient as a fraction in its lowest terms.

37. Reduce to its lowest terms the fraction whose denominator is 5,000, and whose numerator is the square of 5,000.

38. Find the number of greatest fractional units in $\frac{55}{100}$ and express the value of each of the fractional units.

39. Given: Dividend 132 integral units of the lowest order. Divisor 1,320 fractional units, each $\frac{1}{2}$ as great as 1 integral unit. Required: The lowest terms in which the two numbers can be expressed.

108. Fractions to Higher Terms.

40. Without changing the value of the fraction $\frac{15}{18}$, raise the denominator to 180.

ANALYSIS.

The factor 10 raises 18 to 180. The factor 10 must raise 15 also.

$$\frac{15}{18} \times \frac{10}{10} = \frac{150}{180}.$$

The number has been multiplied by a number equal to 1. (See **102.** 5.)

COMPARISON.

The reciprocal of $\frac{1}{180}$ is 180. Since in 1 integral unit there are 180 fractional units, in $\frac{1}{18}$ of an integral unit there are 10 fractional units, because $\frac{1}{18}$ of 180 is 10. Since in $\frac{1}{18}$ of an integral unit there are 10 fractional units, in $\frac{15}{18}$ of an integral unit there are 150 fractional units. $\frac{15}{18} = \frac{150}{180}$.

41. Reduce $\frac{31}{32}$ to a number whose denominator is 160.

42. Reduce $\frac{31}{32}$ to a number which has 217 fractional units.

43. Reduce $\frac{18}{64}$ to a number whose numerator is 90.

44. Reduce $\frac{3}{4}$ to a number whose fractional unit is $\frac{1}{64}$.

45. Change $\frac{1}{16}$ to a number whose numerator is 79.

46. Reduce $\frac{2}{3}$ to a number, $\frac{1}{3}$ of whose denominator is 7.

Ans. $\frac{14}{21}$.

47. Reduce the reciprocal of $\frac{2}{3}$ to a number, $\frac{1}{4}$ of whose numerator is 9.

48. Find the number of fractional units in $\frac{5}{6}$ raised to ninetieths.

49. The denominator is 32; the numerator is 24; find an equivalent fraction whose numerator is 48.

LEAST COMMON DENOMINATOR.

109. The least common denominator of two or more fractions is the least common multiple of the denominators of the fractions. (See **78.**)

50. Reduce to fractions having the greatest common fractional unit the fractions $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{2}{15}$.

PROCESS.

The L. C. D. of $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{2}{15}$ is 30.

$$\frac{1}{2}: \qquad \frac{1}{2} = \frac{15}{30}$$
$$\frac{2}{3}: \quad \frac{1}{3} = \frac{10}{30}; \quad \frac{2}{3} = \frac{20}{30}$$
$$\frac{2}{15}: \quad \frac{1}{15} = \frac{2}{30}; \quad \frac{2}{15} = \frac{4}{30}$$

COMPARISON OF $\frac{2}{3}$ AND $\frac{1}{30}$.

The reciprocal of $\frac{1}{30}$ is 30. Since in 1 integral unit there are 30 fractional units, in $\frac{1}{3}$ of an integral unit there are 10 fractional units, because $\frac{1}{3}$ of 30 is 10.

$$\frac{1}{3} = \frac{10}{30}; \ \frac{2}{3} = \frac{20}{30}.$$

51. Reduce \$$\frac{1}{10}$, \$$\frac{2}{5}$, and \$$\frac{1}{2}$ to dimes.

52. In 1 bushel there are 4 pecks; reduce to pecks $4\frac{3}{4}$ bu.; $8\frac{1}{2}$ bu.; and $6\frac{1}{4}$ bu.

53. Reduce to fractions having the highest common fractional unit $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$.

54. Reduce to fractions having the least common denominator $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{3}{7}$.

55. Reduce to fractions having the least common denominator $\frac{4}{5}$, $\frac{6}{8}$, $\frac{7}{10}$.

Reduce to fractions having the least common denominator:

56. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$. **57.** $\frac{9}{14}$, $\frac{10}{21}$, $\frac{5}{6}$, $\frac{1}{2}$. **58.** $\frac{12}{25}$, $\frac{11}{15}$, $\frac{4}{5}$, $\frac{2}{3}$.

59. $\frac{2}{27}$, $\frac{7}{18}$, $\frac{5}{12}$, $\frac{8}{9}$. **60.** $\frac{5}{24}$, $\frac{3}{16}$, $\frac{1}{7}$, $\frac{1}{2}$. **61.** $\frac{4}{7}$, $\frac{5}{6}$, $\frac{3}{4}$, $\frac{2}{3}$.

62. $\frac{7}{11}$, $\frac{6}{7}$, $\frac{5}{6}$, $\frac{3}{4}$. **63.** $4\frac{1}{2}$, $5\frac{1}{6}$, $\frac{2}{3}$. **64.** $3\frac{1}{3}$, $4\frac{1}{4}$, $5\frac{1}{5}$.

65. $2\frac{4}{9}$, $3\frac{4}{7}$, $1\frac{4}{63}$. **66.** $3\frac{1}{4}$, $2\frac{1}{7}$, $3\frac{1}{5}$, $4\frac{1}{9}$. **67.** $12\frac{5}{14}$, $3\frac{1}{28}$, $7\frac{7}{8}$.

68. Given: A fraction whose fractional unit is a sixteenth, and whose numerator is 7;

Also, a fraction composed of 8 fractional units, and whose denominator is 32.

Express each fraction with the least possible common denominator.

69. Given: A mixed number composed of 4 integral units, and a fraction whose denominator is 7, and which has but 1 fractional unit in the fraction;

Also, a fraction which has two fractional units, each of its fractional units being $\frac{1}{9}$.

Express each fraction with the least possible common denominator.

70. Given: The three denominators 30, 40, and 50; find the least common denominator.

110. To reduce fractions.

To reduce mixed numbers to improper fractions:

To the given fraction add the whole number reduced to a like fraction.

To reduce improper fractions to integers or mixed numbers:

Divide the numerator by the denominator.

To reduce a fraction to its lowest terms:

Cancel the factors common to the terms of the fraction.

To reduce a fraction to higher terms:

Multiply both terms by that number which will raise the denominator to the required denominator.

To reduce fractions to fractions having the least common denominator:

Reduce each fraction to a fraction whose denominator is the least common multiple of the denominators of the given fractions.

ADDITION OF FRACTIONS.

111. To add fractions is to find their sum, which should be expressed in the greatest units possible.

PROBLEMS.

1. Find the sum of $\frac{1}{16}$, $\frac{4}{16}$, and $\frac{3}{16}$.

PROCESS.

$$\tfrac{1}{16}+\tfrac{4}{16}+\tfrac{3}{16}=\tfrac{8}{16}=\tfrac{1}{2}.$$

2. Find the sum of $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{6}$.

EXPLANATION.

The fractions are reduced to fractions having the least common denominator; their sum is found to be an improper fraction, which is reduced to a mixed number.

PROCESS.

$$\tfrac{3}{4}=\tfrac{45}{60}.\quad \tfrac{4}{5}=\tfrac{48}{60}.\quad \tfrac{5}{6}=\tfrac{50}{60}.$$

$$\tfrac{45}{60}+\tfrac{48}{60}+\tfrac{50}{60}=\tfrac{143}{60}.$$

$$\tfrac{143}{60}=2\tfrac{23}{60}.$$

3. The subtrahend is $\frac{3}{8}$; the remainder is $\frac{1}{6}$; write the minuend in the greatest units possible.

4. What integer equals $\frac{1}{3}+\frac{2}{3}+\frac{1}{9}+\frac{5}{6}+\frac{3}{4}+\frac{22}{72}$?

5. What mixed number equals $\frac{3}{4}+\frac{5}{8}+\frac{6}{7}+\frac{2}{3}$?

6. What mixed number equals $\frac{1}{16}+\frac{3}{8}+\frac{3}{4}+\frac{1}{2}$?

7. Find the sum of $4\frac{1}{4}$, $5\frac{1}{3}$, $6\frac{1}{6}$, $3\frac{1}{3}$.

PROCESS.

$$4+5+6+3=18$$
$$\tfrac{1}{4}+\tfrac{1}{3}+\tfrac{1}{6}+\tfrac{1}{3}=\underline{1\tfrac{1}{12}}$$
$$19\tfrac{1}{12}$$

8. The minuend would be $15\frac{7}{8}$ if it were $19\frac{1}{4}$ less; what is the minuend?

9. The subtrahend is $16\frac{3}{16}$; the remainder is $14\frac{3}{32}$; find the minuend.

10. Find the distance around a rectangular field $\frac{7}{8}$ mile in length, and $\frac{3}{4}$ mile in width.

11. The reciprocal of one fraction is $\frac{16}{9}$; the reciprocal of another is $\frac{8}{7}$; find the sum of the two fractions. *Ans.* $1\frac{7}{16}$.

12. The minuend equals the subtrahend plus $\frac{7}{8}$; the subtrahend equals the remainder plus $\frac{3}{32}$; find the sum of the minuend, subtrahend, and remainder. *Ans.* $3\frac{11}{16}$.

13. Express in the simplest form $\frac{3}{8}+\frac{1}{16}+\frac{3}{24}+\frac{1}{48}+\frac{1}{2}$.

14. Find the sum of one thousand ten and three twenty-firsts, three thousand three hundred and five sevenths, and two thousand nine and eight forty-seconds.

15. There are two proper fractions; the denominator of the greater fraction is the least common multiple of 3 and 7; the denominator of the other is the highest common factor of 84 and 42; the number of fractional units in the greater fraction is 9; the number of fractional units in the other fraction is 7. Find the sum of the fractions.

16. The minuend is $401\frac{7}{8}$; find the sum of the minuend, subtrahend, and remainder.

17. A dividend is 16, its divisor 33; another dividend is 9, its divisor 44; find the sum of the two quotients.

18. Express in Roman the sum of $151\frac{3}{8}$, $161\frac{5}{9}$, $1006\frac{10}{144}$.

19. Find the greatest common fractional unit, and the number of such units, in $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}$.

20. Find the sum of \$$\frac{1}{2}$, \$$\frac{1}{4}$, \$$\frac{2}{5}$, \$$\frac{3}{4}$, and \$$\frac{3}{25}$; express the amount in dollars and cents.

21. Add $101\frac{5}{8}$, $617\frac{2}{3}$, $515\frac{3}{7}$, $207\frac{6}{11}$.

22. Add $315\frac{9}{16}$, $718\frac{9}{32}$, $519\frac{5}{24}$, $727\frac{6}{7}$.

Find the sums:

23. $\frac{1}{8}+\frac{1}{9}+\frac{1}{11}+\frac{2}{18}$.

24. $\frac{3}{7}+\frac{4}{9}+\frac{5}{6}+\frac{7}{8}$.

25. $\frac{2}{3}+\frac{3}{4}+\frac{5}{6}+\frac{6}{7}+\frac{8}{9}$.

26. $150\frac{2}{3}+250\frac{3}{4}+350\frac{5}{6}$.

27. $191\frac{6}{7}+218\frac{3}{11}+718\frac{1}{18}$.

28. $3003\frac{12}{13}+51\frac{6}{7}+3\frac{2}{9}$.

29. $191\frac{5}{6}+618\frac{3}{4}+181\frac{7}{8}$.

30. $216\frac{2}{3}+185\frac{2}{9}+17\frac{1}{2}$.

31. $\$106\frac{4}{5}+\$19\frac{3}{20}+\$25$.

32. $\frac{1}{6}+\frac{2}{7}+\frac{3}{8}+\frac{4}{9}+\frac{2}{8}$.

33. $\frac{5}{8}+\frac{3}{4}+\frac{5}{6}+\frac{2}{3}+\frac{1}{12}$.

34. $\frac{4}{9}+\frac{8}{11}+\frac{3}{13}+\frac{4}{17}$.

35. $\frac{18}{8}+\frac{42}{4}+\frac{65}{9}+\frac{72}{11}$.

36. $\$515\frac{1}{5}+\$60\frac{1}{10}+\$45\frac{17}{20}$.

37. $\$30\frac{7}{10}+\$42\frac{1}{2}+\$18\frac{19}{20}$.

38. $\frac{3}{16}+\frac{3}{32}+\frac{3}{8}+\frac{3}{4}+\frac{3}{2}+3$.

39. $4+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\frac{1}{81}$.

40. $5+\frac{1}{5}+\frac{1}{10}+\frac{1}{20}+\frac{1}{50}+\frac{1}{100}$.

41. $6+\frac{1}{6}+\frac{1}{12}+\frac{1}{24}+\frac{1}{48}+\frac{1}{96}$.

42. $7+7\frac{1}{7}+14\frac{1}{14}+21\frac{1}{21}+28\frac{1}{28}$.

43. $8+\frac{1}{8}+16\frac{1}{2}+32\frac{1}{4}+64$.

44. $9+\frac{1}{9}+9\frac{1}{9}+18+\frac{5}{18}+18\frac{5}{18}$.

45. $10+\frac{1}{10}+10\frac{1}{10}+10\frac{2}{5}+10\frac{4}{25}$.

46. $15+\frac{1}{15}+15\frac{1}{15}+30\frac{7}{30}+\frac{7}{30}$.

47. $16\frac{1}{16}+16\frac{3}{16}+16\frac{5}{16}+16\frac{1}{32}+\frac{1}{2}$.

48. $\$500\frac{1}{2}+\$192\frac{3}{4}+\$75\frac{7}{20}$.

SUBTRACTION OF FRACTIONS.

112. To subtract fractions is to find their difference, which should be expressed in the greatest units possible.

PROBLEMS.

1. Express the difference of $\frac{13}{18}$ and $\frac{9}{16}$.

EXPLANATION.	PROCESS.
The fractions are reduced to fractions having the least common denominator, over which the difference of the new numerators is written.	$\frac{13}{18}=\frac{104}{144}$. $\frac{9}{16}=\frac{81}{144}$. $\frac{104}{144}-\frac{81}{144}=\frac{23}{144}$.

2. Find the difference of $\frac{3}{8}$ and $\frac{3}{32}$.

3. Express the fractional units in $\frac{2}{7}-\frac{1}{9}$.

4. The subtrahend is $\frac{3}{4}$; the minuend is $\frac{5}{6}$; find the difference.

5. The minuend is $\frac{31}{32}$; the difference is $\frac{7}{8}$; find the subtrahend.

6. A man had $\$134\frac{1}{2}$, and spent $\$59\frac{3}{4}$; find the remainder.

EXPLANATION.	PROCESS.
One of the dollars of the minuend is changed to quarter-dollars; this changes the minuend to $\$133\frac{6}{4}$.	$\$134\frac{1}{2}=\$133\frac{6}{4}$ $59\frac{3}{4}$ $\$134\frac{1}{2}-\$59\frac{3}{4}=\$74\frac{3}{4}$

7. Find the difference between $\$5\frac{1}{4}$ and $\$4\frac{3}{5}$.

8. The numerator of the minuend is 7; the fractional unit is a twelfth; the numerator of the subtrahend is 3; the fractional unit of the subtrahend is twice as great as that of the minuend; find the difference. *Ans.* $\frac{1}{12}$.

9. The minuend is 7 integral units of tens; the subtrahend is 7 fractional units of tenths; find the difference. *Ans.* $69\frac{3}{10}$.

10. Minuend: 1 unit of thousands, 3 units of hundreds, 2 units of ninths.

Subtrahend: 9 units of hundreds, 9 units of tens, 9 units of ones, 12 units of thirteenths; find the difference.

Ans. $300\frac{35}{117}$.

11. The minuend is the H. C. F. of 17, 18, 19, and 37; the subtrahend is the difference of $\frac{7}{9}$ and $\frac{21714}{27918}$; find the remainder.

Ans. 1.

12. The minuend is $\frac{1}{2}+\frac{3}{4}+\frac{5}{6}+\frac{6}{7}$; the subtrahend is $\frac{74}{75}$; find the remainder.

13. A number whose numerator is 5, and whose fractional unit is a ninth, is changed to $\frac{1}{2}$; what fraction is subtracted?

14. The minuend is half of the G. C. D. of 7, 8, 9, and 12; the subtrahend is $\frac{1}{8}$; find the remainder.

15. The minuend is \$ $8\frac{1}{2}$; the remainder is \$ $5\frac{3}{4}$; what is the subtrahend?

16. If the fractional unit of the minuend were twice as great as it is, the minuend would be $\frac{5}{9}$; if the fractional unit of the subtrahend were half as great as it is, the subtrahend would be $\frac{1}{14}$; find the difference. *Ans.* $\frac{17}{126}$.

17. If the price of cotton should rise \$ $.00\frac{1}{4}$, good middling would sell for \$ $.09\frac{1}{8}$; find the present price of good middling.

18. Find the difference of $100+\frac{2}{3}+100+\frac{3}{4}$, and $199\frac{5}{8}$.

19. Find the difference of \$ 6,700 and \$ $53\frac{19}{20}$.

20. Express in words the difference of $10,905\frac{1}{21}$ and $903\frac{4}{7}$.

21. What number added to $3\frac{5}{7}$ will make $5\frac{1}{9}$?

22. From the difference of $1,001\frac{4}{7}$ and $909\frac{3}{8}$ subtract the difference of $30\frac{1}{9}$ and $31\frac{1}{4}$.

23. $300\frac{1}{7}+18\frac{4}{9}-307\frac{2}{3}$ equals what number?

113 (A). To add fractions that have different denominators:

Reduce the fractions to fractions having the least common denominator.

Write the sum of the new numerators over the common denominator.

113 (B). To subtract fractions that have different denominators:

Reduce the fractions to fractions having the least common denominator.

Write the difference of the new numerators over the common denominator.

FOR ORAL REVIEW.

1. What is an integer? An abstract number?

2. What is the unit of U. S. money? What is a unit?

3. What is the least integral unit?

4. What is multiplication? Give the distinction between multiplication and addition.

5. What is division? Give the distinction between subtractional division and partitive division.

6. How are numbers lower than integers formed? Give examples. Are they the result of subtractional division, or of partitive division?

7. What is a power? Give examples. What is a factor? Give examples. Name the highest integral factor of 10.

8. What is a fractional unit? Name five different fractional units. Name five different fractions all having the same fractional unit.

9. What is a fraction? How is a common fraction expressed?

10. Distinguish integral unit, fractional unit, and fraction.

11. Give the names of the terms of a fraction. How do they correspond with the terms in division? What does the numerator express? What does the denominator show?

12. What is the value of a fraction? What is the quotient of $\frac{2}{3}$? What is the value of $2 \div 3$? What is the value of $\frac{2}{3}$ expressed as twelfths?

13. What is the standard of comparison in numbers?

14. What is a reciprocal?

15. Give the reciprocal of $\frac{1}{4}$; of $\frac{1}{5}$; of $\frac{1}{6}$; of $\frac{3}{5}$; of $\frac{4}{7}$.

16. Compare $\frac{1}{4}$ with 1; compare 1 with 4.

17. Compare $\frac{1}{5}$ with 1; compare 1 with 5.

18. Compare $\frac{1}{6}$ with 1; compare 1 with 6.

19. Compare $\frac{1}{2}$ with 1; compare $\frac{2}{4}$ with 1.
Compare 1 with 2; compare 1 with $\frac{4}{2}$.

20. Compare $\frac{2}{3}$ with 1; compare 1 with $\frac{3}{2}$.

21. Explain the use of the reciprocal.

MULTIPLICATION OF FRACTIONS.

114. To multiply fractions is to find the product of fractional numbers, or of integers by fractional numbers; the product should be expressed in the greatest units possible.

115. To Multiply Fractions by Integers.

PROBLEMS.

1. The multiplicand is $\$\frac{3}{4}$; the multiplier is 12; find the product.

Process.

$$12 \text{ times } \$\tfrac{3}{4} = \$\tfrac{36}{4} = \$9;$$

or,

$$\$\tfrac{3}{4} = 3 \times \$\tfrac{1}{4}, \text{ or } \tfrac{1}{4} \text{ of } \$3. \quad \overset{3}{\cancel{12}} \times \frac{1}{\cancel{4}} \text{ of } \$3 = \$9.$$

2. The multiplicand is $\$\frac{7}{8}$, the multiplier 32; find the product.

3. The multiplier is 72; the multiplicand $\frac{8}{18}$; find the product.

4. Find the cost of 15 bushels of wheat at $\$\frac{4}{5}$ per bushel.

Analysis.

Cost of 1 bu. $= \$\frac{4}{5}$.
Cost of 15 bu. $= 15 \times \$\frac{4}{5}$.
$15 \times \$\frac{4}{5} = \$\frac{60}{5} = \$12$.
Cost of 15 bu. wheat $= \$12$.

5. Find the cost of 7 knives @ \$1.25 each.

6. Find the cost of 8 pairs of shears @ \$1.75 each.

7. Find the cost of 9 pairs of shoes @ $\$2\frac{1}{4}$ each.

$$\begin{array}{ll} \underline{9 \times \$2\tfrac{1}{4} = 9 \times \$2 + 9 \times \$\tfrac{1}{4}.} & \\ 9 \times \$2 = \$18 & \\ 9 \times \$\tfrac{1}{4} = \underline{\$\ \ 2\tfrac{1}{4}} \ (\$\tfrac{9}{4}). & \\ \qquad\qquad \$20\tfrac{1}{4} & \end{array} \qquad \begin{array}{rl} 2\tfrac{1}{4} & \\ \underline{9} & \\ 2\tfrac{1}{4} & = 9 \times \tfrac{1}{4}. \\ \underline{18} & = 9 \times 2. \\ 20\tfrac{1}{4} & = 9 \times 2\tfrac{1}{4}. \end{array}$$

8. Find the cost of 18 rakes at $\$\frac{3}{4}$ each.

9. Find the cost of 21 shovels at $\$1\frac{1}{4}$ each.

10. Find the product of 18 and $2\frac{3}{8}$.

11. Multiply $7\frac{7}{8}$ by 12.

Find the products:

12. $6 \times 18\frac{1}{2}$.	**18.** $38 \times 43\frac{3}{16}$.	**24.** $29 \times 302\frac{3}{4}$.
13. $7 \times 19\frac{3}{4}$.	**19.** $47 \times 44\frac{9}{32}$.	**25.** $38 \times 417\frac{15}{16}$.
14. $9 \times 27\frac{5}{8}$.	**20.** $52 \times 63\frac{7}{8}$.	**26.** $49 \times 710\frac{31}{32}$.
15. $12 \times 34\frac{6}{7}$.	**21.** $71 \times 16\frac{5}{16}$.	**27.** $67 \times 598\frac{6}{7}$.
16. $14 \times 47\frac{5}{9}$.	**22.** $18 \times 99\frac{4}{9}$.	**28.** $48 \times 317\frac{1}{64}$.
17. $27 \times 38\frac{7}{12}$.	**23.** $62 \times 43\frac{59}{64}$.	**29.** $77 \times 80\frac{31}{32}$.

116. To Multiply Integers by Fractions and by Mixed Numbers.

30. A train that is running on time at the rate of 20 miles per hour is due in $\frac{3}{4}$ of an hour; how far away is the train?

ANALYSIS.

Distance run in 1 hour = 20 mi.
Distance run in $\frac{1}{4}$ hour = $\frac{1}{4}$ of 20 mi. = 5 mi.
Distance run in $\frac{3}{4}$ hour = 3×5 mi. = 15 mi.

$$\frac{3}{4} \times 20 \text{ mi.} = \frac{60}{4} \text{ mi.} = 15 \text{ mi.}$$

31. In a mile there are 5,280 feet; how many feet are in $\frac{4}{5}$ of a mile?

32. The weight of 1 barrel of flour is 196 pounds; find the weight of $\frac{3}{7}$ of a barrel.

33. The average weight of a bale of cotton is 450 pounds; find the weight of $\frac{5}{9}$ of an average bale.

34. The weight of a bushel of corn is 56 pounds; find the weight of $2\frac{3}{4}$ bushels.

Wt. 1 bu. = 56 lb.
Wt. 2 bu. = 2×56 lb. = 112 lb.
Wt. $\frac{1}{4}$ bu. = $\frac{1}{4} \times 56$ lb. = 14 lb.
Wt. $\frac{1}{2}$ bu. = 2×14 lb. = 28 lb.
Wt. $2\frac{3}{4}$ bu. = 154 lb.

$$2\tfrac{3}{4} \times 56 = 2 \times 56 + \tfrac{3}{4} \times 56.$$

$$\begin{aligned} 2 \times 56 &= 112 \\ \tfrac{3}{4} \times 56 &= 42 \\ \hline 2\tfrac{3}{4} \times 56 &= 154 \end{aligned}$$

Find the weight:

35. Of $5\frac{7}{8}$ bushels of wheat; 60 pounds to the bushel.

36. Of $6\frac{5}{16}$ bushels of oats; 32 pounds to the bushel.

37. Of $6\frac{9}{16}$ bushels of meal; 48 pounds to the bushel.

38. Of $20\frac{3}{4}$ bushels of cotton-seed; 32 pounds to the bushel.

39. Of $16\frac{5}{6}$ bushels of mixed feed; 42 pounds to the bushel.

40. Find the cost of $32\frac{7}{8}$ bu. seed-wheat, at 96¢ per bushel.

Find the products:

41. $6\frac{3}{4} \times 19$.
42. $7\frac{3}{8} \times 21$.
43. $5\frac{4}{9} \times 27$.
44. $18\frac{3}{7} \times 64$.
45. $25\frac{5}{6} \times 32$.
46. $32\frac{7}{8} \times 41$.
47. $15\frac{3}{16} \times 48$.
48. $17\frac{3}{32} \times 64$.
49. $18\frac{5}{32} \times 96$.
50. $19\frac{7}{32} \times 144$.
51. $7\frac{1}{7} \times 42$.
52. $8\frac{1}{3} \times 65$.
53. $9\frac{7}{8} \times 18$.
54. $20\frac{3}{4} \times 10$.
55. $200\frac{7}{8} \times 10$.

117. To Multiply Fractions by Fractions.

56. The multiplier is $\frac{8}{9}$; the multiplicand is $\frac{9}{10}$; find the product.

ANALYSIS.

$\frac{1}{9}$ of $\frac{9}{10} = \frac{1}{10}$.
$\frac{8}{9}$ of $\frac{9}{10} = 8 \times \frac{1}{10}$.
$8 \times \frac{1}{10} = \frac{8}{10}$ or $\frac{4}{5}$.
$\frac{8}{9} \times \frac{9}{10} = \frac{8}{10}$ or $\frac{4}{5}$.

PROCESS: $\frac{8}{9} \times \frac{9}{10} = \frac{72}{90}$, or $\frac{4}{5}$.

COMPARISON.

The reciprocal of $\frac{8}{9}$ is $\frac{9}{8}$. The product is $\frac{8}{9}$ of the multiplicand. The multiplicand is $\frac{9}{8}$ of the product.
The multiplicand, or $\frac{9}{8}$ of the product $= \frac{9}{10}$.
$\frac{1}{8}$ of the product $= \frac{1}{10}$.
$\frac{8}{8}$ of the product $= \frac{8}{10}$, or $\frac{4}{5}$.

57. The multiplier is $\frac{6}{7}$; the multiplicand is $\frac{14}{48}$; find the product.

58. What part of a bushel is $\frac{3}{4}$ of $\frac{7}{8}$ of a bushel?

59. The factors are $\frac{19}{20}$ and $\frac{18}{38}$; find the product.

60. What part of a mile is $\frac{2}{3}$ of $\frac{4}{5}$ of a mile?

61. The factors are $\frac{4}{7}$ and $\frac{16}{21}$; find the product.

62. Reduce $18\frac{3}{4}$ to an improper fraction, and multiply it by $\frac{2}{3}$.

63. Multiply $\frac{3}{4}$ by $19\frac{1}{2}$; $16\frac{1}{3}$ by $\frac{3}{7}$.

64. Multiply $\frac{5}{8}$ by $17\frac{3}{4}$; $18\frac{3}{7}$ by $\frac{2}{3}$.

65. Multiply $\frac{15}{16}$ by $22\frac{1}{2}$; $37\frac{1}{2}$ by $\frac{3}{4}$.

66. Multiply by cancelling: $\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \frac{5}{6} \times \frac{6}{7}$.

67. Cancel: $\frac{3}{8} \times \frac{3}{7} \times \frac{3}{5} \times \frac{4}{7} \times \frac{4}{9} \times \frac{5}{12} \times \frac{8}{16}$.

68. Cancel: $\frac{4}{5} \times \frac{3}{8} \times \frac{7}{9} \times \frac{11}{12} \times \frac{13}{16} \times \frac{14}{15} \times \frac{28}{75}$.

69. Cancel: $\frac{6}{7} \times \frac{8}{9} \times \frac{3}{5} \times \frac{1}{2} \times \frac{8}{15} \times \frac{40}{45} \times \frac{3}{4}$.

70. Cancel: $\frac{18}{25} \times \frac{19}{26} \times \frac{20}{27} \times \frac{21}{31} \times \frac{22}{23} \times \frac{46}{57}$.

71. Cancel: $\frac{13}{15} \times \frac{29}{30} \times \frac{11}{39} \times \frac{15}{16} \times \frac{1}{32} \times \frac{1}{58} \times \frac{32}{33}$.

72. Multiply $20\frac{3}{4}$ by $15\frac{1}{2}$ without reducing.

EXPLANATION.

(i) The entire multiplicand is multiplied by the fractional unit of the multiplier.

(ii) The fractional part of the multiplicand is multiplied by the integral part of the multiplier.

(iii) The integers of the two factors are multiplied together.

(iv) The partial products are added.

PROCESS.

$$\begin{array}{ll} 20\frac{3}{4} & \\ 15\frac{1}{2} & \\ \hline 10\frac{3}{8} & = \frac{1}{2} \text{ of } 20\frac{3}{4} \\ 11\frac{1}{4} & = 15 \times \frac{3}{4} \\ 300 & = 15 \times 20 \\ \hline 321\frac{5}{8} & = 15\frac{1}{2} \times 20\frac{3}{4} \end{array}$$

73. Multiply $19\frac{3}{4}$ by $16\frac{1}{2}$ without reducing.

74. Multiply $30\frac{5}{8}$ by $6\frac{1}{5}$ without reducing.

75. Multiply $72\frac{3}{4}$ by $19\frac{1}{4}$ without reducing.

76. The multiplicand is composed of 30 integral units, and 2 fractional units; the multiplier is composed of 31 integral units and 4 fractional units; the fractional units of the multiplier are tenths, and each is half as great as the fractional unit of the multiplicand; find the product. *Ans.* $954\frac{14}{25}$.

77. The divisor is 3 times the quotient; the quotient is 8 integral units and 1 fractional unit whose denominator is 4; find the dividend. *Ans.* $204\frac{3}{16}$.

78. The quotient is 4 times the divisor; the divisor is 7 integral units, and 2 fractional units whose denominator is 7; find the dividend. *Ans.* $212\frac{16}{49}$.

79. The divisor is 6 times the quotient; the quotient is an improper fraction whose reciprocal is $\frac{3}{8}$; find the dividend.

80. Given: The fractional unit one-eighth common to both factors; in one factor the number of integral units is 300 and the number of fractional units 7; the other factor twice as great; find the product. *Ans.* $181,051\frac{17}{32}$.

118. To multiply fractions:

Reduce each mixed number to an improper fraction.

Write the product of the numerators for the numerator of the product, and the product of the denominators for the denominator of the product.

REVIEW WORK.

For oral review, pages 56 and 57.

For review problems, pages 49 and 50.

DIVISION OF FRACTIONS.

119. To divide fractions is to find the quotient when either the dividend or the divisor is a fraction, or when both are fractions. The quotient should be expressed in the greatest units possible.

120. To Divide a Fraction by an Integer.

PROBLEMS.

1. The dividend is $\frac{15}{16}$; the divisor is 5; find the quotient.

$$\frac{1}{5} \text{ of } \frac{15}{16} = \frac{3}{16};\quad \frac{5)15}{16} = \frac{3}{16};\quad \frac{1}{\cancel{5}} \times \frac{\overset{3}{\cancel{15}}}{16} = \frac{3}{16}.$$

2. The dividend is $\frac{3}{4}$; the divisor is 5; find the quotient.

ANALYSIS.

To divide by 5 is to find a quotient that is $\frac{1}{5}$ of the dividend.

The dividend is $\frac{3}{4}$.

$\frac{1}{5} \times \frac{3}{4} = \frac{3}{20}$.

$\frac{3}{4} \div 5 = \frac{3}{20}$.

(See **102**, 3.)

COMPARISON.

The reciprocal of 5 is $\frac{1}{5}$. The dividend is 5 times the quotient. The quotient is $\frac{1}{5}$ the dividend.

The dividend is $\frac{3}{4}$.

The quotient is $\frac{1}{5}$ of $\frac{3}{4}$.

$\frac{1}{5}$ of $\frac{3}{4} = \frac{1}{5} \times \frac{3}{4} = \frac{3}{20}$.

3. Divide $\frac{8}{9}$ of an acre into 3 equal lots; how much in each lot?

ANALYSIS.

3 lots $= \frac{8}{9}$ A.

1 lot $= \frac{1}{3}$ of $\frac{8}{9}$ A.

$\frac{1}{3} \times \frac{8}{9}$ A. $= \frac{8}{27}$ A.

1 lot $= \frac{8}{27}$ A.

COMPARISON.

3 lots $= \frac{8}{9}$ A.

The reciprocal of 3 is $\frac{1}{3}$.

1 lot is $\frac{1}{3}$ of $\frac{8}{9}$ A.

$\frac{1}{3} \times \frac{8}{9}$ A. $= \frac{8}{27}$ A.

4. The dividend is $\frac{7}{8}$; the divisor is 6; find the quotient.
5. The dividend is $\frac{15}{16}$; the quotient is 5; find the divisor.

Find the quotients:

6. $\frac{17}{32} \div 6$.	**7.** $\frac{1}{8} \div 7$.	**8.** $\frac{3}{4} \div 8$.	**9.** $\frac{3}{7} \div 16$.
10. $\frac{3}{7} \div 12$.	**11.** $\frac{4}{9} \div 7$.	**12.** $\frac{6}{7} \div 9$.	**13.** $\frac{5}{16} \div 7$.
14. $\frac{5}{8} \div 8$.	**15.** $\frac{5}{7} \div 7$.	**16.** $\frac{9}{16} \div 9$.	**17.** $\frac{3}{10} \div 10$.
18. $\frac{6}{11} \div 22$.	**19.** $\frac{9}{16} \div 16$.	**20.** $\frac{5}{9} \div 9$.	**21.** $\frac{9}{16} \div 1$.

121. To Divide a Mixed Number by an Integer.

22. Divide $7{,}001\frac{3}{8}$ by 4.

There are two partial quotients, 1750 and $\frac{1\frac{3}{8}}{4}$.

$\frac{1\frac{3}{8}}{4}$ is a complex fraction. (See **99.**)

$$4)\underline{7001\tfrac{3}{8}}$$
$$1750 + \frac{1\frac{3}{8}}{4};\quad \frac{1\frac{3}{8}}{4} = \tfrac{11}{8} \div 4 = \tfrac{11}{32}$$

$$7001\tfrac{3}{8} \div 4 = 1750\tfrac{11}{32}.$$

23. Find $\frac{1}{4}$ of $32\frac{1}{2}$ bushels of corn.

24. The dividend is $50\frac{1}{2}$; the divisor is 4; find the quotient.

25. Find $\frac{1}{3}$ of $50\frac{1}{3}$ inches.

26. The dividend is $61\frac{2}{3}$; the divisor is 3; find the quotient.

27. Find $\frac{1}{2}$ of $62\frac{3}{8}$ feet.

28. The dividend is $70\frac{3}{8}$; the quotient is 5; find the divisor.

29. The dividend is $100\frac{3}{7}$; the divisor is 9; find the quotient.

30. The dividend is $1{,}036\frac{7}{8}$; the divisor is 15; find the quotient.

31. A dividend is $3{,}006\frac{3}{4}$; one factor is 10; find the other factor.

32. The dividend is composed of one thousand nine integral units, and 8 fractional units each of which is four times as great as a sixtieth; the divisor is 9; find the quotient. *Ans.* $112\frac{23}{135}$.

33. The dividend is composed of 3,001 integral units, and a fraction whose reciprocal is 4; the divisor is 9; find the quotient.

34. In 9 years a town collected \$1,000,367$\frac{3}{4}$ in taxes; find the yearly average for the time.

Find the quotients:

35. $\frac{7}{8} \div 5$.	**36.** \$$1\frac{15}{25} \div 3$.	**37.** $9{,}000\frac{1}{8} \div 108$.
38. $10{,}900\frac{3}{7} \div 14$.	**39.** $10{,}200\frac{4}{15} \div 30$.	**40.** $41{,}000\frac{5}{6} \div 100$.
41. $160\frac{7}{8} \div 10$.	**42.** $781\frac{1}{9} \div 7$.	**43.** $10{,}701\frac{3}{8} \div 12$.
44. $170\frac{3}{7} \div 10$.	**45.** $1{,}610\frac{3}{7} \div 5$.	**46.** $7{,}963\frac{5}{9} \div 13$.
47. $190\frac{3}{4} \div 20$.	**48.** $1{,}846\frac{7}{8} \div 16$.	**49.** $8{,}002\frac{7}{10} \div 10$.
50. $200\frac{3}{4} \div 20$.	**51.** $3{,}001\frac{4}{9} \div 18$.	**52.** $7{,}156\frac{13}{25} \div 100$.
53. $3{,}001\frac{3}{16} \div 10$.	**54.** $10{,}001\frac{6}{25} \div 50$.	**55.** $9{,}003\frac{7}{8} \div 200$.

122. To Divide an Integer by a Fraction.

First Method.

56. A train runs $\frac{5}{8}$ of a mile per minute; in what time will it run 36 miles?

ANALYSIS.

If the train runs $\frac{5}{8}$ mi. per minute, it will run 36 mi. in as many times 1 min. as 36 mi. contains $\frac{5}{8}$ mi.

$36 \text{ mi.} = \frac{288}{8} \text{ mi.}$
$\frac{288}{8} \text{ mi.} \div \frac{5}{8} \text{ mi.} = 288 \text{ mi.} \div 5 \text{ mi.}$
$288 \text{ mi.} \div 5 \text{ mi.} = 57\frac{3}{5}.$
$57\frac{3}{5}$ times 1 min. $= 57\frac{3}{5}$ min.

COMPARISON.

If the train ran 1 mi. per minute it would run 36 mi. in 36 min. But it runs $\frac{5}{8}$ of a mile per minute. The reciprocal of $\frac{5}{8}$ is $\frac{8}{5}$. 36 is $\frac{5}{8}$ of the number of minutes required.

Number of minutes required is $\frac{8}{5}$ of 36.

$\frac{1}{5}$ of $36 = 7\frac{1}{5}.$
$\frac{8}{5}$ of $36 = 8 \times 7\frac{1}{5} = 57\frac{3}{5}.$
$36 \div \frac{5}{8} = 57\frac{3}{5}.$

PROCESS.

$36 \div \frac{5}{8} = \frac{288}{8} \div \frac{5}{8}.$
$\frac{288}{8} \div \frac{5}{8} = 288 \div 5.$

Both dividend and divisor have been multiplied by 8. (See 60.)

$288 \div 5 = 57\frac{3}{5}.$

The analysis and process are based on **102** (5). If both numerator and denominator of a fraction be multiplied by the same number, the value of the fraction is not changed. 8 is used as the multiplier in order to reduce the dividend to the same fractional unit as the divisor. In effect they are reduced to fractions having the least common denominator.

57. A bushel of corn weighs $\frac{14}{15}$ as much as a bushel of wheat; how many bushels of corn will equal in weight 90 bushels of wheat?

58. A bushel of oats weighs as much as $\frac{8}{15}$ of a bushel of wheat; how many bushels of oats will equal in weight 100 bushels of wheat?

59. A bushel of oats weighs as much as $\frac{4}{7}$ of a bushel of corn; how many bushels of oats will equal in weight 100 bushels of corn?

60. How many times does 10 contain $\frac{4}{7}$?

61. Given: 16 ounces to the pound; cartridges weighing $\frac{5}{8}$ ounce each; required: the number of cartridges to weigh 10 pounds.

62. At $\$\frac{3}{5}$ per bushel, how many bushels of corn can be bought for $30?

63. At $\$\frac{4}{5}$ per bushel, how many bushels of wheat can be bought for $35?

Second Method.

64. The dividend is 40; the divisor is $\frac{6}{7}$; find the quotient.

ANALYSIS.

$40 \div 1 = 40.$
$40 \div 6 = \frac{1}{6}$ of 40.
$40 \div \frac{6}{7} = 7 \times \frac{1}{6}$ of 40.
$7 \times \frac{1}{6}$ of $40 = \frac{7}{6} \times 40.$
$\frac{7}{6} \times 40 = \frac{280}{6}.$
$\frac{280}{6} = 46\frac{2}{3}.$
$40 \div \frac{6}{7} = \frac{7}{6} \times 40 = 46\frac{2}{3}.$

COMPARISON.

The reciprocal of $\frac{6}{7}$ is $\frac{7}{6}$. The dividend is $\frac{6}{7}$ of the quotient; the quotient is $\frac{7}{6}$ of the dividend. The dividend is 40. The quotient is $\frac{7}{6}$ of 40.

$\frac{1}{6}$ of $40 = 6\frac{2}{3}.$
$\frac{7}{6}$ of $40 = 7 \times 6\frac{2}{3} = 46\frac{2}{3}.$
$40 \div \frac{6}{7} = \frac{7}{6} \times 40 = 46\frac{2}{3}.$

65. The dividend is 88; the divisor is $\frac{11}{12}$; find the quotient.
66. The dividend is 75; the divisor is $\frac{4}{15}$; find the quotient.
67. The dividend is 86; the divisor is $\frac{3}{10}$; find the quotient.
68. The dividend is 90; the divisor is $\frac{5}{8}$; find the quotient.
69. The dividend is 31; the divisor is $\frac{7}{8}$; find the quotient.
70. The dividend is 41; the divisor is $\frac{7}{16}$; find the quotient.
71. The dividend is 91; the divisor is $\frac{4}{13}$; find the quotient.
72. The dividend is 82; the divisor is $\frac{1}{3}$ of $\frac{3}{4}$; find the quotient.
73. The dividend is 76; the divisor is $\frac{2}{3}$ of $\frac{3}{4}$; find the quotient.
74. The dividend is 76; the divisor is $\frac{2}{3}$; find the quotient.

SHORT ANALYSIS.

$76 \div \frac{1}{3} = 3 \times 76.$

$76 \div \frac{2}{3} = \frac{3 \times 76}{2}.$

$\frac{3 \times \overset{38}{\cancel{76}}}{\cancel{2}} = 3 \times 38 = 114.$

$76 \div \frac{2}{3} = 114.$

SHORT COMPARISON.

The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$. The dividend is $\frac{2}{3}$ of the quotient. The quotient is $\frac{3}{2}$ of the dividend.

$\frac{3}{2}$ of the dividend $76 = \frac{3}{2} \times 76.$
$\frac{3}{2} \times 76 = 114.$

Find the quotients:

75. $180 \div \frac{3}{4}$.	**81.** $16 \div \frac{5}{8}$.	**87.** $40 \div \frac{3}{8}$.
76. $190 \div \frac{7}{8}$.	**82.** $101 \div \frac{3}{64}$.	**88.** $50 \div \frac{5}{12}$.
77. $210 \div \frac{3}{16}$.	**83.** $113 \div \frac{2}{9}$.	**89.** $50 \div \frac{5}{24}$.
78. $340 \div \frac{3}{32}$.	**84.** $141 \div \frac{8}{9}$.	**90.** $50 \div \frac{1}{96}$.
79. $101 \div \frac{2}{8}$.	**85.** $10 \div \frac{15}{16}$.	**91.** $50 \div \frac{5}{96}$.
80. $102 \div \frac{2}{9}$.	**86.** $10 \div \frac{15}{32}$.	**92.** $100 \div \frac{2}{8}$.

123. To Divide a Fraction by a Fraction.

First Method.

93. How many times can $\frac{3}{32}$ of a bushel of oats be subtracted from $\frac{3}{4}$ of a bushel of oats?

ANALYSIS BY SUBTRACTIONAL DIVISION.

$$\frac{3}{4} \div \frac{3}{32} = \frac{24}{32} \div \frac{3}{32}.$$
$$24 \div 3 = 8.$$

$\frac{3}{32}$ can be subtracted 8 times from $\frac{3}{4}$.

COMPARISON.

The reciprocal of $\frac{3}{32}$ is $\frac{32}{3}$. The number of times that $\frac{3}{4}$ contains $\frac{3}{32}$ is $\frac{3}{4}$ of $\frac{32}{3}$ times.

$$\frac{\cancel{3}}{\cancel{4}} \times \frac{\cancel{32}}{\cancel{3}} = 8.$$

(If it be remembered that 32 lb. oats make a bushel, it will be seen that in this case a reduction to the L. C. D. is also a reduction to a common unit of measure.)

94. The dividend is $\frac{4}{9}$; the divisor is $\frac{1}{6}$; reduce them to a common fractional unit, and find the quotient.

95. The dividend is $\frac{7}{8}$; the divisor is $\frac{1}{14}$; find the quotient.

96. The dividend is $\frac{8}{9}$; the divisor is $\frac{7}{8}$; find the quotient.

97. The dividend is $\frac{5}{12}$; the divisor is $\frac{3}{4}$; find the quotient.

98. What must $\frac{5}{8}$ be multiplied by to make $\frac{3}{7}$?

99. What must $\frac{6}{7}$ be multiplied by to make $\frac{4}{5}$?

100. How many times can $\frac{6}{14}$ be subtracted from $\frac{6}{7}$?

101. How many times can $\frac{7}{16}$ be subtracted from $\frac{7}{8}$?

102. How many times can $\$\frac{5}{20}$ be subtracted from $\$\frac{5}{10}$?

103. What must $\frac{3}{11}$ be multiplied by to make $\frac{8}{22}$?

104. How many times is $\frac{3}{22}$ contained in $\frac{3}{11}$?

105. What must $\frac{3}{8}$ be multiplied by to make $\frac{3}{4}$?

106. What must $\frac{3}{8}$ be divided by to make $\frac{3}{4}$?

107. How many times does $\$1\frac{3}{4}$ contain $\$\frac{1}{4}$?

108. How many half-dollars make $\$2\frac{1}{2}$?

109. Divide $3\frac{1}{2}$ by $\frac{1}{4}$.

110. The dividend is $4\frac{1}{2}$; the divisor is $\frac{2}{3}$; find the quotient.

Find the quotients:

111. $\frac{2}{3} \div \frac{1}{3}$.	**117.** $2\frac{2}{3} \div \frac{3}{4}$.	**123.** $4\frac{1}{4} \div 3\frac{1}{8}$.
112. $\frac{3}{4} \div \frac{2}{5}$.	**118.** $2\frac{3}{4} \div \frac{2}{3}$.	**124.** $5\frac{1}{6} \div 7\frac{1}{8}$.
113. $\frac{5}{8} \div \frac{6}{11}$.	**119.** $3\frac{1}{4} \div \frac{3}{4}$.	**125.** $7\frac{1}{8} \div 5\frac{1}{6}$.
114. $\frac{7}{9} \div \frac{9}{10}$.	**120.** $5\frac{1}{8} \div \frac{3}{8}$.	**126.** $10\frac{1}{2} \div 2\frac{1}{10}$.
115. $\frac{3}{16} \div \frac{3}{32}$.	**121.** $6\frac{1}{4} \div \frac{5}{7}$.	**127.** $2\frac{1}{10} \div 10\frac{1}{2}$.
116. $\frac{8}{15} \div \frac{4}{5}$.	**122.** $8\frac{7}{16} \div \frac{3}{8}$.	**128.** $\frac{14}{3} \div \frac{13}{4}$.

Second Method.

129. The dividend is $\frac{5}{6}$; the divisor is $\frac{4}{9}$; find the quotient.

ANALYSIS.

$1 \div \frac{1}{9} = 9.$
$1 \div \frac{4}{9} = \frac{1}{4} \times 9 = \frac{9}{4}.$
$\frac{1}{6} \div \frac{4}{9} = \frac{1}{6} \times \frac{9}{4} = \frac{9}{24}.$
$\frac{5}{6} \div \frac{4}{9} = 5 \times \frac{9}{24} = \frac{45}{24}.$
$\frac{5}{6} \div \frac{4}{9} = \frac{45}{24} = 1\frac{7}{8}.$

COMPARISON.

The reciprocal of $\frac{4}{9}$ is $\frac{9}{4}$. The dividend is $\frac{4}{9}$ of the quotient. The quotient is $\frac{9}{4}$ of the dividend. The dividend is $\frac{5}{6}$.

$\frac{9}{4}$ of $\frac{5}{6} = \frac{45}{24}$.
$\frac{45}{24}$ is the product of $\frac{9}{4} \times \frac{5}{6}$.
$\frac{45}{24}$ is the quotient of $\frac{5}{6} \div \frac{4}{9}$.

130. The dividend is $\frac{7}{8}$; the divisor is $\frac{5}{6}$; find the quotient.

131. What must $\frac{6}{7}$ be multiplied by to make $\frac{7}{8}$?

132. What part of $\frac{2}{3}$ is $\frac{3}{8}$?

133. The dividend is $\frac{5}{16}$; the divisor is $\frac{5}{8}$; find the quotient.

134. What must $13\frac{1}{2}$ be multiplied by to make $26\frac{3}{4}$?

135. What part of $62\frac{1}{2}$ is $12\frac{1}{2}$?

136. How many times does $\frac{15}{16}$ contain $\frac{5}{32}$?

137. How many times does $\frac{9}{25}$ contain $\frac{9}{75}$?

138. The dividend is $3\frac{1}{3}$; the divisor is $6\frac{2}{3}$; find the quotient.

139. What part of $\frac{32}{75}$ is $\frac{6}{25}$?

140. What part of $37\frac{1}{2}$ is $6\frac{3}{4}$?

141. What must $\frac{4}{9}$ be multiplied by to make $\frac{5}{6}$?

142. The dividend is $\frac{17}{18}$; the divisor is $\frac{18}{17}$; find the quotient.

143. What part of $\frac{1}{6}$ is $\frac{1}{9}$?

144. Find the number of times $\frac{15}{16}$ contains $\frac{8}{15}$.

145. What part of $\frac{15}{16}$ is $\frac{8}{15}$?

146. The dividend is $100\frac{2}{3}$; the divisor is $\frac{1}{3}$; find the quotient.

147. What part of $87\frac{1}{2}$ is $18\frac{3}{4}$?

148. The dividend is $66\frac{2}{3}$; the divisor is $3\frac{1}{3}$; find the quotient.

149. The dividend is $33\frac{1}{3}$; the divisor $6\frac{2}{3}$; find the quotient.

150. What part of $\frac{7}{8}$ is $\frac{3}{4}$?

151. Find the quotient of $\frac{17}{18} \div \frac{4}{7}$.

152. The dividend is 100; the divisor is $12\frac{1}{2}$; find the quotient.

153. The dividend is $12\frac{1}{2}$; the divisor is 100; find the quotient.

154. The dividend is 100; the divisor is $37\frac{1}{2}$; find the quotient.

155. The dividend is 100; the divisor is $87\frac{1}{2}$; find the quotient.

156. The dividend is 100; the divisor is $66\frac{2}{3}$; find the quotient.

157. The dividend is 100; the divisor is $16\frac{2}{3}$; find the quotient.

158. The dividend is $33\frac{1}{3}$; the divisor is 100; find the quotient.

Find the quotients:

159. $66\frac{2}{3} \div 33\frac{1}{3}$.	**164.** $16\frac{2}{3} \div 66\frac{2}{3}$.	**169.** $83\frac{1}{3} \div 6\frac{1}{4}$.
160. $87\frac{1}{2} \div 16\frac{2}{3}$.	**165.** $56\frac{1}{4} \div 6\frac{3}{4}$.	**170.** $62\frac{1}{2} \div 18\frac{3}{4}$.
161. $93\frac{3}{4} \div 6\frac{1}{4}$.	**166.** $1\frac{1}{4} \div 12\frac{1}{2}$.	**171.** $\frac{15}{16} \div 18\frac{3}{4}$.
162. $6\frac{1}{4} \div 93\frac{3}{4}$.	**167.** $3\frac{3}{4} \div 1\frac{1}{3}$.	**172.** $33\frac{1}{3} \div 56\frac{1}{4}$.
163. $37\frac{1}{2} \div 87\frac{1}{2}$.	**168.** $31\frac{1}{4} \div 56\frac{1}{4}$.	**173.** $6\frac{1}{4} \div 93\frac{3}{4}$.

124. To divide fractions:

When the divisor is concrete:

Regard both dividend and divisor as abstract numbers and multiply the dividend by the reciprocal of the divisor; the product will be equal to the abstract quotient required. (See **53**.)

When the divisor is abstract:

Multiply the dividend by the reciprocal of the divisor.

REVIEW PROBLEMS.

1. Find the product of $\frac{3}{4}$, $\frac{1}{2}$, $\frac{1}{10}$, and $\frac{1}{8}$.
2. Find the difference of $\frac{2}{3} \times \frac{1}{4}$ and $\frac{1}{3} \times \frac{1}{2}$.
3. Add $\frac{7}{8}$, $\frac{5}{6}$, $\frac{4}{5}$, $\frac{9}{10}$; add $\frac{5}{9}$, $\frac{3}{8}$, $\frac{4}{7}$, $\frac{2}{3}$.
4. What part of $18\frac{3}{4}$ is $6\frac{1}{4}$?
5. Find the sum of $\frac{5}{8}$ of $\frac{7}{9}$ and $\frac{6}{7}$ of $\frac{7}{8}$.
6. The dividend is $49{,}019\frac{1}{6}$; the divisor is $16{,}339\frac{13}{18}$; find the quotient.

7. The product is $\frac{1}{18}$; one factor is $301\frac{1}{6}$; find the other factor.

8. What part of $\frac{15}{16}$ is $\frac{3}{4}$? of $\frac{2}{3}$ is $\frac{5}{8}$?

9. Find the factor that with $\frac{3}{4}$ will produce $\frac{7}{8}$.

10. Find the quotient of the expression $\frac{1}{2}$ of $\frac{7}{8} \div 5$.

11. What part of $\frac{7}{8}$ is $\frac{3}{4}$? of $\frac{7}{8}$ is $\frac{5}{8}$?

12. Find the quotient of $\frac{7}{8}$ of $\frac{9}{10} \div \frac{1}{6}$ of 3.

13. What must $\frac{1}{2}$ of $\frac{7}{8}$ be multiplied by to make $\frac{14}{16}$?

14. Find the quotient of $\frac{3}{16}$ of $\frac{7}{8} \div \frac{2}{3}$ of $\frac{5}{12}$.

15. What part of $133\frac{1}{3}$ is 100?

16. What part of $166\frac{2}{3}$ is $66\frac{2}{3}$?

17. What part of 400 is $133\frac{1}{3}$?

18. Find the value of $\frac{3}{4} \div \frac{1}{2} \times 5$.

19. Find the value of $\frac{7}{8} \div \frac{3}{4} \times \frac{6}{7}$.

{ First multiply; then divide. }

20. The dividend is $\frac{3}{9} \times \frac{3}{8}$; the divisor is $\frac{1}{9} \times \frac{5}{8}$; find the quotient.

21. What part of one thousand is $66\frac{2}{3}$?

22. Dividend, $\frac{4}{7} \times \frac{5}{9}$; divisor, $\frac{3}{8} \times \frac{2}{3}$; quotient = ?

23. Dividend, $\frac{5}{8} \times \frac{3}{7}$; divisor, $\frac{1}{16} \times \frac{2}{3}$; quotient = ?

24. Dividend, $\frac{2}{3} \div \frac{3}{4}$; divisor, $\frac{6}{7} - \frac{1}{8}$; quotient = ?

25. Product, $\frac{7}{8}$; multiplicand, $\frac{1}{8}$ of 6; multiplier = ?

26. Minuend, $\frac{5}{7} \div 16$; subtrahend, $\frac{1}{8} \times \frac{1}{7}$; difference = ?

27. What part of five thousand is $1{,}666\frac{2}{3}$?

28. What part of one thousand is $133\frac{1}{3}$?

29. What part of one hundred is $3\frac{1}{3}$?

30. Dividend, $6\frac{1}{8} \div \frac{3}{8}$; divisor, $5\frac{1}{9} \div \frac{3}{7}$; quotient = ?

31. Dividend, $7\frac{3}{4} \div 6\frac{2}{3}$; divisor, $7\frac{2}{3} \div 6\frac{3}{4}$; quotient = ?

32. Product, $\frac{4}{7} \times \frac{5}{9}$; multiplicand, $\frac{1}{8} \times \frac{3}{7}$; multiplier = ?

33. Multiplier, $\frac{4}{9} \times \frac{5}{7}$; multiplicand, $\frac{1}{4} \times \frac{3}{7}$; product = ?

34. What part of $\frac{1}{3} \times 100$ is $\frac{1}{3}$ of $33\frac{1}{3}$?

35. What part of $\frac{1}{6} \times 100$ is $\frac{1}{6}$ of $16\frac{2}{3}$?

36. Multiplier, $\frac{3}{4} \div \frac{1}{8}$; product, $\frac{1}{8}$; multiplicand = ?

37. Find the value of $\frac{15}{16} \div \frac{3}{8} \times \frac{1}{2}$.

38. Find the value of $17\frac{1}{2}$ bushels of corn at \$$\frac{3}{5}$ per bushel.

39. Find the value of $21\frac{1}{2}$ bushels of wheat at \$$\frac{4}{5}$ per bushel.

40. What number is equal to $\frac{1}{3}$ of $\frac{1}{4} \div \frac{1}{4}$ of $\frac{2}{3}$?

41. Dividend, $16\frac{1}{2} \div 8$; divisor, $15\frac{1}{2} \div 7$; quotient = ?
42. Divisor, $2\frac{1}{6} \times 2$; quotient, $7\frac{1}{8} \div 3\frac{1}{4}$; dividend = ?
43. How many fifths are in $30 \times 18\frac{1}{10}$?
44. Multiplier, $\frac{2}{9} \times \frac{5}{8}$; multiplicand, $\frac{3}{4}$ of \$75; product = ?
45. Divisor, $\frac{1}{6} \times \frac{7}{8}$; dividend, $\frac{2}{9}$ of 7; quotient = ?
46. Dividend, $\frac{3}{8} \times 40$; quotient, $\frac{3}{16} \times \frac{3}{8}$; divisor = ?
47. How many eighths are in $40 \times 3\frac{1}{8}$?
48. What part of 25×25 is $3\frac{1}{8}$?
49. What part of 5^3 is $12\frac{1}{2}$?
50. Find a number equal to $5^5 \div \frac{1}{5} \times 5^2$.
51. Divisor, $\frac{3}{8} \times \frac{2}{3} \times \frac{1}{9}$; quotient, $\frac{3}{7} \times \frac{21}{35}$; dividend = ?
52. Dividend, $\frac{3}{4} \times \frac{5}{8} \times \frac{1}{2}$; divisor, $\frac{5}{9} \times \frac{4}{5} \times \frac{6}{7}$; quotient = ?
53. Dividend, $\frac{2}{7} \times \frac{5}{8}$; divisor, $\frac{3}{8} - \frac{1}{16}$; quotient = ?

54. A farmer gives, twice a day, to each of his two horses $\frac{1}{8}$ bushel of corn; how long will $8\frac{1}{2}$ bushels of corn last?

55. Find the sum of the sum and the difference of $\frac{3}{4}$ and $\frac{7}{8}$.

56. A farmer harvested $17\frac{7}{8}$ bushels of wheat per acre from a field of $29\frac{3}{4}$ acres, and sold the crop @ \$$\frac{4}{5}$ per bushel. Find the amount of sales.

57. Find the sum of the sum and the difference of $\frac{7}{8}$ and $\frac{5}{16}$.

58. Find the cost of 17 bales of cotton, averaging $461\frac{3}{34}$ pounds to the bale, at \$.08$\frac{1}{4}$ per pound.

59. Find the sum of the sum and the product of $\frac{15}{16}$ and $\frac{31}{32}$.

60. Find the cost of $\frac{1}{4}$ bu. meal and $3\frac{1}{2}$ lb. bacon; meal selling at \$$\frac{4}{5}$ per bushel, and bacon @ 10¢ per pound.

61. Find the sum of the sum and the product of $\frac{5}{8}$ and $\frac{15}{16}$.

62. In three fields there are $319\frac{1}{4}$ A.; in one there are $101\frac{3}{4}$ A.; in another $58\frac{7}{8}$ A.; how many acres in the third field?

63. Find the sum of the sum and the product of $\frac{3}{8}$ and $\frac{3}{16}$.

64. Counting 52 weeks to the year, and 5 school days to the week, what part of the year's time is a session of 40 weeks? *Ans.* $\frac{50}{91}$.

65. At 32 pounds to the bushel, what must be paid for 648 pounds of cotton seed @ \$$\frac{1}{5}$ per bushel?

66. At 32 pounds to the bushel, what must be paid for 1,620 pounds of oats @ \$$\frac{7}{20}$ per bushel?

67. Find the difference of the quotient and the product of the multiple $\frac{14}{15}$ and the factor $\frac{1}{80}$.

68. At 56 pounds to the bushel, what must be paid for 2,825 pounds of corn @ \$ $\frac{5}{8}$ per bushel?

69. Find the sum of the quotient and the product of the multiple $\frac{8}{9}$, and the factor $\frac{4}{27}$.

70. At 48 pounds to the bushel, what must be paid for 972 pounds of meal @ \$ $\frac{9}{20}$ per bushel?

71. Find the product of the quotient and the product of the multiple $\frac{3}{4}$ and the factor $\frac{3}{8}$.

72. If $\frac{3}{8}$ of a mile is 660 yd., what part of a mile is 990 yd.?

73. Find the product of the quotient and the sum of the dividend $2\frac{1}{2}$ and the divisor $\frac{7}{8}$.

74. Find the cost of $30\frac{5}{12}$ bushels of wheat @ \$ $\frac{4}{5}$ per bushel.

75. If the dividend were multiplied by $3\frac{1}{8}$, its divisor would be $\frac{3}{16}$; find the divisor.

76. Given: $2\frac{5}{8}$ miles from the depot; a train to arrive in 7 minutes; find the rate of speed required to meet the train.

77. In $\frac{1}{16}$ of a mile there are 110 yards; how many yards in $\frac{3}{4}$ of a mile?

78. If the multiplier were $1\frac{7}{8}$ times as great as it is, the product would be $\frac{3}{4}$; find the product.

79. Find the product of the sum and the product of $\frac{5}{16}$ and $\frac{5}{8}$.

80. Given: $33\frac{1}{2}$ acres, yielding $21\frac{1}{8}$ bu. wheat per acre. Required: Total yield in pounds, 60 pounds to the bushel.

81. Given: $46\frac{3}{8}$ acres, yielding $37\frac{1}{2}$ bushels of corn per acre. Required: Total yield in pounds, 56 pounds to the bushel.

82. What part of $\frac{3}{4}$ of a mile is $\frac{5}{8}$ of a mile?

83. There are 5,280 ft. in 1 mile; find the difference in feet between $\frac{3}{4}$ of a mile and $\frac{5}{6}$ of a mile.

84. If a cistern had contained twice as much water, and the water had been used only half as fast as it was used, the water would have lasted $3\frac{3}{7}$ weeks; how many days did the water last? *Ans.* 6 days.

85. If with 2 ploughs a field is worked in $3\frac{1}{2}$ days, in what time could it be worked with 3 ploughs?

86. In a field the rows of cotton are 4 feet apart; a hand hoes $1\frac{3}{16}$ acres per day for 6 days; how much would he have hoed if the rows had been 5 feet apart? *Ans.* $8\frac{29}{32}$ A.

87. In three weeks a man ploughed a field; the first week he ploughed $\frac{3}{8}$ of it; the second week $\frac{1}{4}$ of it; what part of the field did he plough the third week?

88. Mr. Thomas's farm contains 560 acres; $\frac{1}{8}$ of it is in corn, $\frac{3}{8}$ in oats, and $\frac{1}{7}$ in cotton; of the remainder $\frac{1}{3}$ is woodland, and $\frac{2}{3}$ pasture; find the size of the pasture.

89. What part of Mr. Thomas's farm is woodland?

90. Given: 24 lb. bagging and ties to cover 1 bale of cotton; 1,350 lb. seed-cotton to make 450 lb. lint-cotton; $35\frac{1}{2}$ acres yielding enough for 19 475-lb. bales; required: the average yield per acre in seed-cotton. *Ans.* $724\frac{10}{71}$ pounds.

91. Required: The weight of the cotton-seed produced by the $35\frac{1}{2}$ acres given in problem 90.

92. A bushel of oats weighs as much as $\frac{2}{3}$ of 1 bu. meal; 1 bushel of meal weighs as much as $\frac{6}{7}$ of 1 bu. corn; 1 bushel of corn weighs as much as $\frac{14}{15}$ of 1 bu. wheat; what part of a bushel of wheat equals the weight of a bushel of oats?

93. A miller keeps as toll $\frac{1}{7}$ of the corn to be ground. The toll a man pays equals in weight what part of the meal he receives? *Ans.* $\frac{1}{6}$.

94. The men could have hoed the field in $4\frac{1}{2}$ days; but they worked only $\frac{7}{8}$ of each day; in how many days did they hoe the field?

95. Required: The average product per acre of a field of $44\frac{1}{2}$ acres yielding a total of $923\frac{3}{8}$ bushels of wheat.

96. If $1\frac{7}{8}$ pounds of tea cost $1, what will 16 pounds cost?

97. If $\frac{5}{16}$ of a farm is in crops, and $\frac{5}{16}$ of the remainder is woodland, what part of the farm is woodland?

98. When beef sells at $12\frac{1}{2}$ cents, and pork at 15 cents, how much beef equals in cost 1 pound of pork?

99. Running 50 miles at 20 miles per hour a train loses $\frac{3}{4}$ of an hour; find the train's regular rate of speed per hour.

100. Find the product of the sum and the product of $\frac{1}{7}$, $\frac{3}{8}$, $\frac{2}{3}$.

DECIMAL FRACTIONS.

DEFINITIONS AND PRINCIPLES.

125. A fractional unit resulting from the division of the unit 1 by 10 or by any power of 10 is called a **Decimal Unit.**

The fractional units $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, etc., are decimal units; they may be expressed by the decimal scale thus: .1, .01, .001, etc.

126. A **Decimal Fraction** is one or more decimal units.

To express a decimal fraction, the fractional sign of division is not required; fifteen hundredths is usually expressed thus: .15.

127. The **Decimal Point** (.) distinguishes decimal fractions from integers.

But for the decimal point, 1 and 2 tenths (1.2) would be read *twelve.* A decimal point between an integer and a fraction, is read *and.*

128. The **Numerator** of a decimal fraction is the number of decimal units in the fraction.

The numerator of each of the fractions .1, .01, and .001 is 1.
The numerator of each of the fractions .09 and .009 is 9.

129. The **Denominator** of a decimal fraction is the power of 10 shown by the number of figures in the expression.

The denominator of .1 is 10; of .01 is 100; of .001 is 1000.
If the fraction is expressed with one figure, its denominator is the first power of 10; if it is expressed with two figures, its denominator is the second power of 10; and so on.

130. Any decimal fraction can be written in the form of a common fraction.

131. A common fraction whose unit is the result of the division of the unit 1 by a factor of a power of 10 can be reduced to a decimal fraction.

In the fraction $\frac{3}{4}$, the fractional unit $\frac{1}{4}$ is the result of the division of the unit 1 by 4; 4 is a factor of 100, which is the second power of 10. $\frac{3}{4}$ may be reduced to $\frac{75}{100}$, or .75.

NOTATION OF DECIMAL FRACTIONS.

132. The values of the orders of units written by the decimal scale decrease from left to right without regard to the decimal point. From the lowest integral order through successive higher orders the value of the unit increases by successive powers of 10. From the lowest integral order through successive lower orders the value of the unit decreases by successive powers of 10.

133. From the lowest integral order all higher and lower orders are derived.

The Lowest Integral Order is Units of Ones.

THE HIGHER ORDERS.		THE LOWER ORDERS.	
1st	tens.	tenths	1st.
2d	hundreds.	hundredths	2d.
3d	thousands.	thousandths	3d.
4th	ten-thousands.	ten-thousandths	4th.
5th	hundred-thousands.	hundred-thousandths	5th.
6th	millions.	millionths	6th.
7th	ten-millions.	ten-millionths	7th.
8th	hundred-millions.	hundred-millionths	8th.
9th	billions.	billionths	9th.
	etc., etc.	etc., etc.	

EXERCISES.

Express decimally:

1. A tenth; five tenths; seven tenths; nine tenths.

2. Twelve hundredths; eleven hundredths; one hundredth.

3. 125 thousandths; 25 thousandths; 5 thousandths.

4. 2,569 ten-thousandths; 569 ten-thousandths; 69 ten-thousandths.

5. 9 hundred-thousandths; 25 hundred-thousandths; 25 millionths.

6. Write in words .05; .06; .01; .015; .0155; .01675.

7. Write in words .105; .1005; .10005; .005; .0015; .0105.

8. Write in words 2.105; 2.015; 2.0105; 2.00105.

9. Write in words 123.123; 105.501; 510.015; .0001.

10. Write in words 201.1021; 2001.01; 10.00012; 2.10002.

Express decimally:

11. Two and two hundredths; three hundred-thousandths.

12. One and one thousandth; three millionths.

13. One and two millionths; two ten-millionths.

14. One and twenty-one millionths; two hundred one ten-thousandths.

15. Three and three hundred nine ten-millionths.

16. Three and three thousand nineteen ten-millionths.

17. Three and three thousand nineteen billionths.

18. Three and two hundred one hundred-millionths.

19. Express 109 hundred-thousandths.

20. Express 109 hundred-millionths.

Write in words:

21. .015	**26.** .01305	**31.** .000105	**36.** .0000105
22. .0125	**27.** .01035	**32.** .100015	**37.** .1000015
23. .0025	**28.** .03005	**33.** .010005	**38.** .0100005
24. .1025	**29.** .00005	**34.** .001005	**39.** .0010015
25. .2105	**30.** .10005	**35.** .010105	**40.** .00100105

134. To write a decimal fraction:

After the decimal point write the numerator with as many noughts prefixed as are required to make the number of figures in the expression equal to the number of noughts in the denominator.

REDUCTION.

135. Decimal to Common Fractions.

136. As the denominator of a decimal fraction is a power of 10, any factor used in reducing a decimal fraction to lowest terms must be a power of 10, or a factor of a power of 10.

If the numerator of the decimal fraction contains neither the factor 2 nor the factor 5, the fraction is in its lowest terms.

PROBLEMS.

1. Write .5 as a common fraction.

(i) .5 is $\frac{5}{10}$. This is not reduction. $\frac{5}{10}$ is the same number as .5.
(ii) $\frac{5}{10} = \frac{1}{2}$. This is reduction. Both terms have been divided by 5.

2. Reduce .75 to its lowest terms.

75 is not a factor of any power of 10. .75 is $\frac{75}{100}$.
25 is the H. C. F. of 75 and 100. $\frac{75}{100} = \frac{3}{4}$.

3. Reduce .125 to its lowest terms.

4. Find the lowest power of 10 that contains the factor 4. Reduce .04 to its lowest terms.

Reduce to lowest terms:

5.	.08	.16	.185	.54	.155
6.	.1025	.2015	.0125	.6015	.016
7.	.0034	.0134	.0505	.16015	.01615
8.	.96875	.00375	.00075	.15625	.625
9.	.3125	.4375	.0625	.9375	.0375

10. Reduce $.83\frac{1}{3}$ to its lowest terms.

As 3, the denominator of $\frac{1}{3}$, is not a factor of a power of 10, the expression cannot be written decimally. $.83\frac{1}{3}$ is called a *complex decimal*.

$.83\frac{1}{3}$ is $\frac{83\frac{1}{3}}{100} = \frac{250}{3} \div 100.$
$\frac{250}{3} \div 100 = \frac{250}{300} = \frac{5}{6}.$

11. Reduce $.33\frac{1}{3}$ to a common fraction.

12. Reduce $.66\frac{2}{3}$ to its lowest terms.

13. Reduce $.16\frac{2}{3}$ to its lowest terms.

14. Reduce $.555\frac{5}{9}$ to its lowest terms.

15. Reduce $.0\frac{2}{3}$ to its lowest terms.

16. Reduce $.93\frac{3}{4}$; $.06\frac{2}{3}$; $.08\frac{1}{3}$; $.43\frac{3}{4}$; $.06\frac{1}{4}$; $.12\frac{1}{2}$.

137. Common to Decimal Fractions.

First Method.

17. Reduce $\frac{1}{2}$ to a decimal fraction.

The new denominator will be the least power of 10 that contains the denominator 2 as a factor.

The first power of 10 is the new denominator.

$\frac{1}{2} \times \frac{5}{5} = \frac{5}{10}$, which is .5.
The number has been multiplied by a factor equal to 1.

18. Reduce $\frac{9}{50}$ to a decimal fraction.

The factors 2 and 5 are the only prime factors that can be used in reducing common to decimal fractions.

$\frac{9}{50} \times \frac{2}{2} = \frac{18}{100}$, or .18.

$(10 = 2 \times 5 : 10^2 = 2^2 \times 5^2 : 10^3 = 2^3 \times 5^3$, etc.)

19. Reduce $\frac{3}{4}$ and $\frac{3}{25}$ to decimal fractions.

It is evident that in either case 10^2 is the denominator required:

$10^2 = 2^2 \times 5^2$: as the denominator 4 is 2^2, it must be multiplied by 5^2; as the denominator 25 is 5^2, it must be multiplied by 2^2.

$$\frac{3}{4} \times \frac{5 \times 5}{5 \times 5} = \frac{75}{100}, \text{ or } .75.$$

$$\frac{3}{25} \times \frac{2 \times 2}{2 \times 2} = \frac{12}{100}, \text{ or } .12.$$

20. Reduce to decimal fractions $\frac{3}{5}$; $\frac{3}{40}$; $\frac{3}{16}$.

21. Reduce to decimal fractions $\frac{23}{40}$; $\frac{39}{80}$; $\frac{7}{8}$.

22. Reduce to decimal fractions $\frac{24}{125}$; $\frac{19}{32}$; $\frac{9}{16}$.

Second Method.

23. Reduce $\frac{7}{40}$ to a decimal fraction.

EXPLANATION.

$\frac{7}{40} = \frac{70}{400} = \frac{700}{4000} = \frac{700}{4000} \div \frac{4}{4} = \frac{175}{1000}$, or .175.

Cancelling the common factors of 700 and 4000, reduces $\frac{700}{4000}$ to $\frac{175}{1000}$, which is a decimal fraction; the factors cancelled are 2 and 2.

The same result is obtained by dividing the numerator by the denominator, hence:

$$\frac{700}{4000} = \frac{7.00}{40.00}$$

$$\frac{7.00}{40.00} = \frac{7.00}{40}$$

$$\frac{7.00}{40} = \frac{.700}{4}$$

$$4\overline{).700}$$
$$.175$$

138. To reduce a common fraction to a decimal:

Divide the numerator by the denominator.

24. Reduce $\frac{9}{20}$ to a decimal fraction.

25. Express $5\frac{3}{20}$ as a mixed decimal number. *Ans.* 5.15.

26. Express $4\frac{7}{20}$ as a mixed decimal number.

27. Express $48\frac{9}{16}$ as a mixed decimal number.

28. Express $50\frac{1}{3}$ as a mixed decimal number. *Ans.* 50.3333+.

As the denominator of $\frac{1}{3}$ is not a factor of any power of 10, no exact decimal will express the fraction; carried to four places, it approximates its exact value.

29. Reduce $\frac{2}{3}$ to an approximate decimal fraction.

30. Reduce to approximate decimal fractions $\frac{3}{7}$; $\frac{8}{9}$; $\frac{5}{6}$; $\frac{1}{12}$.

31. Reduce to approximate decimal fractions $\frac{11}{12}$; $\frac{2}{17}$; $\frac{3}{11}$; $\frac{4}{9}$; $\frac{2}{13}$.

139. ADDITION OF DECIMAL FRACTIONS.

PROBLEMS.

1. Add 75 cents, 3 dollars, $4\frac{1}{4}$ dollars, 15 cents, and 5 cents,

EXPLANATION.

The fractions are reduced to fractions having a common denominator while being written in columns for addition. 75 cents is 75 hundredths of a dollar; \$3 is \$3 and no hundredth; \$$4\frac{1}{4}$ is \$4 and 25 hundredths of a dollar; 15 cents is 15 hundredths of a dollar, and 5 cents is 5 hundredths of a dollar. The adding is done as with integers.

\$ .75
3.00
4.25
.15
.05
\$8.20

2. Add five and one hundredth, six and one tenth, seven and one thousandth, and eight and one ten-thousandth.

In adding abstract decimal fractions it is not necessary to reduce the fractions to common decimal units; but each column must consist of fractions that have a common decimal unit; tenths are added to tenths, hundredths to hundredths, and so on. The mere reading of the sum does, in effect, reduce the different fractions composing the sum to common decimal units.

5.01
6.1
7.001
8.0001
26.1111

3. Add .1001, 20.01, 4.0015, 7.10705, 3.1.

4. Add 202, 5.05, 71.051, 5.75651, 2.15.

5. Add 7, .07, .7, .175, 2.175155, 1.05.

6. Add 103.1, 30.105, 3.625, 3.16625, and 3.

7. Add 5.5, 7.051, 1.0005, 3.105, 155.5.

8. Add 75, 18.5, 3,910.0625, 7.875, 5.375.

140. SUBTRACTION OF DECIMAL FRACTIONS.

PROBLEMS.

1. Find the difference between \$5 and 5 cents.

The minuend is expressed as 5 and no hundredth dollars; the subtrahend is 5 hundredth-dollars; the difference is \$4 and 95 hundredths of a dollar, or \$4.95.

$$\begin{array}{r} \$5.00 \\ .05 \\ \hline \$4.95 \end{array}$$

2. The minuend is .4; the subtrahend is .004; find the difference.

The subtrahend is thousandths; the minuend is reduced to thousandths. 4 tenths = 400 thousandths; annexing ciphers does not alter the value of a decimal fraction.

$$\begin{array}{r} .400 \\ .004 \\ \hline .396 \end{array}$$

3. Find the difference of .55 and .3956.

One of the units of hundredths in the minuend is reduced to 9 thousandths and 10 ten-thousandths, from which the 5 thousandths and 6 ten-thousandths of the subtrahend are readily subtracted. The result is found still more readily by Left-hand Subtraction. (See page 13.)

$$\begin{array}{l} .55 \\ .3956 \\ \hline \end{array}$$

4. Find the difference of 17.015 and 15.0075.
5. Express as a decimal fraction the difference of $\frac{3}{4}$ and .755
6. Find the difference of $\frac{7}{8}$ and 876 thousandths.
7. Find the difference of $\frac{1}{8}$ and 124 thousandths.
8. Find the difference of $\frac{3}{8}$ and 4 tenths.
9. Find the difference of $12\frac{3}{8}$ and 12.376.
10. Find the difference of $66\frac{1}{4}$ and 70.25.
11. Find the difference of 3.105 and 1.000315.

141. MULTIPLICATION OF DECIMAL FRACTIONS.

PROBLEMS.

1. Find the product of $\frac{1}{10}$ and $\frac{1}{10}$.
2. Find the product of $\frac{1}{10}$ and $\frac{1}{100}$.
3. Find the product of $\frac{1}{10}$ and $\frac{1}{1000}$.

4. Find the product of $2\frac{3}{10}$ and $2\frac{13}{100}$.

COMMON FRACTION PROCESS.

$2\frac{3}{10} = \frac{23}{10} : \frac{23}{10} \times \frac{213}{100} = \frac{4899}{1000}$.

$2\frac{13}{100} = \frac{213}{100} : \frac{4899}{1000} = 4\frac{899}{1000}$.

DECIMAL PROCESS.

$$\begin{array}{r} 2.13 \\ \underline{2.3} \\ 639 \\ \underline{426} \\ 4.899 \end{array}$$

142. The product of decimal fractions contains as many decimal places or figures as both factors contain.

The successive steps of the decimal process show this still more clearly. The 1st, 2d, and 4th steps show that sometimes it is necessary to prefix ciphers.

		2.13	
		2.3	
1st . . .	$.3 \times .03 = .009$ . . .	$.009$	$= \frac{3}{10} \times \frac{3}{100}$.
2d . . .	$.3 \times .1 = .03$ . . .	$.03$	$= \frac{3}{10} \times \frac{1}{10}$.
3d . . .	$.3 \times 2. = .6$ . . .	$.6$	$= \frac{3}{10} \times 2.$
4th . . .	$2 \times .03 = .06$ . . .	$.06$	$= 2 \times \frac{3}{100}$.
5th . . .	$2 \times .1 = .2$ . . .	$.2$	$= 2 \times \frac{1}{10}$.
6th . . .	$2 \times 2. = 4.$ . . .	$4.$	$= 2 \times 2.$
		4.899	$= 2.3 \times 2.13$.

5. Find the product of 5.105 and 6.01.

6. Multiply 1.25678 by 10; by 100; by 1000.

143. *Each removal of the decimal point in a number one place to the right multiplies the number by 10.*

7. Find the cost of 60.25 bu. wheat at \$.85 per bushel.

8. Find the cost of 70.75 bushels of oats at \$.35 per bushel.

Find the products of:

9. 7.875 and 9.7075.

10. 10.001 and 10.1.

11. 10.01, 10.001, and 9.155.

12. 7.75, 8.875, and 19.125.

13. 1.105, 7.175, and 3.25.

14. .009, .09, and .9.

15. .19, 1.29, and 179.

16. 2.55, 2.5, and $\frac{1}{4}$.

144. DIVISION OF DECIMAL FRACTIONS.

PROBLEMS.

1. A farmer paid \$225 for 10 acres of land; find the cost per acre.

2. A broker paid $9,825 for 100 shares of railroad stock; find the cost per share.

145. Each removal of the decimal point in a number one place to the left divides the number by 10.

3. Divide 1 by $\frac{1}{10}$; $\frac{1}{10}$ by $\frac{1}{10}$; $\frac{1}{10}$ by $\frac{1}{100}$; $\frac{1}{10}$ by $\frac{1}{1000}$.

4. Prove by division that $2.3 \times 2.13 = 4.899$.

5. Prove by division that $2.3 \times 2.1 = 4.83$.

6. One factor is 3.007; the other factor is 1.07; find the product.

7. Multiply 5.45 by .016, and prove the work by dividing the product by the multiplicand.

$$\begin{array}{r} 5.45 \\ \underline{.016} \\ 3270 \\ \underline{545} \\ .08720 \end{array} \qquad\qquad \begin{array}{l} 5.45).08720(.016 \\ \underline{545} \\ 3270 \\ \underline{3270} \end{array}$$

146. The dividend contains as many decimal places or figures as both factors contain; hence, *The quotient contains as many decimal places as the number of decimal places in the dividend exceeds the number of decimal places in the divisor.*

If both dividend and divisor are multiplied or divided by the same number, the quotient is not affected. (See 5 and 6, Art. 102.)

8. Divide .0872 by 5.45.

Remove the decimal point from the divisor in order to make the divisor a whole number.

$\frac{.0872}{5.45} \times \frac{100}{100} = \frac{8.72}{545}$, *or* to multiply both dividend and divisor by 100, remove the decimal point in each two places to the right.

545)8.720(.016

To divide decimals by decimals:

Divide as in whole numbers.

Point off in the quotient as many decimal places as the number of decimal places in the dividend exceeds the number of decimal places in the divisor, prefixing noughts when necessary.

9. Find the quotient of $780.516 \div 2.43$.
10. Find the quotient of $7.25406 \div 3.19$.
11. Find the quotient of $.034016 \div 1{,}063$.
12. Find the quotient of $34{,}016 \div 10.63$.
13. Find the quotient of $10.9536 \div 22.82$.
14. Find the quotient of $107.648 \div 5.8$.

Find the quotients of:

15. $9.675 \div 22.5$.
16. $294{,}630 \div 8.54$.
17. $3{,}375 \div 280.25$.
18. $8.7796 \div 4.25$.
19. $5.4768 \div 228.2$.
20. $6 \div 150$.
21. $3 \div .75$.
22. $4 \div .005$.
23. $12 \div .875$.
24. $33 \div .6$
25. $25 \div .25$.
26. $50 \div .005$.
27. $.75 \div 1{,}500$.
28. 87.5 by $\frac{7}{8}$.
29. 99 by $.0003$.

MISCELLANEOUS PROBLEMS.

1. Reduce to a decimal fraction $\frac{2}{3}$ of $\frac{5}{6}$.
2. Reduce to a common fraction $.9175$.
3. Reduce to a mixed decimal $2 \times \frac{7}{8} \times \frac{3}{4}$.
4. Express decimally $3 \times \frac{2}{5} \times \frac{1}{4}$.
5. Express as a mixed number 2.3×5.005.
6. Add $\frac{1}{8}$, $1\frac{1}{8}$, and 6.065; express the sum decimally.
7. Express decimally the difference of $\frac{3}{4}$ and $2\frac{1}{8}$.
8. Express decimally the quotient of $5 \div \frac{4}{7}$.
9. Express decimally the quotient of $8 \div \frac{5}{9}$.
10. Express decimally the quotient of $25 \div \frac{4}{5}$.
11. Express decimally the quotient of $6\frac{3}{4} \div 9$.
12. Express decimally the quotient of $\frac{18}{25} \div 9$.
13. Express decimally the quotient of $67\frac{3}{8} \div 9\frac{4}{5}$.
14. Express decimally the quotient of $27.72 \div 12\frac{5}{6}$.
15. Express decimally the product of 8.4 and $5\frac{4}{7}$.
16. Express decimally the sum of $\frac{3}{8}$, $\frac{3}{4}$, $\frac{1}{2}$, $\frac{1}{16}$.
17. Express decimally the product of $1\frac{3}{8}$ and $1\frac{7}{8}$.
18. Express decimally the quotient of $\$5.05 \div \frac{2}{5}$.
19. Express decimally the product of $\$10.25$ and $\frac{3}{4}$.
20. Express decimally the quotient of $1.8 \div 33\frac{1}{3}$.
21. Express decimally the difference of $\frac{7}{12}$ and $\frac{5}{6}$.
22. Express decimally the sum of $.01$, $\frac{2}{5}$, $1\frac{3}{4}$, and $.005$.

REVIEW PROBLEMS.

1. Find the sum of the prime factors of 840.

2. Find the sum of the composite factors of 36.

3. Find the cost of 8 bales of cotton, averaging 465 pounds, @ \$.08$\frac{1}{8}$.

4. Find the decimal fraction which equals $\frac{59}{64} - \frac{7}{8}$.

5. Find the cost of 10$\frac{7}{8}$ bushels of wheat @ \$.85.

6. A lot cost $\frac{1}{8}$ as much as the house upon it; the house and lot cost \$4,000; find the cost of the lot.

7. Find the cost of 16,370 ft. pine lumber @ \$15 per M.

8. If 3$\frac{1}{2}$ bu. oats cost \$1.05, what should 2 bushels cost?

9. Find the sum of all the numbers less than 100, each of which is divisible by 9. *Ans.* 594.

10. The multiplier is 11; the multiplicand is $\frac{1}{2}$ of the sum of 99 and 9; find the product.

11. The corn produced on an acre of land was sold for \$22.95; the fodder was sold for $\frac{1}{9}$ as much as the corn; and the shucks were sold for $\frac{1}{3}$ as much as the fodder; find the total proceeds.

12. D bought 94 M. shingles for \$352.50; find the cost per thousand.

13. Find the cost of 38 cords of wood @ \$5.75 per cord.

14. A farmer sold 2 bales of cotton, differently classed, for \$78.85; for one bale weighing 540 pounds he received \$.08$\frac{1}{4}$ per pound; for the other bale he received \$.07 per pound; how much did it weigh? *Ans.* 490 pounds.

15. At 1$\frac{3}{7}$ bushels of seed to the acre, how many bushels of oats will be required to sow 14$\frac{2}{5}$ acres?

16. At $\frac{3}{16}$ bushel of seed to the acre, how many acres can be planted with 51 bushels of corn?

17. A farmer exchanged with a neighbor 50 bushels of oats for corn, giving 7 bushels of oats for 4 bushels of corn; how many bushels of corn did he get? *Ans.* 28$\frac{4}{7}$ bushels.

18. Find the difference in weight of 38$\frac{1}{2}$ bu. corn at 56 lb. to the bushel and 67$\frac{3}{8}$ bu. oats at 32 lb. to the bushel.

19. If 7 bushels of oats is worth 4 bushels of corn, and 20 bushels of corn is worth 15 bushels of wheat, how much oats should equal in value 20 bushels of wheat? *Ans.* $46\frac{2}{3}$ bushels.

20. Find the difference in cost of $38\frac{1}{2}$ bushels of corn @ $.45 and $67\frac{3}{8}$ bushels of oats @ $.32$\frac{1}{2}$.

21. Into how many lots, each to consist of $\frac{7}{16}$ of an acre, could 112 acres of land be divided?

22. A tract of 2,561 acres of land was purchased @ $2.50 per acre; it was then cut up into 13 farms of equal size, and sold @ $18.75 per acre; find the selling price of one of the farms. *Ans.* $3,693.75.

Find the profit on the entire tract. *Ans.* ——.

23. Mr. White, ginning cotton for a toll of $\frac{1}{14}$, received toll in one week amounting, when sold @ $.08$\frac{1}{2}$, to $153; how many bales averaging 450 pounds did Mr. White gin that week? *Ans.* 56 bales.

24. If a Mexican dollar is worth $$\frac{17}{20}$, how many Mexican dollars equal the value of $680 in U. S. money?

25. If, where the cotton rows are 4 feet apart, a man in 12 hours can plough four acres, 2 furrows to the row, in how many hours could he plough 2 furrows in 5-foot rows on four acres? *Ans.* $9\frac{3}{5}$ hours.

26. If a load for 6 yoke of oxen is 7,500 pounds, what should be a load for 8 yoke of oxen?

27. Last year a field of $31\frac{1}{4}$ acres yielded per acre $24\frac{1}{2}$ bushels of wheat, which was sold @ $.95; this year the yield was $\frac{7}{8}$ as great, and the price $\frac{9}{10}$ as much; find the value of this year's crop.

28. If the land that produces a bale of cotton yields 30 bu. cotton-seed, what is the value @ $.20 per bushel of the cotton-seed produced by the land that yields 21 bales of cotton?

29. If $\frac{7}{16}$ of a rice-planter's crop sells for $15,400, what is the value of the remainder at the same price per pound?

30. A ranch comprising 170,000 acres cost in 1880 $190,400, which was $\frac{4}{7}$ as much as it was sold for in 1890; find the price paid per acre in 1890.

31. A dealer paid $16.80 for some coffee, which, proving to be damaged, he sold for $\frac{5}{8}$ of its cost, losing thereby $.06 per pound; find the weight of the coffee.

32. Shelled corn weighs 56 pounds to the bushel; corn in the ear weighs $1\frac{1}{4}$ times as much; find the weight of the cobs from which $35\frac{1}{2}$ bushels of corn was shelled.

33. Shelled corn weighs $\frac{7}{9}$ as much as corn in the shuck; find the weight of the shucks from 25 bushels of corn.

34. If there are 3,109 four-penny nails in 12 pounds, how many four-penny nails make 60 pounds?

35. Find the cost of 7 bales of wool, averaging 308 pounds, @ $.28½ per pound.

36. If 42 barrels resin weigh 7 tons, how many tons will 49 barrels weigh?

37. In 4-foot cotton rows, a stalk to every 12 inches in the row, there are 10,890 stalks to the acre; find the number of stalks to the acre in 5-foot rows, a stalk to every 8 inches in the row. *Ans.* 13,068 stalks.

38. Find the cost @ $.06¾ apiece, of the 2-bushel bags that contain the crop of 16 acres yielding $22\frac{1}{2}$ bushels of wheat per acre. *Ans.* $12.15.

39. A quarter-barrel of flour is 49 pounds; find the weight of 16 barrels of flour.

40. When wheat is selling at $.75, and corn at $.60, how many bushels of corn are equal in value to 40 bushels of wheat?

41. How much meal @ $.55 per bushel can a man buy with a week's wages, when he earns $1.50 per day?

42. If 3 yards of yard-wide cloth cost $.90, what should be the cost of 4 yards of cloth of the same kind, but 2 yards wide? *Ans.* $2.40.

43. A butcher sold the fore-quarters of a beef, each weighing $77\frac{1}{2}$ pounds, @ $.08½ per pound; find the amount he received. *Ans.* $13.17 +.

44. Find the cost, at $.87½ per hundred, of the rails required for building a fence 12 rails high and 300 panels long. *Ans.* $31.50.

45. A sugar plantation was sold for $72,000, which was $\frac{8}{9}$ of the owner's estimate of its value; find the owner's estimate.

46. Find the freight charges on 2,125 pounds of goods shipped from New York to New Orleans @ $.78 per hundred pounds.

47. On similar acres a farmer tested two kinds of fertilizers: on acre #1 he put 300 pounds of fertilizer A at a cost of $.01$\frac{1}{4}$ per pound; on acre #2 he put 300 pounds of fertilizer B at a cost of $.02$\frac{1}{4}$ per pound; find the difference in the cost of fertilizing the two acres.

48. Acre #1 yielded 256 pounds and acre #2, 300 pounds of lint-cotton which sold at $.08$\frac{1}{2}$; counting the cotton-seed worth the expense of picking, etc., which fertilizer paid more, and how much more?

49. If the cotton had sold at $.06$\frac{3}{4}$, which fertilizer would have paid more, and how much more? *Ans*. A; $.03.

50. From a field of 76 acres in corn, $\frac{1}{3}$ of the crop was sold @ $.40 per bushel, and brought $119.70; find the yield per acre.

51. From a field yielding 27$\frac{1}{2}$ bushels of wheat per acre, $\frac{3}{4}$ of the crop was sold at $.72 per bushel, and brought $475.20; find the number of acres in the field.

52. If 1 day's work give $\frac{3}{22}$ of a field its first ploughing, in how many more days can the first ploughing of the field be completed? *Ans*. 6$\frac{1}{3}$ more days.

53. A speculator sold a lot for $\frac{7}{8}$ of the amount he paid for it; if he had sold it for $200 more, he would have gained $60; what did he pay for the lot?

54. A merchant bought coffee @ 18¢ per pound; in selling it he received as much for 6 pounds as he paid for 8 pounds; find the selling price.

55. A dealer bought 12 boxes of oranges for $33; 2 boxes spoiled; at what price per box must he sell the others in order to gain $.25 per box on the 12 boxes?

ADDITIONAL REVIEW PROBLEMS: Pages 48 and 49.

FOR ORAL WORK.

Review. — *Induction.*

1. What is a factor? A multiple? A power?

2. Show the difference between a product and a sum. Show the difference between a multiple and a power.

3. Could 3,597 be raised to the 100th power by addition?

4. Show the difference between multiplication and addition.

5. Could the prime factors of $3{,}597^{100}$ be found by subtraction?

6. Show the difference between subtraction and division.

7. Can 3 dollars be multiplied by 3 dollars? Why?

8. Show the difference between an abstract number and a concrete number.

9. Can 3 tenths be multiplied by 3 tenths? Why?

10. What is a reciprocal? Give examples.

11. What is a common fraction? A decimal fraction?

12. What is an integral unit? A fractional unit? A unit?

13. What is the unit of grain measure?

14. What is the unit of cloth measure?

15. Give three units of time measure.

16. Give two natural units of time measure.

Ex. 1. **432 = 4 hundreds, 3 tens, 2 units.**
Ex. 2. **432 = 4 bushels, 3 pecks, 2 quarts.**

17. Is it necessary to write abstract numbers as in example 1, with the name of each of the orders of units? Why?

18. Is it necessary to write units of measure as in example 2, with the name of each of the orders of units? Why?

19. Why cannot units of measure be written on the same scale, as in decimal notation?

20. What number of pecks makes a bushel? What quantity of corn can be bought for $ 1, when a peck sells for 25 cents?

21. What quantity of corn equals one-fourth of the quantity of corn that fills a bushel-measure?

22. If a peck of potatoes weigh 15 pounds, what is the weight of $2\frac{3}{4}$ bushels of potatoes?

COMPOUND NUMBERS.

MEASURES.

DEFINITIONS AND PRINCIPLES.

147. Measures are Concrete Units of Quantity.

148. A quantity consisting of several quantities, each of which has a different concrete unit, is called a **Compound Quantity**.

4 bu. 3 pk. is a compound quantity: the unit of the first quantity is the bushel; the unit of the second quantity is the peck; written together they express one quantity, but seven units of quantity of two different orders, each of the units called bushels being 4 times as great as each of the units called pecks.

149. A **Compound Number** is a number of concrete units of two or more orders whose higher units are not constant multiples of lower units.

$5.55 is a compound quantity, but not a compound number; it is composed of different concrete units (cents, dimes, and dollars), but the higher orders of its units are constant multiples of lower orders; its scale does not vary.

6 bu. 3 pk. 5 qt. is a compound number; its scale varies, and is therefore called a compound, or mixed, scale; 1 unit of the highest order equals 4 units of the order of pecks, while 1 unit of the order of pecks equals 8 units of the order of quarts.

150. The operations of compound numbers are **Reduction, Addition, Subtraction, Multiplication,** and **Division.**

The principles of compound numbers are the same as those which govern simple numbers.

TABLES OF COMPOUND NUMBERS.

ENGLISH MONEY.

151. English money is the currency of Great Britain. The measures are farthings, pence, shillings, and pounds.

TABLE.

4 farthings	= 1 penny (1*d.*).		
12 pence	= 1 shilling (1*s.*)		= 48 far.
20 shillings	= 1 pound (£1)	= 240*d.*	= 960 far.

A farthing is usually expressed $\frac{1}{4}d$.

DRY MEASURES.

152. The measures are pints, quarts, pecks, and bushels; they are used in estimating quantities of grain, fruit, and many other dry products.

TABLE.

2 pt.	= 1 qt.		
8 qt.	= 1 pk.		= 16 pt.
4 pk.	= 1 bu.	= 32 qt.	= 64 pt.

For convenience, as well as for greater accuracy in measuring, many dry commodities are sold by weight, so many pounds to the bushel (see **168**).

LIQUID MEASURES.

153. The measures in common use are gills, pints, quarts, and gallons. They are used in estimating quantities of liquids; such as water, oil, milk, wine, vinegar, etc.

TABLE.

4 gi.	= 1 pt.		
2 pt.	= 1 qt.		= 8 gi.
4 qt.	= 1 gal.	= 8 pt.	= 32 gi.

A barrel (bbl.) is $31\frac{1}{2}$ gal. A hogshead (hhd.) is 63 gal., or 2 bbl.

AVOIRDUPOIS WEIGHTS.

154. The measures in common use are ounces, pounds, hundredweight, and tons. They are used in estimating the weight of heavy articles.

TABLE.

16 oz.	= 1 lb.		
100 lb.	= 1 cwt.		= 1,600 oz.
20 cwt.	= 1 T.	= 2,000 lb.	= 32,000 oz.

The long ton (2,240 lb.) is used at the U. S. custom houses, and in weighing coal, iron, etc., at the mines.

TROY WEIGHTS.

155. The measures are grains, pennyweights, ounces, and pounds. They are used in coinage and in chemistry, and in finding the weight of jewels, precious stones, etc.

TABLE.

24 gr.	= 1 pwt.		
20 pwt.	= 1 oz.		= 480 gr.
12 oz.	= 1 lb.	= 240 pwt.	= 5,760 gr.

The Troy pound is the standard at the U. S. mints.

156. *Comparison of the Troy ounce and pound with the Avoirdupois ounce and pound.*

$437\frac{1}{2}$	Troy grains	= 1 oz. Avoirdupois.
480	Troy grains	= 1 oz. Troy.
1	oz. Avoirdupois	= $\frac{175}{192}$ oz. Troy.
7,000	Troy grains	= 1 lb. Avoirdupois.
5,760	Troy grains	= 1 lb. Troy.
1	lb. Troy	= $\frac{144}{175}$ lb. Avoirdupois.

APOTHECARIES' WEIGHTS.

157. The measures are grains, scruples, drams, ounces, and pounds. They are used only by physicians and apothecaries in prescribing and mixing dry medicines.

TABLE.

20 grains	= 1 scruple (℈).		
3 scruples	= 1 dram (ʒ); 1 ʒ		= 60 gr.
8 drams	= 1 ounce (℥); 1 ℥	= 24 ℈	= 480 gr.
12 ounces	= 1 pound (℔); 1 ℔ = 96 ʒ	= 288 ℈	= 5,760 gr.

The Apothecaries' pound equals the Troy pound; the Apothecaries' ounce equals the Troy ounce.

APOTHECARIES' FLUID MEASURES.

158. The measures are minims, fluid drachms, fluid ounces, pints, and gallons. They are used as measures for liquids in preparing medical prescriptions.

TABLE.

60 minims	= 1 fluid drachm (*f*ʒ).
8 fluid drachms	= 1 fluid ounce (*f*℥).
16 fluid ounces	= 1 pint (O).
8 pints	= 1 gallon (Cong.).

LINEAR MEASURES.

159. The measures are inches, feet, yards, rods (also called perches and poles), and miles. They are used in estimating distances, length, breadth, etc.

TABLE.

12 in. = 1 ft.			
3 ft. = 1 yd.			= 36 in.
$5\frac{1}{2}$ yd. = 1 rd.		= $16\frac{1}{2}$ ft.	= 198 in.
320 rd. = 1 mi.	= 1,760 yd.	= 5,280 ft.	= 63,360 in.

Other linear measures are: 1 degree of the equator = 69.16 mi.
1 hand = 4 in.; 1 fathom = 6 ft.; 8 furlongs = 1 mi.
880 fathoms = 1 mi.; 3 miles = 1 league.

SURVEYORS' LINEAR MEASURES.

160. The measures are inches, links, chains, rods or poles, and miles. They are used by surveyors and engineers in measuring roads, distances over land, etc.

TABLE.

7.92 in.	= 1 link.			
25 links	= 1 rd. or pole		=	198 in.
4 poles	= 1 chain	=	100 links =	792 in.
80 chains	= 1 mi. = 320 P.	=	8,000 links =	63,360 in.

The foregoing table is based upon Gunter's Chain, which has 100 links, and is 66 feet long. Another chain is also much used: it is 100 feet long, and is composed of 120 10-inch links.

SQUARE MEASURES.

161. The measures are square inches, square feet, square yards, square rods, acres, and square miles. They are used in estimating areas of land and other surfaces.

TABLE.

144 sq. in.	= 1 sq. ft.		
9 sq. ft.	= 1 sq. yd.	=	1,296 sq. in.
$30\frac{1}{4}$ sq. yd.	= 1 sq. rd.	=	$272\frac{1}{4}$ sq. ft.
160 sq. rd.	= 1 A. = 4,840 sq. yd.	=	43,560 sq. ft.
640 A.	= 1 sq. mi.		

A section of Government land = 640 A.; a township = 36 sq. mi.

SURVEYORS' SQUARE MEASURES.

162. The measures are square links, square chains, acres, etc. They are used by surveyors in estimating areas of land.

TABLE.

10,000 square links	= 1 square chain.
10 sq. ch.	= 1 acre.
640 acres	= 1 square mile.
36 sq. mi.	= 1 township.

CUBIC MEASURES.

163. The measures are cubic inches, cubic feet, cubic yards, and cords. They are used in estimating solids and capacities.

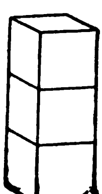
TABLE.

1,728 cu. in. = 1 cu. ft.
27 cu. ft. = 1 cu. yd. = 46,656 cu. in.
128 cu. ft. = 1 cord.

A cord of wood is a pile 8 ft. long, 4 ft. wide, 4 ft. high.

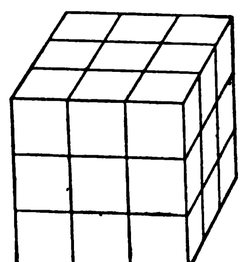
A **Cube** has three dimensions: length, breadth, and thickness. It is a solid body having six equal square surfaces. Cubic measure applies to volume rather than to form, the solid bodies and vessels, etc., whose capacities are sought, not necessarily having the form of cubes.

FIG. 1.



3 cubic feet.

FIG. 2.



27 cu. ft. = 1 cu. yd.

The column (Fig. 1.) shows three small cubes.

Each layer of the large cube (Fig. 2) shows 3 columns of small cubes; there are 3 layers of 3 columns of 3 cubes.

$$3 \times 3 \times 3 = 3^3 = 27.$$

CIRCULAR MEASURES.

164. The measures are seconds ($''$), minutes ($'$), degrees ($^\circ$), and signs (S); they are used in estimating latitude and longitude, direction, angles, etc.

TABLE.

60 seconds ($''$)	= 1 minute ($'$).
60 minutes	= 1 degree ($^\circ$).
30 degrees	= 1 sign (S).
360° or 12 signs	= 1 circumference of a circle.

TIME MEASURES.

TABLE.

165. 60 seconds = 1 minute.

60 minutes = 1 hour;	1 hr. =	3,600 sec.
24 hours = 1 day;	1 da. = 1,440 min. =	86,400 sec.

365 da. = 1 common year; 366 da. = 1 leap-year.

7 da. = 1 week; 12 months = 1 year; 100 yr. = 1 century.

In business transactions 30 da. = 1 mo.; 360 da. = 1 yr.

The time required for the earth to make its revolution around the sun is nearly 6 hours more than 365 days. This accounts for leap-year. If every year were accounted 365 days, in 4 years there would lack nearly 1 day, and in 400 years nearly 100 days of the constant accord of the year with the seasons; and the first of the year would begin in the early autumn. One day is therefore added to almost every fourth year. If the earth's annual revolution were performed in exactly $365\frac{1}{4}$ days, adding one day to every fourth year would forever preserve the harmony of the year with the seasons; but the fraction of the 366th day is less than $\frac{1}{4}$ day by about 11 min. $10\frac{8}{10}$ sec., or about $\frac{1}{33}$ of $\frac{1}{4}$ day, which would amount to about 3 days in 400 years. To offset this difference, there are in 400 years only 97, instead of 100, leap-years.

Except three in a hundred, every year whose number is exactly divisible by 4 is a leap-year. The number of a leap-year is a multiple of 4, and, if it be a multiple of 100, it is a multiple of 400 also. The last year of the 19th century will not be a leap-year; 1900 is a multiple of 4 and of 100, but it is not a multiple of 400. The years 1700, 1800, 1900, not being leap-years, the three days otherwise gained in 400 years are offset. The year 2000 will be a leap-year.

THE SEASONS.

Spring.		Autumn or Fall.	
March,	3rd month, 31 days.	September,	9th month, 30 days.
April,	4th month, 30 days.	October,	10th month, 31 days.
May,	5th month, 31 days.	November,	11th month, 30 days.
Summer.		**Winter.**	
June,	6th month, 30 days.	December,	12th month, 31 days.
July,	7th month, 31 days.	January,	1st month, 31 days.
August,	8th month, 31 days.	February,	2nd month, 28 days.

In a leap-year February has 29 days.

LONGITUDE AND TIME.

166. The earth's annual revolution was formerly supposed to require but 360 days; hence the division of the circumference of the circle into 360 equal parts called degrees. The earth's daily rotation upon its axis causes the sunlight to move over the 360 degrees of the earth's circumference in 24 hours; therefore, in 1 hour the sunlight traverses the space of $\frac{1}{24}$ of 360 degrees, which is 15 degrees. In 1 minute the sunlight traverses $\frac{1}{60}$ of 15 degrees, which is $\frac{1}{4}$ of 1 degree, or 15 minutes of longitude; in 1 second the sunlight traverses $\frac{1}{60}$ of 15 minutes of longitude, which is $\frac{1}{4}$ of 1 minute, or 15 seconds of longitude.

TABLE.

Time for sunlight to traverse $15''$ of longitude $=$ 1 second of time.
Time for sunlight to traverse $15'$ of longitude $=$ 1 minute of time.
Time for sunlight to traverse 15° of longitude $=$ 1 hour.

167. MISCELLANEOUS MEASURES.

Paper Measures.

24 sheets	=	1 quire.
20 quires	=	1 ream.
2 reams	=	1 bundle.
5 bundles	=	1 bale.

Counting.

20 in number	=	1 score.
12 in number	=	1 dozen.
12 dozen	=	1 gross.
12 gross	=	1 great gross.

Various.

1 qt. dry measure	= 67.20 cu. in.	160 A.	=	1 quarter-section land.
1 qt. liquid measure	= 57.75 cu. in.	320 A.	=	1 half-section land.
1 gallon	= 231 cu. in.	640 A.	=	1 whole section land.
1 bushel	= 2,150.42 cu. in.	100 sq. ft.	=	1 square, on roofs, etc.

The Weight of Grain.

168. Many of the States have fixed by statutes the minimum weight of a bushel of each of the principal kinds of grain.

The following list gives the weights commonly used:

1 bu. oats	= 32 lb.	1 bu. clover seed	= 60 lb.
1 bu. rye	= 56 lb.	1 bu. shelled corn	= 56 lb.
1 bu. barley	= 48 lb.	1 bu. ear-corn	= 70 lb.
1 bu. wheat	= 60 lb.	1 bu. corn-meal	= 48 lb.

REDUCTION.

169. Reduction of a **compound number** is the process of finding a number which will express the value of the compound quantity as one quantity.

A compound number may be reduced to a simple number whose unit is the highest unit of the compound number. 1 wk. 3 da. = $1\frac{3}{7}$ wk. This is sometimes called reduction ascending.

A compound number may be reduced to a simple number whose unit is the lowest unit of the compound number. 1 wk. 3 da. = 10 da. This is sometimes called reduction descending.

A compound number may be reduced to a simple number whose unit is the unit of any of the quantities composing the compound quantity. 1 wk. 3 da. 6 hr. = $10\frac{1}{4}$ da.

170. Reduction of a simple number to a compound number is the process of finding a quantity's value, expressed as several quantities. 8 da. = 1 wk. 1 da.

This also is sometimes called reduction ascending.

PROBLEMS.

1. Find the number of inches in 2 yd. 2 ft. 6 in.

ANALYSIS.

$$\begin{array}{lrlr}
1\text{ yd.} = 36\text{ in.}; & 2\text{ yd.} & = 2 \times 36\text{ in.} = & 72\text{ in.} \\
1\text{ ft.} = 12\text{ in.}; & 2\text{ ft.} & = 2 \times 12\text{ in.} = & 24\text{ in.} \\
 & \underline{6\text{ in.}} & = & \underline{6\text{ in.}} \\
 & 2\text{ yd. } 2\text{ ft. } 6\text{ in.} & = & 102\text{ in.}
\end{array}$$

PROCESS.

$$\begin{array}{l}
\quad\quad 3 \quad 12 \\
2\text{ yd. } 2\text{ ft. } 6\text{ in.} \\
\underline{3} \\
6\text{ ft.} = 2 \times 3\text{ ft.} \\
\underline{2} \\
8\text{ ft.} = 6\text{ ft.} + 2\text{ ft.} \\
\underline{12} \\
96\text{ in.} = 8 \times 12\text{ in.} \\
\underline{\ \ 6} \\
102\text{ in.} = 96\text{ in.} + 6\text{ in.}
\end{array}$$

2. Reduce 6 bu. 3 pk. 2 qt. 1 pt. to pints.
3. Reduce £20 18*s.* 10*d.* to pence. *Ans.* 5,026*d.*
4. Reduce 7 gal. 3 qt. 1 pt. 1 gi. to gills.
5. Reduce 3 T. 18 cwt. 5 lb. 4 oz. to ounces.
6. Reduce 1 lb. 7 oz. 15 pwt. 12 gr. to grains.
7. Reduce 3 ℔. 10 ℥ 7 ʒ 1 ℈ 12 gr. to grains.
8. Reduce 4 mi. 40 rd. 2 ft. 8 in. to inches.
9. Reduce 80 chains 3 poles 4 links to links. *Ans.* 8,079 l.
10. Reduce 4 sq. mi. 40 A. 70 sq. rd. to square rods.
11. How many acres are in 6 townships?
12. How many cubic inches are in 30 cubic yards?
13. How many seconds are in 1 leap-year?
14. Reduce 180° 40′ 40″ to seconds. *Ans.* 650,440″.
15. How many minutes were in February, 1892?
16. How many sheets of paper are in 6 bales?
17. How many cubic inches are in 7 gallons?

18. Reduce 167 pints to bushels, pecks, etc.

PROCESS.

2	167 pt.
8	83 qt. 1 pt.
4	10 pk. 3 qt.
	2 bu. 2 pk.

ANALYSIS.

167 pt. = $\frac{1}{2}$ as many quarts.
$\frac{1}{2}$ of 167 qt. = $83\frac{1}{2}$ qt. = 83 qt. + 1 pt.
83 qt. = $\frac{1}{8}$ as many pecks.
$\frac{1}{8}$ of 83 pk. = $10\frac{3}{8}$ pk. = 10 pk. + 3 qt.
10 pk. = $\frac{1}{4}$ as many bushels.
$\frac{1}{4}$ of 10 bu. = $2\frac{1}{2}$ bu. = 2 bu. + 2 pk.
167 pints = 2 bu. 2 pk. 3 qt. 1 pt.

19. Reduce 402 farthings to £, *s.*, *d.*

20. Reduce 317 gills to gallons, etc.

21. Reduce 43,090 lb. to tons, etc. *Ans.* 21 T. 10 cwt. 90 lb.

22. Reduce 14,171 gr. to pounds, ounces, pennyweights, etc.

23. Reduce 15,972 ft. to miles, etc. *Ans.* 3 mi. 8 rd.

24. Reduce 579 pints to higher units.

25. Reduce 680 rods to higher units.

26. Reduce 66 feet to rods. *Ans.* 4 rd.

27. How many rods are in $4\frac{1}{2}$ miles?

28. How many yards are in 4 mi. 2 rd.? *Ans.* 7,051 yd.

29. How many tons, etc., in 37,232 oz.?

30. Reduce 40 rd. to yards. *Ans.* 220 yd.

31. Reduce 5,960 ft. to miles, rods, yards, and feet.

ANALYSIS.

5960 feet = $\frac{1}{3}$ of 5960 yards = $1986\frac{2}{3}$ yd.
Of the $1986\frac{2}{3}$ yd. the $\frac{2}{3}$ yd. = $\frac{2}{3}$ of 3 ft. = 2 ft.
The 1986 yd. = 1986 rd. ÷ $5\frac{1}{2}$, or $\frac{2}{11}$ of 1986 rd. = $361\frac{1}{11}$ rd.
Of the $361\frac{1}{11}$ rd., the $\frac{1}{11}$ rd. = $5\frac{1}{2}$ times $\frac{1}{11}$ yd., or $\frac{11}{2}$ of $\frac{1}{11}$ yd. = $\frac{1}{2}$ yd.
The 361 rd. = $\frac{1}{320}$ of 361 mi. = $1\frac{41}{320}$ mi.
Of the $1\frac{41}{320}$ mi., the $\frac{41}{320}$ mi. = $\frac{41}{320}$ of 320 rd. = 41 rd.
The highest quantity required is 1 mi.

The results are	1 mi.	41 rd.	$\frac{1}{2}$ yd.	2 ft.
			$\frac{1}{2}$ yd. =	1 ft. 6 in.
The results now are	1 mi.	41 rd.	0 yd.	3 ft. 6 in.
			1 yd. =	3 ft.
The final result is	1 mi.	41 rd.	1 yd.	0 ft. 6 in.

32. Prove the foregoing. How many feet in 1 mi. 41 rd. 1 yd. 6 in.?

33. Reduce 39 yards to rods, etc. *Ans.* 7 rd. 1 ft. 6 in.

34. Reduce 121 square yards to square rods.

35. Reduce 8 square rods to square yards.

36. Reduce 79 yards to rods, etc. *Ans.* 14 rd. 2 yd.

37. Reduce 80 yards to rods, etc.

38. Reduce 12,500 square feet to square rods, square yards, square feet, and square inches.

EXPLANATION.

(1) 12500 sq. ft. = $\frac{1}{9}$ of 12500 sq. yd.; $\frac{1}{9}$ of 12500 sq. yd. = $1388\frac{8}{9}$ sq. yd.
$\frac{8}{9}$ sq. yd. = $\frac{8}{9}$ of 9 sq. ft. = 8 sq. ft.

(2) 1388 sq. yd. = as many times 1 sq. rd. as 1388 contains $30\frac{1}{4}$; 1388 contains $30\frac{1}{4}$ as many times as 4×1388 contains $4 \times 30\frac{1}{4}$, or as many times as 5552 contains 121 = $45\frac{107}{121}$ times; $45\frac{107}{121} \times 1$ sq. rd. = $45\frac{107}{121}$ sq. rd.; $\frac{107}{121}$ sq. rd. = $\frac{107}{121}$ of $\frac{121}{4}$ sq. yd. = $26\frac{3}{4}$ sq. yd. The highest quantity required is 45 sq. rd.

(3) The results now are 45 sq. rd. $26\frac{3}{4}$ sq. yd 8 sq. ft. 0 sq. in.
$\frac{3}{4}$ sq. yd. = 6 sq. ft. 108 sq. in.

(4) The results now are 45 sq. rd. 26 sq. yd. 14 sq. ft. 108 sq. in.
14 sq. ft. = 1 sq. yd. 5 sq. ft.

(5) The final result is 45 sq. rd. 27 sq. yd. 5 sq. ft. 108 sq. in.

NUMERICAL PROCESS.

$$9\underline{)12500}$$
$$1388:8$$

$$30\tfrac{1}{4})1388$$
$$\underline{\ \ 4}\qquad\underline{\ \ 4}$$
$$121\ )5552(45$$
$$\underline{484}$$
$$712$$
$$\underline{605}$$
$$107 = \frac{107}{4} = 26\frac{3}{4}$$

The remainder, 107, is $\frac{107}{4}$, because the dividend is fourths.

39. Reduce 550 square feet to square inches.

40. Reduce 3 cu. yd. 26 cu. ft. 1,049 cu. in. to cubic inches.

41. How many square yards in 3 square miles?

42. How many square feet in 4 square rods?

43. Reduce 81 yards to rods, etc. *Ans.* 14 rd. 4 yd.

44. Reduce 83 yards to rods, etc.

45. Reduce 15 rd. 1 ft. 6 in. to inches.

46. Find the number of square rods, etc., in 17,000 square feet.

47. Reduce 62 sq. rd. 13 sq. yd. 3 sq. ft. 72 sq. in. to square feet. *Ans.* 17,000 sq. ft.

48. Prove Problem 38.

49. Reduce 3 A. 14 sq. rd. 5 sq. yd. to square feet.

50. Reduce 28,468$\frac{1}{4}$ square yards to acres, etc.

51. How many gallons of water are in a full cistern whose capacity is 346,500 cubic inches? *Ans.* 1,500 gal.

52. How many ounces are in 4 pounds of iron?

53. How many pounds are in 48 ounces of silver?

54. Find the cost of 100 pecks of meal at \$.60 per bushel.

55. How many Avoirdupois ounces will balance the weight of 2,625 Troy ounces? *Ans.* 2,880 Av. oz.

56. How many Troy pounds will balance the weight of 432 Avoirdupois pounds? *Ans.* 525 T. lb.

57. How many square chains in 5 A. 50 sq. rd.?

58. Find the difference in cubic inches between 19 gallons and 9 bushels.

59. Find the difference between a surface of 30 square feet and a surface of 30 feet square.

60. Find the difference between a six-inch cube and a cube whose volume is 6 cubic inches.

61. Reduce 520 yards to rods, etc.

62. Reduce 900 square yards to square rods, etc.

63. How many quarts of water are there in a cistern containing 924 cu. ft. of water?

64. Find the number of acres in a rectangular field 880 yards wide and 440 yards long.

65. At \$5.50 per cord, what would be the cost of a pile of wood 16 feet long, 8 feet wide, and 8 feet high?

66. Find the cost of 15,300 lb. fodder @ \$.90 per cwt.

67. Find the value of a rectangular field 80 rods long, 40 rods wide, @ \$26.50 per acre.

68. A room 18 ft. by 16 ft. by 11 ft. is closely packed with stove-wood; find the value of the wood @ \$6 per cord.

69. How many Troy pounds would it take to balance the weight of a ton?

70. Reduce 1,893 yards to miles, rods, etc.

71. Reduce 1 mi. 8 rd. 1 yd. to yards.

72. Reduce 1 mi. 7 rd. 2 ft. 6 in. to inches.

73. How many yards are in $\frac{4}{5}$ of a mile?

74. Find the number of square feet in $2\frac{1}{2}$ acres.

75. What part of a degree is 40′? *Ans.* $\frac{2}{3}$°.

76. What part of a day is 18 hours?

FRACTIONAL QUANTITIES.

PROBLEMS.

1. Reduce $\frac{1}{40}$ of a bushel to a fraction of a quart.

$\frac{1}{40}$ bu. = $\frac{1}{40}$ of 4 pk. = $\frac{1}{10}$ pk.
$\frac{1}{10}$ pk. = $\frac{1}{10}$ of 8 qt. = $\frac{4}{5}$ qt.

$$\frac{1}{\underset{5}{\cancel{40}}} \times 4 \times \cancel{8} \text{ qt.} = \frac{4}{5} \text{ qt.}$$

2. Reduce $\frac{1}{4}$ pint to a fraction of a gallon.

$\frac{1}{4}$ pt. = $\frac{1}{4} \times \frac{1}{2}$ qt. = $\frac{1}{8}$ qt.
$\frac{1}{8}$ qt. = $\frac{1}{4} \times \frac{1}{8}$ gal. = $\frac{1}{32}$ gal.

$$\frac{1}{4} \times \frac{1}{2} \times \frac{1}{4} \text{ gal.} = \frac{1}{32} \text{ gal.}$$

3. Reduce $\frac{1}{2}$ acre to a fraction of a square mile.

4. Reduce $\frac{1}{2}$ ounce to a fraction of a ton.

5. Reduce $\frac{1}{2}$ hour to a fraction of a day.

6. Reduce $\frac{1}{64}$ of a day to a fraction of an hour.

7. Reduce $\frac{7}{9}$ of a gill to a fraction of a gallon.

8. What part of a degree is 40″? *Ans.* $\frac{1}{90}$°.

9. What part of a day is 20 hours?

10. Reduce $\frac{1}{9}$ of a day to hours and minutes.

1 da. = 24 hr.; $\frac{1}{9}$ da. = $\frac{1}{9}$ of 24 hr. = 2 hr. + $\frac{2}{3}$ hr.
1 hr. = 60 min.; $\frac{2}{3}$ hr. = $\frac{2}{3}$ of 60 min. = 40 min.
$\frac{1}{9}$ da. = 2 hr. 40 min.

11. Reduce $\frac{4}{9}$ of a day to hours and minutes.

12. Reduce $\frac{7}{11}$ of an acre to square rods, etc.

13. Reduce $\frac{3}{8}$ of £1 to shillings, pence, etc.

14. Reduce $\frac{5}{9}$ of a pound Troy to ounces, pennyweights, etc.

15. Reduce $\frac{1}{16}$ of a ton to pounds, ounces, etc.

16. Reduce $\frac{4}{9}$ of a mile to yards, feet, and inches.

17. Reduce $\frac{2}{3}$ of a week to days and hours.

18. Reduce $\frac{1}{9}$ of a leap-year to days and hours.

19. Reduce $\frac{4}{9}$ of a common year to days, hours, etc.

20. Reduce $\frac{5}{6}$ of a bushel to pecks, quarts, etc.

21. Reduce $\frac{1}{15}$ of a ton to pounds, etc.

22. Reduce 3 pk. 5 qt. 1 pt. to a fraction of a bushel.

> 3 pk. 5 qt. 1 pt. = 59 pt.; 1 bu. = 64 pt.
> 1 pt. = $\frac{1}{64}$ bu.; 59 pt. = $\frac{59}{64}$ bu.

23. Reduce 586 yd. 2 ft. to a fraction of a mile. *Ans.* $\frac{1}{3}$ mi.

24. Reduce 2 ft. 2 in. to a fraction of a yard.

25. What part of a day is 2 hr. 40 min.?

26. What part of a day is 4 hr. 20 min.?

27. What part of a ton is 666 lb. $10\frac{2}{3}$ oz.?

> 666 lb. $10\frac{2}{3}$ oz. = $\frac{32000}{3}$ oz.; 1 T. = $\frac{96000}{3}$ oz.
> $\frac{32000}{3} \div \frac{96000}{3} = \frac{1}{3}$. *Ans.* $\frac{1}{3}$ T.

28. What part of a common year is equal to 7 wk. 6 da.?

29. What part of a rod is 5 yd. 1 ft.? *Ans.* $\frac{32}{33}$ rd.

30. What part of a week is 21 hours?

31. What part of a week is 6 da. 3 hr.?

32. What part of a yard is 2 ft. 7 in.?

33. What part of 3 weeks is $1\frac{2}{3}$ days?

34. What part of 5 miles is 3 mi. 880 yd.?

35. What part of a common year is $18\frac{1}{4}$ days?

36. What part of a leap-year is $18\frac{1}{2}$ days?

37. What part of $15 is $1.25?

38. What part of 2 bushels is 1.25 pecks? *Ans.* .15625 of 2 bu.

39. How many yards are in .75 of a mile?

40. What part of a mile is 1,540 yards?

41. What part of a day is .75 of an hour?

42. What part of a mile is 76 rods?

43. How much more than an acre is a lot that is 210 feet square? *Ans.* $\frac{3}{242}$ A.

44. What part of a gallon is 3 qt. 1 pt. 2 gi.? Express the result decimally.

Express decimally in higher quantities:

45. 3 qt. 1 pt.

46. 2 pk. 2 qt.

47. 40 min. 48 sec.

48. 2 hr. 24 min.

49. 7 cwt. 50 lb.

50. 54 square inches.

51. 560 acres.

52. 216 cubic inches.

ADDITION OF COMPOUND NUMBERS.

171. Addition of Compound Numbers is the process of finding the sum of several compound numbers.

PROBLEMS.

1. Find the sum of 7 gal. 3 qt. 1 pt. 3 gi., 5 gal. 1 pt. 2 gi., and 6 gal. 3 qt. 1 pt.

EXPLANATION.

The sum of the gills is 5 gi.; 5 gi. = 1 pt. 1 gi. 1 is written under gills, and 1 pt. is added to the column of pints.

The sum of the pints is 4 pt.; 4 pt. = 2 qt. 0 is written under pints, and 2 qt. is added to the column of quarts.

The sum of the quarts is 8 qt.; 8 qt. = 2 gal. 0 is written under quarts, and 2 gal. is added to the column of gallons.

The sum of the gallons is 20 gal.

PROCESS.

gal.	qt.	pt.	gi.
7	3	1	3
5	0	1	2
6	3	1	0
20	0	0	1

2. Find the sum of £8 13*s.* 6*d.*, £7 8*s.* 4*d.*, £6 9*s.*, and £20 18*s.* 7*d.*

3. Find the sum of 21 mi. 40 rd. 4 yd. 2 in., 13 mi. 250 rd. 3 yd. 2 ft., and 5 mi. 2 yd. 3 in.

4. Find the sum of 4 bu. $3\frac{1}{8}$ pk., 3 bu. $2\frac{7}{8}$ pk., 2 bu. $2\frac{1}{2}$ pk., and 6 bu. $\frac{1}{2}$ pk. *Ans.* 17 bu. 1 pk.

5. Find the sum of 3 T. 15 cwt. 65 lb. 4 oz., 5 T. 14 cwt. 34 lb. 12 oz., and 9 T. 19 cwt.

6. A farmer has three fields: in one there are $46\frac{1}{2}$ acres, in another $27\frac{3}{4}$ acres, and in the third 39 A. 4 sq. rd. Express the sum in acres and square rods.

7. Find the sum of 3 yr. 40 da. 20 hr. 20 min., 2 yr. 325 da. 18 hr. 40 min., and 2 yr. $40\frac{1}{4}$ da.

8.

rd.	yd.	ft.	in.
18	1	2	10
19	4	2	8
17	3	1	6
	$4\frac{1}{2}$		
	$\frac{1}{2}=$	1	6

9.

A.	sq. rd.	sq. yd.	sq. ft.
220	20	18	3
119	89	10	6
29	121	3	5
		$1\frac{3}{4}$	
		$\frac{3}{4}=$	6 + 108 sq. in.

10. Find the sum of $\frac{1}{4}$ mile and $\frac{1}{8}$ mile. *Ans.* —— ft.

11. Find the sum of $\frac{1}{4}$ mile and $\frac{1}{2}$ rod.

12. Find the sum of $\frac{3}{4}$ yard and $\frac{3}{4}$ rod.

13. Add 1 mi. 16 rd. 4 yd. 3 in., 2 mi. 115 rd. $4\frac{1}{4}$ yd., and $\frac{1}{2}$ mile.

14. Find the sum of $17\frac{1}{2}$ pounds and $14\frac{1}{2}$ ounces, Avoirdupois.

15. Find the weight, at 60 lb. to the bushel, of 2 bags of wheat, one containing 1 bu. $3\frac{1}{2}$ pk., the other 1 bu. 1 pk. $3\frac{1}{2}$ qt.

16. From a bag of corn a farmer fed to his hogs $1\frac{3}{4}$ bushels and then had $\frac{1}{2}$ peck in the bag; how much corn was in the bag at first? *Ans.* 1 bu. 3 pk. 4 qt.

17. Find the sum of $2\frac{3}{4}$ A., $3\frac{3}{4}$ sq. rd., and $4\frac{1}{2}$ sq. yd.

18. A man gave a miller $1\frac{3}{8}$ pecks of corn for toll, and had 2 bu. 2 qt. corn ground into meal; how much corn did the man carry to mill?

19. For 6 days a man lost time from work as follows: 15 minutes, $\frac{3}{4}$ hour, 1 hour, $2\frac{1}{2}$ hours, 20 minutes, $3\frac{1}{2}$ hours; find the total loss.

20. After shipping $70\frac{1}{2}$ tons of cotton-seed, an agent for an oil mill has on hand 13 T. $4\frac{1}{2}$ cwt.; find the quantity he had at first.

21. A man walked for 3 hours: the first hour $2\frac{1}{2}$ miles, the second 2 mi. $100\frac{1}{2}$ rd., the third 3 mi. 4 rd.; find the total distance.

22. Washington is 77° west of Greenwich, and San Francisco is 45° 26′ 15″ west of Washington; how far west of Greenwich is San Francisco?

23. Express as a mixed decimal the sum of 4 bu. $3\frac{1}{2}$ pk. and 3 bu. 1 pk. $4\frac{1}{2}$ qt. *Ans.* 8.265625 bu.

24. Express decimally the sum of $1\frac{1}{4}$ miles and $40\frac{1}{2}$ yards. *Ans.* 1.273 + miles.

25. Express decimally, in days, the sum of 6 wk. $4\frac{1}{2}$ da., 2 wk. 3 da. $4\frac{1}{2}$ hr., 4 da. 4 hr. 30 min., and $\frac{1}{5}$ hr. *Ans.* $67.883\frac{1}{3}$ days.

SUBTRACTION OF COMPOUND NUMBERS.

172. Subtraction of Compound Numbers is the process of finding the difference between two compound numbers.

PROBLEMS.

1. Find the difference between 6 rd. 4 yd. 1 ft. and 5 rd. 5 yd. 2 ft.

EXPLANATION.

From 1 ft. 2 ft. cannot be subtracted. Reducing 1 of the yards of the minuend to feet, 3 ft. + 1 ft. = 4 ft. 4 ft. − 2 ft. = 2 ft. 2 is written as the remainder of feet.

There remain but 3 yards in the minuend. 5 yd. cannot be subtracted from 3 yd. Reducing 1 of the rods of the minuend to yards, $5\frac{1}{2}$ yd. + 3 yd. = $8\frac{1}{2}$ yd. $8\frac{1}{2}$ yd. − 5 yd. = $3\frac{1}{2}$ yards. $3\frac{1}{2}$ is written as the remainder of yards.

There remain but 5 rods in the minuend. 5 rd. − 5 rd. = 0. There is no remainder of rods.

The remainder is $3\frac{1}{2}$ yd. 2 ft. Reducing the $\frac{1}{2}$ yd. to feet and inches, the remainder becomes 3 yd. 3 ft. 6 in. Reducing the 3 feet to a yard, the remainder, expressed in its highest units, is 4 yd. 6 in.

PROCESS.

rd.	yd.	ft.	
6	4	1	
5	5	2	
	$3\frac{1}{2}$	2	
	$\frac{1}{2}$	= 1	6 in.
	3	3	6 in.
	1		
	4	0	6 in.

2. Solve Problem 1 by Left-Hand Subtraction.

3. Find the difference of 21 mi. 40 rd. 4 yd. and 13 mi. 50 rd. 5 yd.

4. Find the difference of 4 bu. $2\frac{1}{8}$ pk. and 3 bu. $3\frac{7}{8}$ pk.

5. Find the difference of $46\frac{1}{2}$ acres and 27 A. 120 sq. rd.

6. Find the difference of $3\frac{1}{2}$ weeks and $3\frac{3}{4}$ days.

7. Find the difference of 7 rd. 3 yd. 1 ft. and 6 rd. 2 ft.

8. A brook divides a farm of $167\frac{1}{2}$ acres into two parts: on one side of the brook there are 98 A. 60 sq. rd.; how much of the farm is on the other side of the brook?

9. A county 30 miles square has a county-seat containing 4 sq. mi.; how much of the county lies outside of the city?

10. A wagon with its load of corn weighed 2 T. 1 cwt. 14 lb.; the empty wagon weighs 8 cwt. 98 lb.; find the weight of the corn.

11. From a barrel of $31\frac{1}{2}$ gallons of vinegar 7 gal. 3 qt. 1 pt. was used; how much remained in the barrel?

12. Find the difference of $\frac{3}{4}$ mile and $\frac{15}{16}$ mile. *Ans.* — ft.

13. Find the difference of $\frac{1}{2}$ rod and 2 yd. 6 in.

14. Find the difference of $\frac{5}{6}$ hour and 40 min. 10 sec.

15. Find the difference of $\frac{1}{8}$ acre and $\frac{3}{4}$ square rod.

16. A note, given Oct. 20, 1892, was paid April 10, 1893. Find the time from the date until the payment of the note.

EXPLANATION.

Each of the six numbers in the margin is 1 greater than the past time it represents: for example, on the 10th day only 9 days have passed. As the same numbers have been added to the minuend and the subtrahend, the remainder is correct.

PROCESS.

	12	30
yr.	mo.	da.
1893	4	10
1892	10	20
	5	20

17. Find the difference in time between July 4, 1776, and Feb. 22, 1893.

18. Find the difference in time between Sept. 17, 1862, and March 4, 1893.

19. If 3 cwt. 96 lb. $7\frac{1}{10}$ oz. be added to a number, the sum will be 5 hundredweight; find the number.

20. If the numbers 4 bu. $3\frac{1}{8}$ pk. and $3\frac{1}{4}$ bu. be added to a third number, the sum will be 10 bu. 1 qt.; find the third number.

21. If the two numbers 4 gal. 2 qt. 1 pt. and 3 gal. 2 qt. $1\frac{1}{2}$ pt. be added to a third number, the sum will be 10 gallons. Express decimally the third number.

22. If a decimal number be added to 15.355 tons, the sum will be 18 T. 9 cwt. 58 lb.; find the decimal number.

23. From a farm consisting of $506\frac{3}{4}$ acres the owner sold to one man a field, and to another man a piece of woodland containing 40 A. $30\frac{1}{2}$ sq. rd., and had then in his farm 412 A. 120 sq. rd.; find the size of the field. *Ans.* 53 A. $129\frac{1}{2}$ sq. rd.

24. A train is running $\frac{5}{8}$ mile per minute; in one minute it loses $6\frac{1}{2}$ rods: it regains the lost distance in the next minute; how many yards does it run the second minute?

Ans. $1{,}135\frac{3}{4}$ yd.

MULTIPLICATION OF COMPOUND NUMBERS.

173. Multiplication of Compound Numbers is a short process of finding the sum of as many equal compound numbers as there are units in a given abstract number.

PROBLEMS.

1. Multiply 5 bu. 3 pk. 6 qt. by 8.

PROCESS.

bu.	pk.	qt.
5	3	6
		8
47	2	

EXPLANATION.

8 times 6 qt. = 48 qt.; 48 qt. = 6 pk. 0 qt. There is no product to be written for quarts. 8 times 3 pk. = 24 pk.; 24 pk. + 6 pk. = 30 pk.; 30 pk. = 7 bu. + 2 pk. 2 is written for the product of pecks.

8 times 5 bu. = 40 bu.; 40 bu. + 7 bu. = 47 bu. 47 is written for the product of bushels.

2. Multiply 3 mi. 40 rd. 4 yd. 2 ft. 6 in. by 12.

3. Multiply 7 A. 100 sq. rd. 29 sq. yd. 7 sq. ft. by 12.

4. How many bushels of oats are in 128 sacks, each containing 2 bu. 2 pk.?

5. Find the cost of 7 tons of coal @ £1 12*s*. 6*d*. per ton.

6. Find the cost of 4 loads of wheat, averaging 40 bu. 3 pk. 6 qt. to the load, @ $1 per bushel. *Ans.* $163.75.

7. Find the cost of 6 loads of hay, averaging 1 T. 5 cwt. 45 lb. to the load, @ $8 per ton.

8. Find the yield of rye from a field of $22\frac{1}{2}$ acres producing 36 bu. 3 pk. to the acre. *Ans.* $826\frac{7}{8}$ bu.

9. Find the yield of corn from a field of $37\frac{1}{4}$ acres producing 34 bu. 2 pk. to the acre.

10. Find the yield of wheat from a field of $95\frac{7}{8}$ acres producing 26 bu. 1 pk. to the acre.

Find the value of the crop @ $.85 per bushel.

11. Find the number of acres and square rods in a farm $\frac{1}{8}$ of which is a field consisting of 76.15 acres.

Ans. 609 A. 32 sq. rd.

12. At a cost of \$25,000 per mile, what is the total cost of building a railroad $\frac{1}{8}$ of whose length is 15 mi. 15 rd.?

13. At \$.35 per bushel, find the value of the yield of oats from a field of 36 acres, producing 54 bu. 3 pk. to the acre.

Ans. \$689.85.

14. The multiplicand is 3 mi. 200 rd. 8 in.; the multiplier is 13; find the product.

15. A compound number is divided by 8, and the quotient is 7 bu. 3 pk. 5 qt. 1 pt.; find the number.

16. A compound number is divided by 5 cwt. 54 lb. 8 oz., and the quotient is 9; find the number.

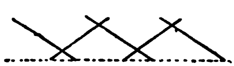
17. There are 7 equal compound numbers; $\frac{1}{7}$ of one of the numbers is 1 cwt. 4 lb. 5 oz.; find the sum of the seven numbers. *Ans.* 2 T. 11 cwt. 11 lb. 5 oz.

18. A lot 209 feet square is planted in fine seed-corn; it yields at the rate of 45 bushels per acre; find the value of the crop @ \$2 per bushel. *Ans.* \$90.25.

19. If a man walks at the rate of 16 rods per minute, how far will he walk in 4 hr. 30 min.? *Ans.* 13 mi. 160 rd.

20. A farmer has 9 head of mules, to each of which he feeds $\frac{7}{8}$ pk. corn per day; how much corn will he feed to the mules in 365 days? *Ans.* 718 bu. 2 pk. 3 qt.

21. A farmer builds a zigzag fence 12 rails high; every 16 ft. 6 in. needs 2 panels; in the whole fence there are 7,680 rails; how long is the fence? *Ans.* 1 mi.



22. What decimal number expresses in pounds the product of $17\frac{1}{2}$ times 6 cwt. 45 lb. 8 oz.? *Ans.* 11,296.25 lb.

23. Find the decimal number that expresses the number of pecks in the total yield of $30\frac{3}{4}$ acres producing 27 bu. 3 pk. corn per acre. *Ans.* ——.

Find the value of the crop @ 58 cents per bushel.

Ans. \$494.92+.

24. A pioneer corps, making a road through a swamp, averaged 5 rd. 4 yd. of road per day, and completed the work in 16 days; find the length of the road.

DIVISION OF COMPOUND NUMBERS.

174. Division of Compound Numbers is the process of finding an abstract quotient or a concrete quotient when the dividend is a compound number.

I. *The Divisor Abstract.*

PROBLEMS.

1. Find $\frac{1}{9}$ of 1 wk. 6 da. 18 hr. 9 min.

Explanation.

$\frac{1}{9}$ of 1 wk. = $\frac{1}{9}$ of 7 da.
7 da. + 6 da. = 13 da.
$\frac{1}{9}$ of 13 da. = 1 da. + $\frac{4}{9}$ da.
1 is written for days in the quotient.
$\frac{4}{9}$ da. = $\frac{1}{9}$ of 4 da.; 4 da. = 4 × 24 hr. = 96 hr.
96 hr. + 18 hr. = 114 hr.
$\frac{1}{9}$ of 114 hr. = 12 hr. + $\frac{6}{9}$ hr.
12 is written for hours in the quotient.
$\frac{6}{9}$ hr. = $\frac{1}{9}$ of 6 hr.; 6 hr. = 6 × 60 min. = 360 min.
360 min. + 9 min. = 369 min.
$\frac{1}{9}$ of 369 min. = 41 min.; 41 is written for minutes in the quotient.

Process.

	wk.	da.	hr.	min.
9)	1	6	18	9
		1	12	41

2. Find $\frac{1}{5}$ of 3 wk. 5 da. 20 hr. 40 min. 30 sec.

3. Divide 1 mi. 240 rd. 5 yd. 2 ft. by 7.

4. Divide 1 T. 3 cwt. 49 lb. 6 oz. by 8.

5. Find $\frac{1}{14}$ of 315 bushels. *Ans.* 22 bu. 2 pk.

6. If \$5 will pay for 7 bu. 2 pk. 5 qt. corn, how much corn can be bought for \$1?

II. *The Divisor Concrete.*

7. Divide 7 bu. 2 pk. 5 qt. by 1 bu. 2 pk. 1 qt.
(Reduce dividend and divisor to quarts.)

8. Find the price of corn per bushel in Problem 6.

9. A garden that contains an acre has walks 3 feet wide and 884 yards long. What part of the garden are the walks?

10. How many sheets of paper 6 inches square would cover one side of a wall 16 feet long and 11 feet high?

11. How many widths of paper 18 inches wide and 288 inches long would be required for a wall 24 feet by 12 feet?

MISCELLANEOUS.

12. Divide 7 mi. 16 rd. 4 yd. 1 ft. by 7.

13. If 1 peck of apples produce 1 qt. vinegar, how many bushels of apples will be required to produce $3\frac{1}{2}$ gal. vinegar?

14. What part of 4 bushels is equal to 3 pecks?

15. If a man walk 16 rods per minute, how long will it take him to walk 20 miles?

16. What part of 2 tons is 9 cwt. 20 lb.?

17. If the product was 3 sq. mi. 280 A., and the multiplier was 25, what was the multiplicand?

18. What part of 3 cwt. is 50 lb. 8 oz.?

19. If the product was 5 T. 6 cwt. 50 lb. and the multiplicand was 4 cwt. 52 lb., what was the multiplier?

20. What part of a bushel is $\frac{1}{2}$ pint?

21. The dividend is 7 wk. 4 da. 10 hr. 16 min.; the divisor is 5; find the quotient.

22. What part of a day is 8 hr. 40 min.?

23. The distance around a square field is 153 rd. 5 yd. 6 in.; find the length of a side.

24. What part of an acre is 65 square rods?

25. The dividend is 1 T. 10 cwt. 50 lb. 8 oz.; the divisor is 1.5; find the quotient.

26. Find $\frac{1}{5}$ of $\frac{7}{8}$ bushel. *Ans.* 5 qt. $1\frac{1}{5}$ pt.

27. If a farmer's plough-hands can plough an acre in 3 hr. 10 min., how many acres could they plough in 3 weeks, working 10 hours per day? *Ans.* $56\frac{16}{19}$ A.

28. Divide 8.325 miles by 9. *Ans.* 296 rd.

29. A farmer uses 16 pecks of meal every 5 days; if he has his own corn ground at a cost of $\frac{1}{7}$ for toll, how many bushels does it cost him a year?

30. The dividend is $6\frac{8}{10}$ miles; the divisor is $4\frac{3}{4}$; find the quotient.

31. The distance around a field is 1 mi. 874 ft.; a twelve-rail fence encloses the field, there being a panel to every $8\frac{1}{2}$ feet; how many rails in the fence? *Ans.* 8,688 rails.

LONGITUDE AND TIME.

175. Given two places, the first of which is 15° east of the second: Then it is noon at the first place 1 hour earlier than at the second place.

The second place is 15′ east of a third place: Then it is noon at the second place 1 minute earlier than at the third place.

The third place is 15″ east of a fourth place: Then it is noon at the third place 1 second earlier than at the fourth place. (See **166.**)

PROBLEMS.

1. Boston is in 71° 3′ 30″ west longitude; New Orleans is in 90° west longitude; find the difference of sun-time (noon).

PROCESS.

	°	′	″
	90	00	00
	71	3	30
15)	18	56′	30″
	1 hr.	15 m.	46 sec.

EXPLANATION.

The difference of longitude is found to be 18° 56′ 30″.

The process consists in dividing the number by 15, and in regarding the quotient as hours, minutes, and seconds instead of °, ′, and ″.

ANALYSIS.

For every 15° there is 1 hr. of difference; for 18° there are as many times 1 hr. of difference as 18° contains 15°; 18° contains 15° $1\frac{1}{5}$ times;
$1\frac{1}{5}$ times 1 hr. = $1\frac{1}{5}$ hr. = 1 hr. 12 min.

For every 15′ there is 1 min. of difference; for 56′ there are as many times 1 min. of difference as 56′ contains 15′; 56′ contains 15′ $3\frac{11}{15}$ times;
$3\frac{11}{15}$ times 1 min. = $3\frac{11}{15}$ min. = 3 min. 44 sec.

For every 15″ there is 1 sec. of difference; for 30″ there are as many times 1 sec. of difference as 30″ contains 15″;
30″ contains 15″ twice; twice 1 sec. = 2 sec.

The total difference of time = 1 hr. 15 min. 46 sec.

SHORT ANALYSIS.

18° 56′ 30″ = 68,190″.

1″ makes a difference of $\frac{1}{15}$ sec. of time.
68,190″ makes a difference of 68,190 times $\frac{1}{15}$ sec. of time.
68,190 × $\frac{1}{15}$ sec. = 4,546 sec. = 1 hr. 15 min. 46 sec.

2. The longitude of Denver is 105° 4′ W.; the longitude of Vicksburg is 90° 54′ W.; find the difference of sun-time.

3. Nashville is in 86° 49′ W. longitude; Santa Fé is in 106° 10′ W. longitude; find the difference of sun-time.

4. Charleston is in 79° 57′ W. longitude; San Francisco is in 122° 26′ W. longitude; how much later is the sun-time at San Francisco than at Charleston?

5. Longitude of Montgomery, 86° 23′ W.; longitude of El Paso, 106° 28′ W.; find the difference of sun-time.

6. Longitude of St. Petersburg, 30° 45′ E.; longitude of Philadelphia, 75° 10′ W.; find the difference of sun-time.

Longitude may be measured east and west from any place used as a starting-point. The meridian of Greenwich is recognized by the U. S. maps and charts. East longitude means east from Greenwich; west longitude means west from Greenwich. In finding the difference of longitude in Problem 6, the two numbers are added.

7. Berlin is in 13° 24′ E. longitude; Havana is in 82° 22′ W. longitude; find the difference of sun-time.

8. Tokio is in 139° 40′ E. longitude; Montreal is in 45° 35′ W. longitude; find the difference of sun-time.

The difference of longitude between two places cannot be greater than 180°, because 180° is half-way round the earth. East from Tokio to Montreal is a shorter distance than west from Tokio to Montreal. 360° less the sum of the two numbers is the shorter distance, and is their true difference.

9. Baltimore is in 76° 37′ W. longitude; St. Paul is in 93° 5′ W. longitude; find the difference of sun-time.

10. The difference of sun-time between two cities is 2 hr. 16 min.; find the difference of longitude.

Process.		Analysis.
hr.	min.	For 1 min. difference of time there are 15′ difference of longitude; for 16 min. there are 16 × 15′ = 240′ = 4° 0′.
2	16	For 1 hr. difference, there are 15° difference; for 2 hr. difference, there are 2 × 15° difference = 30°; 30° + 4° = 34°.
	15	
34°	0′	

11. The difference between the sun-time of Washington and that of Cincinnati is 30 minutes; find the difference of longitude. *Ans.* 7° 30′.

REVIEW PROBLEMS.

1. At \$5.70 per cord, find the cost of $16\frac{1}{2}$ cords of wood.

2. The dividend is 150.005; the divisor is 5,050; find the quotient.

3. How many times can a man spend \$7.50 out of \$150?

4. At what rate per hundred does a man split rails when he receives \$21 for splitting 3,500?

5. The two factors are the sum and the difference of .04 and $\frac{3}{4}$; find the product.

6. At 85 cents to the panel, a fence cost \$224.40; how many panels are in the fence?

7. Each of four numbers is a prime number; their product is 3,003; find the four numbers.

8. Find the difference of the cost of 140 bushels of corn @ \$.65, and the cost of 120 bushels of wheat @ \$.85.

9. At what rate per yard does a man buy cloth, when he pays \$6.75 for $7\frac{1}{2}$ yards?

10. Regard $2\frac{1}{2}$ as a unit; how many such units does the number 1,000 contain?

11. At what rate per hundred does a man pick cotton, when he receives \$9.88 for picking 1,520 pounds?

12. The two factors are the sum and the product of $2\frac{1}{4}$ and .005; find the product of the two factors.

13. Find the profit on 14 bales of cotton, averaging 460 lb., bought @ \$.07$\frac{7}{8}$ and sold @ \$.08$\frac{1}{16}$.

14. The dividend is $\frac{1}{4}$; the divisor is .00025; find the quotient.

15. At what rate per bushel is corn exchanged for wheat, when 280 bushels of corn are given for 224 bushels of wheat?

Ans. 5 corn to 4 wheat, or $1\frac{1}{4}$ to 1.

16. Regard 15″ as a unit; how many such units does 15° contain?

17. Find the cost of $\frac{1}{4}$ ton of cotton-seed meal @ \$1.25 per hundredweight.

18. The product of two numbers is $1.8333\frac{1}{3}$; one of the numbers is 2.5; find the other number.

19. The dividend is 1 mi. 78 rd. 5 yd. 2 ft.; $\frac{4}{5}$ of the divisor is 8; find the quotient.

20. When there is a telegraph pole to every 58 yd. 2 ft., how many are there to the mile?

21. Regard $66\frac{2}{3}$ feet as a unit; how many such units are there in 5 miles?

22. At what price per bushel is corn sold when a man pays \$15.30 for $25\frac{1}{2}$ bushels?

23. Find the value of the cotton in a toll of $\frac{1}{14}$ for ginning 42,000 lb. seed-cotton yielding $\frac{1}{3}$ lint which sells @ \$.08$\frac{1}{8}$.

24. The product is the sum of \$$1\frac{9}{16}$ and \$$3\frac{1}{4}$; the multiplicand is \$1; find the multiplier.

25. From the data given in Problem 23, find the value of the cotton-seed @ \$9 per ton.

26. When there is a cross-tie for every $2\frac{1}{2}$ feet, how many cross-ties are there for $2\frac{1}{2}$ miles?

27. The dividend is 2 sq. mi. 500 A. 50 sq. rd. 12 sq. yd.; the divisor is $1\frac{1}{2}$ square feet; find the quotient.

28. At what rate per acre is a yield of $137\frac{1}{4}$ bushels of grain from $3\frac{1}{2}$ acres?

29. The minuend is $\frac{27}{29}$; express approximately a decimal number equal to the sum of the subtrahend and remainder.

Ans. .931.

30. Take $\frac{3}{8}$ as a unit; how many such units are in $33\frac{1}{8}$?

31. A man bought a farm for \$2,013 and sold it for \$2,376, gaining \$2.75 per acre; find the number of acres.

32. At what rate of speed per hour did a man travel who rode $14\frac{1}{2}$ miles in 25 minutes?

33. The product is 7 wk. 3 da. 14 hr. 40 min.; the multiplicand is 1 week; find the multiplier.

34. A man bought a bale of cotton for \$36 and sold it for \$38.40, gaining $\frac{1}{2}$ cent per pound; at what price per pound did he sell it? *Ans.* \$.08 per lb.

35. Find the difference, in cubic inches, between the capacity of a box filled with 10 bushels of wheat and the capacity of a cask filled with 10 gallons of water. *Ans.* 19,194.2 cu. in.

36. The multiplier is $\frac{7}{8}$; the multiplicand is 1 day; find the value of the product.

37. Consider $\frac{3}{8}$ yd. as a unit; how many such units are there in a mile?

38. A field of 20 acres, that produced 11 bu. 1 pk. corn per acre, was sold at \$7 per acre; how much greater was the value of the crop, which was sold at \$.70 per bushel?

39. At what rate per mile does a man buy a railroad ticket from A to B, 132 miles, the ticket costing \$3.30?

40. The product is 4 cwt. 39 lb. 9 oz.; the multiplier is 8; find the multiplicand.

41. A man sold 45 horses at a profit per head of \$9, which was $\frac{1}{10}$ of the price he paid per head; how much did he get for the horses?

42. At what rate per week does a man pay board, when he pays \$71.50 for 91 days?

43. Regard $\frac{7}{8}$ as a unit; how many such units are in 350?

44. From A to B is 28 miles; leaving A, the mail-rider makes $\frac{2}{7}$ of the distance and waters his horse; he makes $\frac{2}{7}$ of the remaining distance and waters again; how far is he now from B?

45. C's farm contains 488 acres; it is $\frac{7}{8}$ as large as half of D's; how many acres are in D's farm?

46. The dividend is $\frac{3}{8}$ mile; the divisor is $\frac{3}{16}$ feet; find the quotient.

47. At what rate per second does the sunlight move at the equator, a degree being $69\frac{1}{6}$ miles?

48. A man sold a coop of chickens at the rate of \$2.50 per dozen, and received \$6.25; how many chickens did he sell?

49. Regard $66\frac{2}{3}$ as a unit; how many such units are there in 10,200?

50. What must 3.006 be multiplied by to make a number equal to $\frac{3}{4}$ of 4,008?

51. Regard a wagon-body filled with $19\frac{1}{2}$ bushels of ear-corn as a unit; how many such units are there in the crop of corn from $60\frac{2}{3}$ acres, yielding 36 bushels per acre?

52. At a cost of \$.75 per head for shoeing all around, find the cost of shoeing 24 horses all around, and 12 mules in front.

Ans. \$22.50.

53. The multiplier is a fraction whose numerator is 7, and whose denominator is 8; the multiplicand is 3 miles; find the value of the product.

54. An express train runs 201 miles in 5 hr. 1 min. 30 sec.

At what rate per minute does it run?

At what rate per hour does it run?

In what time does it run a mile?

55. Regard $1\frac{1}{2}$ minutes as a unit; how many such units are there in 18,090 seconds?

56. From B to C is 70 miles; the regular rate is 3 cents per mile; the ticket agent at C receives \$43.05 for half-rate tickets to B; how many half-rate tickets does he sell?

57. The dividend is 1 wk. 6 da. 12 hr. 30 min.; the divisor is $\frac{1}{8}$; find the quotient. *Ans.* 15 wk. 3 da. 4 hr.

58. A farmer has $47\frac{3}{4}$ acres in cotton which yields 486 pounds of seed-cotton per acre; find the cost of picking the crop @ 55 cents per hundred pounds.

59. The product is the sum of 5^3 and 6^4; the multiplicand is 9^3; find the multiplier.

60. Find the rate per hundred pounds when a wagoner charges \$7.25 for hauling 2,900 pounds of freight.

61. A farmer needs a 12-rail fence for a square field; the distance around the field is 1,320 yd.; every 33 ft. requires 4 panels; how many rails must be split?

62. If the farmer can join fences with a neighbor for $\frac{3}{8}$ of one side, how many rails can he save?

63. Ten months ago a farmer was offered \$$.08\frac{3}{4}$ per pound for 45 bales of cotton averaging 480 pounds; but he stored it in a warehouse at a cost per month of 25 cents a bale for storage and 10 cents a bale for insurance; he now sells @ \$$.09\frac{1}{4}$, but the cotton has lost in weight 9 pounds per bale. Not counting the use of the money, has he gained or lost, and how much? *Ans.* Lost \$86.96.

FOR ORAL WORK.

Review. — Induction.

1. How many square feet make a square yard?

2. What part of a square yard is a square foot?

3. If a square yard of slate costs $2.25, what is the rate per square foot?

4. How many cubic feet make a cord? What part of a cord is a cubic foot? If a cord of wood costs $6.40, what is the rate per cubic foot?

5. What part of an acre is a square rod? If an acre yields 1,600 pounds of wheat, what is the rate per square rod?

6. What part of 60 is 15? If sunlight moves 15 degrees in 60 minutes, in what time does it move a degree?

7. If $1 will buy 1 bushel of wheat, what part of a bushel would 25 cents buy at the same rate?

8. What part of $1 is $33\frac{1}{3}$ cents? If a pound of butter costs $33\frac{1}{3}$ cents, how much butter will $1 buy?

9. What part of $1 is $16\frac{2}{3}$ cents? If a yard of cloth costs $16\frac{2}{3}$¢, how many yards can be bought for $2?

10. What part of $1 is 75 cents? If a bushel of corn costs 75¢, how many bushels can be bought for $3?

11. If 8 bushels of corn can be bought for $6, what is the rate per bushel? If 8 bushels can be bought for $6, how many bushels can be bought for $12? How many for $24?

12. What part of $1 is $66\frac{2}{3}$ cents? If 12 bu. corn is bought for $8, at what price per bushel is it bought?

13. If 12 bu. corn cost $8, what would 24 bushels cost? What would 30 bushels cost? What would 48 bushels cost?

14. If $8 buy 12 bu. corn, how many bushels would $24 buy? How many bushels would $20 buy?

15. There are as many eights in twelve as there are fours in what number? There are as many nines in twelve as there are eighteens in what number? There are as many fives in fifteen as there are sevens in what number?

PROPORTION OF NUMBERS.

RATIOS.

176. A quotient, used to express the result of a comparison, is called a **Ratio**.

In comparing one number with another number, the result of the comparison is obtained by dividing the one number by the other.

A dividend is 24; its divisor is 6; the quotient, or ratio of 24 to 6, is 4. Regarding 6 as a unit, there are four such units in 24.

PROBLEMS.

1. Regard $3\frac{1}{2}$ as a unit; how many such units are in 7?
2. What is the ratio of 6 to 7? *Ans.* $\frac{6}{7}$.
3. If $\frac{1}{3}$ is a unit, how many such units are in 7?
4. What is the ratio of 15 to 27? *Ans.* $\frac{5}{9}$.
5. Divide 50 into 10 units; what is each unit?
6. A dime is a unit; how many such units are in $1?
7. What is the ratio of 1 to $\frac{1}{10}$? Of 100 to 10? *Ans.* 10.
8. What is the ratio of 2 feet to 1 yard?
9. What is the ratio of 6 hours to 1 day?
10. What is the ratio of 1 week to 1 day?
11. What is the ratio of $\frac{8}{9}$ to $\frac{2}{9}$? Of $\frac{12}{15}$ to $\frac{3}{15}$? *Ans.* 4.
12. What is the ratio of $\frac{1}{2}$ to $\frac{1}{4}$? Of $\frac{1}{3}$ to $\frac{1}{6}$?
13. Divide $\frac{8}{9}$ into 4 units; what is each unit?
14. Find the ratio of $\frac{8}{9}$ to $\frac{1}{4}$. *Ans.* $3\frac{5}{9}$.
15. What is the ratio of $\frac{8}{9}$ to $\frac{1}{2}$?
16. Divide $2\frac{2}{3}$ into 3 units; what is each unit? *Ans.* $\frac{8}{9}$.
17. What is the ratio of $3\frac{3}{4}$ to $1\frac{1}{4}$?
18. What is the ratio of $7 to $3? Of $20 to $9?
19. What is the ratio of $7 to $$3\frac{1}{2}$? Of $11 to $$5\frac{1}{2}$?
20. What is the ratio of $15 to $$1\frac{1}{2}$? Of $25 to $$12\frac{1}{2}$?
21. What is the ratio of $1 to 75 cents? Of $1.75 to $1?

22. What is the ratio of 75 cents to \$1? Of \$.75 to \$1.75?

23. What is the ratio of 60 cents to \$1? *Ans.* $\frac{3}{5}$.

24. What is the ratio of 50 cents to 1 dollar?

25. What is the ratio of $33\frac{1}{3}$ cents to 1 dollar?

26. What is the ratio of 25 cents to 1 dollar?

27. What is the ratio of 20 cents to 1 dollar?

28. What is the ratio of $16\frac{2}{3}$ cents to 1 dollar?

29. What is the ratio of $12\frac{1}{2}$ cents to 1 dollar?

30. What is the ratio of 10 cents to 1 dollar?

31. What is the ratio of $8\frac{1}{3}$ cents to 1 dollar? Of $3\frac{1}{3}$¢ to \$1?

32. What is the ratio of $6\frac{1}{4}$ cents to 1 dollar? Of $1\frac{1}{3}$¢ to \$1?

33. What is the ratio of 5 cents to 1 dollar? Of 2¢ to \$1?

177. The exact divisors of a number are called the **Aliquot Parts** of the number.

Aliquot parts of \$1 are shown in Problems 24–33.

PROBLEMS.

1. At 50 cents apiece, find the cost of 48 hoes.

PROCESS.	ANALYSIS.
$\frac{1}{2}$ of \$48 = \$24, or $2\underline{)\$48}$ \$24	50 cts. = \$$\frac{1}{2}$. At \$1 the cost would be 48 × \$1 = \$48. At \$$\frac{1}{2}$ the cost is $\frac{1}{2}$ of \$48 = \$24.
In the process, the cost at \$1 is divided by the ratio of \$1 to the actual cost.	

2. What is the cost of 76 chickens at 25 cents apiece?

$\frac{1}{4}$ × \$76 = \$19.

3. Find the cost of 168 pounds of coffee @ \$.25.

4. Find the cost of 369 bushels of oats @ \$.$33\frac{1}{3}$.

5. Find the cost of 486 bushels of cotton-seed @ \$.$16\frac{2}{3}$.

Find the cost of the following:

6. 612 lb. butter @ $33\frac{1}{3}$¢.	**12.** 666 lb. wool @ \$.$16\frac{2}{3}$.
7. 96 doz. eggs @ 25¢.	**13.** 480 lb. sugar @ \$.$06\frac{1}{4}$.
8. 86 bu. corn @ 50¢.	**14.** 360 lb. bacon @ \$.$08\frac{1}{3}$.
9. 120 yd. cloth @ $33\frac{1}{3}$¢.	**15.** 320 lb. shot @ \$.$06\frac{1}{4}$.
10. 500 bu. cotton-seed @ 20¢.	**16.** 160 yd. lawn @ \$.$12\frac{1}{2}$.
11. 888 lb. beef @ $12\frac{1}{2}$¢.	**17.** 240 lb. pork @ \$.$16\frac{2}{3}$.

Find the cost of the following:

18. 482 bu. meal @ \$.50.
19. 584 bu. bran @ \$.25.
20. 400 bu. bran @ \$.20.
21. 630 bu. oats @ \$.33$\frac{1}{3}$.
22. 440 bu. potatoes @ \$.50.
23. 57 bu. turnips @ \$.25.
24. 84 bu. peaches @ \$.33$\frac{1}{3}$.
25. 168 bu. charcoal @ \$.12$\frac{1}{2}$.
26. 36 yd. ribbon @ 8$\frac{1}{3}$¢.
27. 48 yd. muslin @ 16$\frac{2}{3}$¢.
28. 45 yd. flannel @ 33$\frac{1}{3}$¢.
29. 74 yd. cambric @ 12$\frac{1}{2}$¢.
30. 64 yd. calico @ 6$\frac{1}{4}$¢.
31. 164 yd. lace @ 25¢.
32. 78 yd. cottonade @ 16$\frac{2}{3}$¢.
33. 27 yd. sheeting @ 33$\frac{1}{3}$¢.

34. Find the cost of 150 bu. corn @ \$.75 per bushel.

PROCESS.

150 bu. @ \$1 = \$150.
150 bu. @ \$$\frac{1}{2}$ = \$ 75.
" " " \$$\frac{1}{4}$ = 37.50
150 bu. @ \$$\frac{3}{4}$ = \$112.50

PROCESS BY SUBTRACTION.

150 bu. @ \$1 = \$150.
150 bu. @ \$$\frac{1}{4}$ = 37.50
150 bu. @ \$$\frac{3}{4}$ = \$112.50

35. Find the cost of 250 bu. oats @ 37$\frac{1}{2}$ cts. per bushel.

\$.37$\frac{1}{2}$ is \$$\frac{1}{4}$ + \$$\frac{1}{8}$, or \$.37$\frac{1}{2}$ is \$$\frac{1}{2}$ − \$$\frac{1}{8}$.

36. Find the cost of 248 bu. wheat @ 87$\frac{1}{2}$¢ per bushel.

37. Find the cost of 45 yd. linen @ 66$\frac{2}{3}$¢ per yard.

38. Find the cost of 240 bu. wheat @ 93$\frac{3}{4}$¢.

PROCESS.

240 bu. @ \$ 1 = \$240
240 bu. @ \$ $\frac{1}{2}$ = \$120
" " " \$ $\frac{1}{4}$ = 60
" " " \$ $\frac{1}{8}$ = 30
" " " \$$\frac{1}{16}$ = 15
240 bu. @ \$$\frac{15}{16}$ = \$225

PROCESS BY SUBTRACTION.

240 bu. @ \$ 1 = \$240
240 bu. @ \$$\frac{1}{16}$ = 15
240 bu. @ \$$\frac{15}{16}$ = \$225

39. Find the cost of 360 bu. corn @ \$.62$\frac{1}{2}$.

40. Find the cost of 240 lb. wool @ \$.18$\frac{3}{4}$. *Ans.* \$45.

41. Find the cost of 240 yd. cloth @ \$1.18$\frac{3}{4}$.

42. Find the cost of 36 lb. tea @ \$.62$\frac{1}{2}$. *Ans.* \$22.50.

43. Find the cost of 36 yd. cloth @ \$1.62$\frac{1}{2}$.

44. Find the cost of 150 axes @ \$.75.

45. Find the cost of 150 hats @ \$1.75.

46. Find the cost of 64 gal. cider @ \$.31$\frac{1}{4}$.

47. Find the cost of 64 boxes of tomatoes @ \$1.31$\frac{1}{4}$. *Ans.* \$84.

48. The difference of longitude between two cities is 32° 20′ 30″; find the difference of sun-time.

EXPLANATION.	PROCESS.		
In 60 min. the sunlight moves 15°.	32°	20′	30″
What is the ratio of 60 to 15?			4
If in 60 min. the sunlight moves 15°, its rate is 1° in 4 min.; its rate is 1′ in 4 sec.; and 1″ in 4 sixtieth-seconds.	2 hr.	9 min.	22 sec.

At 1° to 4 min., 32° requires 32 times 4 min. = 2 hr. 8 min.
At 1′ to 4 sec., 20′ requires 20 times 4 sec. = 1 min. 20 sec.
At 1″ to $\frac{4}{60}$ sec., 30″ requires 30 times $\frac{4}{60}$ sec. = 2 sec.

At the rate of 1 to 4, 32° 20′ 30″ requires 2 hr. 9 min. 22 sec.

The process does not require the use of the three multiplicands, 4 minutes, 4 seconds, and 4 sixtieth-seconds. Longitude and time are written by the same unvarying scale. 1 higher unit = 60 of the next lower units.

32 × 4, the product regarded as minutes of time, equals
32 ÷ 15, the quotient regarded as hours, because 4 minutes = $\frac{1}{15}$ hour.

49. The difference of longitude between two cities is 18° 16′; find the difference of sun-time. *Ans.* 1 hr. 13 min. 4 sec.

50. The difference of longitude between two cities is 21° 15′; find the difference of sun-time. *Ans.* 1 hr. 25 min.

51. Between two cities the difference of sun-time is 2 hr. 16 min.; find the difference of longitude.

EXPLANATION.	PROCESS.	
	hr.	min.
For 15° there are 60 min. difference; what is the ratio of 15 to 60?	4)2	16
$\frac{1}{4}$ of the number, regarded as degrees, equals 15 times the number, regarded as minutes of longitude.		34°

52. Between two cities the difference of longitude is 30° 16′; find the difference of sun-time.

53. Between two cities the difference of sun-time is 4 hr. 30 min.; find the difference of longitude. *Ans.* 67° 30′.

54. Find the cost of 75 lb. cheese @ \$.18$\frac{3}{4}$ per pound.

55. Between two cities the difference of sun-time is 8 hr. 40 min.; find the difference of longitude.

56. Find the cost of 150 lb. bacon @ \$.08$\frac{1}{8}$ per pound.

57. Find the cost of 480 lb. cotton @ \$.06$\frac{1}{4}$ per pound.

178. In an operation which results in a ratio, the dividend is called the **Antecedent**; the divisor is called the **Consequent**; the two terms taken together are called a **Couplet.**

179. The ratio of the antecedent to the consequent is called the **Direct Ratio**: ratio unqualified means a direct ratio.

180. The ratio of the consequent to the antecedent is called a **Reciprocal Ratio,** or an **Inverse Ratio.**

The ratio of 12 to 2 is 6; the ratio of 2 to 12 is $\frac{1}{6}$, the reciprocal of 6.

181. The **Sign of Division** used in operations with ratios is the colon (:) which is read *to* or *is to.*

PROBLEMS.

1. Find the reciprocal of the ratio of 12 : 3. *Ans.* $\frac{1}{4}$.

2. The antecedent is 6; the consequent is 3; find the inverse ratio.

3. The antecedent is 5; the consequent is 2; find the direct ratio.

4. Find the reciprocal ratio of 7 : 8. *Ans.* $1\frac{1}{7}$.

5. The antecedent is 6; the direct ratio is 2; find the consequent.

6. Find the ratio of 13 to 4; of 21 to 5; of 3 to 19.

7. The antecedent is $\frac{3}{4}$ of 7; the consequent $\frac{3}{8}$ of 8; find the ratio. *Ans.* $\frac{63}{64}$.

8. The ratio is 9; the consequent is 7; find the antecedent.

9. Find the ratio of $7\frac{1}{2}$ to 15; of $8\frac{1}{3}$ to 25.

10. Find the ratio of $1\frac{1}{2}$ acres to 80 square rods. *Ans.* 3.

11. Find the reciprocal of the ratio of $5\frac{1}{2} : 8\frac{1}{4}$. *Ans.* $1\frac{1}{2}$.

12. If corn is sold at \$1 for 2 bushels, what will 12 bushels cost?

13. What is the ratio of 12 bushels to 2 bushels?

14. If a man walks at the rate of 6 miles in 2 hours, how far does he walk in 6 hours?

15. What is the ratio of 6 hours to 2 hours?

16. If beef is sold at the rate of 15 pounds for \$2, how many pounds of beef can be bought for \$12?

17. What is the ratio of 12 dollars to 2 dollars?

18. When eggs are sold at the rate of 6 dozen to the dollar, how many dozen can be bought for $\$2\frac{1}{2}$?

19. What is the ratio of \$2.50 to \$1? of \$6.75 to \$1?

20. What is the ratio of 15 dozen to 6 dozen?

21. If it costs 15 cents to ride 5 miles, what will it cost to ride 20 miles?

22. What is the ratio of 20 miles to 5 miles?

23. What is the ratio of 60 cents to 15 cents?

24. When the ratio is 5, and the antecedent is 60, what is the consequent?

25. There are two couplets that have the same ratio: the first couplet is 15 : 5; the antecedent of the second couplet is 30; find the consequent of the second couplet. *Ans.* 10.

SIMPLE PROPORTION, OR RULE OF THREE.

182. Couplets that have the same ratio are equal. Two couplets united by a sign of equality form a **Simple Proportion.**

183. The **Sign of Equality** in **Proportion** is the double colon (: :). Thus, the expression $4 \div 6 = 8 \div 12$ is written as a proportion

4 : 6 : : 8 : 12, and is read, four *is to* six *as* eight *is to* twelve, or, the ratio of 4 to 6 equals the ratio of 8 to 12.

184. In a proportion the first and last terms are called the **Extremes,** and the intermediate terms are called the **Means.**

In the proportion 4 : 6 : : 8 : 12 or $4 \div 6 = 8 \div 12$, the extremes are 4 and 12; the means are 6 and 8.

Compare the terms in the example.

$$4 : 6 : : 8 : 12 \text{ or } 4 \div 6 = 8 \div 12.$$

Couplets that have the same ratio are equal (see **182**).

Hence, in the proportion 4 : 6 : : 8 : 12,

$$\frac{4}{6} = \frac{8}{12}; \text{ or } \frac{4}{6} \div \frac{8}{12} = 1; \text{ or } \frac{12}{8} \times \frac{4}{6} = 1.$$

For the last equality to be true 12×4 must equal 8×6. 4 and 12 are the extremes, and 6 and 8 are the means in the proportion, hence:

185. The numerical product of the **Extremes** equals the numerical product of the **Means.**

PROBLEMS.

1. Reduce $\frac{3}{8}$ to a fraction whose numerator is 12.
2. Regard $\frac{3}{4} = \frac{9}{\ }$; supply the denominator needed.
3. Supply the fourth term in this proportion; 6 : 8 : : 18 : —.

COMPARISON.

The ratio of 6 to 8 is $\frac{3}{4}$.
The ratio of 18 to the fourth term is $\frac{3}{4}$.
If 18 is $\frac{3}{4}$ of the fourth term, then the fourth term is $\frac{4}{3}$ of 18, or $18 \times \frac{4}{3}$.
$18 \times \frac{4}{3} = 24 =$ fourth term.
6 : 8 : : 18 : 24.

4. Supply the fourth term; 5 : 15 : : 7 : —.
5. Supply the fourth term; 7 : 28 : : 2 : —.
6. Reduce $\frac{5}{9}$ to a fraction whose numerator is 40.
7. Reduce $\frac{1}{3}$ to a fraction whose numerator is 21.
8. Supply the fourth term; $5\frac{1}{2} : 7\frac{1}{2}$: : 6 : —.
9. Reduce $\frac{2}{3}$ to a fraction whose numerator is 36.
10. Reduce $\frac{2}{9}$ to a fraction whose numerator is 18.
11. Supply the fourth term; $3\frac{1}{4} : 6\frac{1}{2}$: : $6\frac{1}{2}$: —.
12. Regard $\frac{6}{8} = \frac{\ }{32}$; supply the numerator needed.
13. Supply the third term; 6 : 8 : : — is to 32.

COMPARISON.

The ratio of 6 to 8 is $\frac{3}{4}$.
The ratio of the third term to 32 is $\frac{3}{4}$.
The third term is $\frac{3}{4}$ of 32; $\frac{3}{4} \times 32 = 24$.

14. Supply the fourth term; \$427 : \$3 : : 54 bushels : —.

Supply the missing term in each proportion:

15. $2\frac{1}{2}$: 10 : : — : 12.
16. $3\frac{1}{3}$: 25 : : 25 : —.
17. $6\frac{1}{4}$: 100 : : $\frac{1}{2}$: —.
18. $6\frac{1}{4}$: — : : 8 : 16.
19. — : 75 : : $3\frac{1}{2} : 10\frac{1}{2}$.
20. \$1.50 : \$2 : : \$$7\frac{1}{2}$: —.
21. $8\frac{1}{3} : 33\frac{1}{3}$: : $66\frac{2}{3}$: —.
22. $12\frac{1}{2} : 37\frac{1}{2}$: : — : $66\frac{2}{3}$.
23. $37\frac{1}{2} : 87\frac{1}{2}$: : $12\frac{1}{2}$: —.
24. $\frac{1}{2}$ pk. : 1 bu. : : 1 bu. : —.
25. $\frac{1}{2}$ bu. : 3 bu. : : 2 bu. : —.
26. $\frac{1}{2}$ rd. : $\frac{1}{2}$ mi. : : 1 mi. : —.
27. 2 da. : 1 wk. : : — : 17 da.
28. 6 min. : 1 hr. : : 6 hr. : —.
29. 15° : 360° : : 1 hr. : —.
30. 4′ : 1° : : 4 min. : — hr.
31. $2\frac{3}{4}$ yd. : 1 rd. : : — : 3 mi.
32. $3^2 : 4^2$: : 5^2 : —.

33. The first term is $\frac{5}{8}$ bushel; the third term is $\frac{7}{8}$; the ratio is $\frac{1}{8}$; find the second and fourth terms.

34. The first term is 9; the product of the extremes is 108; the ratio is 4; find the second, third, and fourth terms.

35. The second term is 8; the product of the means is 24; the ratio is $2\frac{1}{2}$; find the first, third, and fourth terms.

RULE OF THREE.

PROBLEMS.

1. If 6 acres yield 126 bushels of wheat, how many acres at the same rate will yield 252 bushels of wheat?

PROCESS.

126 bushels : 252 bushels : : 6 acres : the number of acres required.
The ratio is $\frac{1}{2}$; therefore 6 acres $= \frac{1}{2}$ the number of acres required.
The number of acres required is 2×6 acres $= 12$ acres.

186. To find the fourth term of a proportion:

Multiply the third term by the reciprocal of the ratio.

2. If 20 panels of a fence are 160 feet long, how many times as long are 100 such panels? What is the length of the 100 panels?

3. If a man can plough 7 acres in 2 days, in what time can he plough 40 acres at the same rate?

The Order of the Terms.

A ratio is a quotient which is the result of comparing two numbers that are alike. No ratio can result from comparing 2 days with 7 acres, or 2 days with 40 acres. But ratios do result from comparing 7 acres with 40 acres, and 40 acres with 7 acres.

If the couplet is not 7 A. : 40 A., it is 40 A. : 7 A.; in either case the 2 days is an odd term; therefore, the 2 days is written for the third term, or antecedent, of a couplet whose consequent, yet to be found, is the fourth term of a proportion (— : — : : 2 da. : —), of which the fourth term is likewise a number of days. (— : — : : 2 da. : — da.)

From the conditions of the problem it is seen that the fourth term must be greater than 2 days; for, if to plough only 7 acres require 2 days, to plough 40 acres would require more than 2 days.

As the ratio of the second couplet is, therefore, the ratio of a less quantity to a greater quantity, so is the ratio of the first couplet the ratio of a less quantity to a greater quantity, for the two couplets have the same ratio; hence, the ratio of the first couplet is the ratio of 7 acres to 40 acres (7 A. : 40 A. : : 2 da. : $11\frac{3}{7}$ da.).

187. To place the terms in Rule of Three:

Write for the third term the quantity that is of the same nature as the quantity required.

If the quantity required be greater than the third term, write for the second term the greater of the two terms of the given couplet; if the quantity required be less than the third term, write for the second term the less of the two terms of the given couplet.

4. There are 97 leap-years in 4 centuries; how many leap-years are in 8 centuries?

5. If 20 bushels of corn cost \$15, what should be the cost of 15 bushels of corn?

6. If $\frac{4}{5}$ of a crop is 1,100 bu. corn, what is $\frac{8}{11}$ of the crop?

7. Find the cost of 16 dozen eggs that are bought at the rate of 5 dozen for \$1.

8. If $2\frac{1}{2}$ yards of calico cost 20 cents, how many yards can be bought for a dollar?

9. A farmer uses 25 bushels of corn every 15 days; how much does he use in 365 days?

10. A freight train, at 14 miles per hour, runs from A to B in 24 hours; in what time does an express train make the run at 35 miles per hour?

11. A bushel of corn weighs 56 pounds; a bushel of wheat weighs 60 pounds; how many bushels of corn will weigh as much as 75 bushels of wheat?

12. Find the cost of 448 pounds of corn at \$.65 per bushel.

56 lb. corn : 448 lb. corn : : \$.65 : —.

13. Find the cost of 4,500 lb. wheat @ \$.95 per bushel.

14. The shadow of a tree is 35 feet long at the same time that the shadow of a gate-post measures 3 feet; the post is 8 feet high; find the height of the tree. *Ans.* $93\frac{1}{3}$ ft.

15. B owns 60 of every 100 head of cattle in a herd of 3,450 head; how many cattle are in B's share of the herd?

16. Two partners, D and C, have together invested \$22,000; of every hundred dollars, D invested \$55; find the amount of money that C invested.

17. For every \$100 in \$75,000 a man pays a tax of 75 cents; find his total tax.

18. Find the number of bushels of grain produced on a field of 20 acres, yielding 80 bushels to every 3 acres.

19. A man walks $6\frac{1}{2}$ miles in 2 hours; how far can he walk in 9 hours at the same rate?

20. Walking at the rate of $9\frac{3}{8}$ miles in $3\frac{1}{8}$ hours, in what time can a man walk 20 miles?

21. If a farmer feeds his 8 mules 14 bushels of corn in 6 days, how much corn does he save on Sunday by turning 6 mules into his pasture? *Ans.* $1\frac{3}{4}$ bu.

22. If 12 bundles of fodder weigh 21 pounds, what is the weight of 100 such bundles?

23. At the rate of 14 bales to 42 acres, how many bales of cotton are produced on 126 acres of land?

24. A fence is 960 yards long; the panels are of equal length; there are 12 panels to every 96 feet; find the number of panels in the fence.

25. A farmer with his own force of 6 ploughs can lay by his crop in 10 days, but before he begins he borrows a force of 3 ploughs from a neighbor; in how many days can he complete the work? *Ans.* $6\frac{2}{3}$ days.

26. If 10 bu. cotton-seed be required to plant $3\frac{1}{2}$ acres, how many bushels will be needed to plant a field of 77 acres?

27. If a watermelon weighing 20 pounds cost \$$\frac{7}{20}$, how many cents should be paid for a watermelon weighing 32 pounds?

28. When the cost for mailing a package weighing $\frac{3}{4}$ pound is 6 cents, what is the cost for mailing a package weighing 4 pounds?

29. If 144 cotton-ties bind 24 bales of cotton, how many ties are required for 144 bales?

30. If £20 be worth \$97.33, what is the value in U. S. money of £240?

31. If a gully, averaging $17\frac{1}{2}$ feet wide, has ruined $2\frac{1}{4}$ acres, how many acres would be ruined if it should become 37 feet wide?

32. In $\frac{3}{8}$ of a wall there are 12,600 bricks; how many bricks are there in $\frac{7}{8}$ of the wall? *Ans.* 29,400 bricks.

33. By the scholastic census of a state there are 556,722 pupils; counting 7 persons to 2 pupils, how many people are in the state?

34. When $2\frac{9}{20}$ bushels of wheat make $\frac{1}{2}$ bbl. flour, how many barrels of flour can be made from 490 bushels of wheat?

35. When 15 bushels of wheat make 3 barrels of flour, how many bushels of wheat will make $9\frac{1}{2}$ barrels of flour?

36. If $4\frac{1}{2}$ pounds of nails are required for $\frac{1}{4}$ of a fence, how many pounds are required for $\frac{7}{8}$ of the fence? *Ans.* $15\frac{3}{4}$ pounds.

37. If 9 pounds of nails are required for $\frac{1}{2}$ of a fence, what part of the fence can be built with 7 pounds of nails?

38. If 2,000 lb. cotton-seed yield 35 gallons of oil, how many pounds of cotton-seed will yield 100 gallons of oil?

39. If 17 gal. 2 qt. oil be produced from $\frac{1}{2}$ ton of cotton-seed, how much oil can be produced from 500 lb. cotton-seed?

40. If 5 cords of wood cost $22, what should be the cost of $5\frac{1}{2}$ cords of wood?

41. A man borrowed money, promising to pay it in 4 months, and to pay also $5 for the use of it, but he kept it for 7 months; how much should he pay for the use of it?

42. When 360 pounds of bagging and ties are required for every 15 bales of cotton, what is the weight of the bagging and ties on 1,000 bales?

43. If 36 yards of carpet 36 inches wide be required for a room, how many yards of carpet 27 inches wide would be required? *Ans.* 48 yards.

44. The cost of digging a ditch 40 rods long was $3.00 for every 4 rods; find the whole cost.

45. If $\frac{5}{8}$ bushel of pecans cost $2, what should be the cost of 2 bushels of pecans?

46. If $1\frac{7}{8}$ bushels of corn cost $1.50, what quantity of corn could be bought for $\$\frac{4}{5}$?

47. If a locomotive consume 500 lb. coal for every 20 miles, how many pounds will it consume in running 76 miles?

48. In what time does a man walk 880 yards, when he walks at the rate of 2 miles in 40 minutes?

49. Solve the proportion, $5\frac{1}{2} : \frac{1}{8} :: 3\frac{1}{8} : —$.

COMPOUND PROPORTION.

188. When the product of the ratios of two or more couplets is equal to the ratio of another couplet, the couplets form a **Compound Proportion.**

Thus, $\left\{\begin{matrix} 3:6 \\ 4:5 \end{matrix}\right\} :: 24:60$ is a compound proportion, and means that the product of the ratio of 3 to 6 and the ratio of 4 to 5 is equal to the ratio of 24 to 60.

$$3:6 = \tfrac{1}{2};\ 4:5 = \tfrac{4}{5};\ \tfrac{1}{2} \times \tfrac{4}{5} = \tfrac{2}{5};\ 24:60 = \tfrac{2}{5}.$$

189. The product of two or more ratios is called a **Compound Ratio.**

PROBLEMS.

1. If $60 will pay 20 hands for 2 days' work, how much money will be required to pay 40 hands for 3 days' work?

EXPLANATION.

(1) In the problem there are 2 ratios:
(i) 20 hands to 40 hands: Ratio $= \frac{1}{2}$.
(ii) 2 days to 3 days: Ratio $= \frac{2}{3}$.

(2) Consider the question in regard to the number of hands: 40 hands for 2 days' work should receive twice as much as 20 hands for 2 days' work. The odd term is multiplied by the reciprocal of the first ratio.

(3) Consider now the question in regard to the number of days. 40 hands for 3 days' work should receive $1\frac{1}{2}$ times as much as 40 hands for 2 days' work. The product of the odd term and the reciprocal of the first ratio is multiplied by the reciprocal of the second ratio.

PROCESS.

$$\text{20 hands} : \text{40 hands} = \tfrac{1}{2}.$$
$$\text{2 days} : \text{3 days} = \tfrac{2}{3}.$$

$$\text{20 hands} : \text{40 hands} = \tfrac{1}{2}.$$
$$2 \times \$60 = \$120.$$

$$\text{2 days} : \text{3 days} = \tfrac{2}{3}.$$
$$\tfrac{3}{2} \times \$120 = \$180.$$

SHORT PROCESS.

$$\left\{\begin{matrix} 20:40 = \frac{1}{2} \\ 2:\ 3 = \frac{2}{3} \end{matrix}\right\} \tfrac{1}{2} \times \tfrac{2}{3} = \tfrac{1}{3} \qquad 3 \times \$60 = \$180.$$

190. To find the fourth term of a compound proportion: *Multiply the odd term by the reciprocal of the compound ratio.*

2. If, at 80 cents per bushel, the corn to feed 30 horses for 2 days costs \$12, what is the cost, at 60 cents per bushel, of the corn to feed 8 horses for 15 days?

The Order of the Terms.

The question demands that the answer be a quantity of money. The answer is the fourth term of the incomplete proportion; then the third term must be a quantity of money, else there can be no ratio and no couplet. But the problem gives three quantities of money.

To determine which of the three quantities of money is the third term, either of two courses of reasoning will serve.

1. There is an odd term given; the odd term must, with the fourth term, form a couplet; the odd term is \$12; therefore \$12 is the third term.

2. Of the three quantities of money given, only one is of the same nature as the required answer, which must be, not the cost of a bushel of corn but, the sum of money required to purchase sufficient corn to feed a given number of horses for a given number of days at a given price per bushel; therefore \$12 is the third term, because it also is a sum of money required by all such conditions.

There are three couplets, which, with the third term found and the known unit of quantity of the fourth term, may be represented thus:—

$$\left\{\begin{array}{c}\text{Horses : horses}\\ \text{Days : days}\\ \text{Cost of 1 bu. : cost of 1 bu.}\end{array}\right\} :: \$12 : \$\text{—}.$$

The conditions of the problem determine the ratio of each couplet.

(I) If it costs \$12 to feed 30 horses, it costs less than \$12 to feed 8 horses; the ratio of \$12 to a less sum governs this couplet, whose ratio, therefore, is the ratio of a greater quantity to a less quantity; 30 horses : 8 horses.

(II) If it costs \$12 in 2 days, it costs more than \$12 in 15 days; the ratio of \$12 to a greater sum governs this couplet, whose ratio, therefore, is the ratio of a less quantity to a greater quantity; 2 days : 15 days.

(III) If it costs \$12 for corn @ 80¢ per bushel, it costs less than \$12 for corn @ 60¢ per bushel; the ratio of \$12 to a less sum governs this couplet, whose ratio, therefore, is the ratio of a greater quantity to a less quantity; 80¢ : 60¢.

$$\left\{\begin{array}{r}\text{30 horses : 8 horses} = \frac{15}{4}\\ \text{2 days : 15 days} = \frac{2}{15}\\ \text{80 cents : 60 cents} = \frac{4}{3}\end{array}\right\} \frac{15}{4} \times \frac{2}{15} \times \frac{4}{3} = \frac{2}{3};\ \frac{3}{2} \times \$12 = \$18.\ \textit{Ans.}$$

3. If 10 mules require 180 bundles of fodder for 6 days, how many bundles will be required for 45 mules for 18 days?

4. If 6 ploughs can work 24 acres of corn in 4 days, in how many days can 8 ploughs work 30 acres?

5. If \$1,000 gain \$60 in 12 months, what sum will gain \$120 in 6 months at the same rate? *Ans.* \$4,000.

6. If the interest on \$100 for 12 months is \$6, what is the interest on \$500 for 3 years at the same rate? *Ans.* \$90.

7. If it cost \$45 to keep 20 horses 10 days, how much should it cost to keep 30 horses 8 days?

8. If 30 horses can be fed 8 days at a cost of \$60, how many horses can be fed for 12 days at a cost of \$12?

9. If a single-track railroad require 400 ties for 1,000 feet, how many ties will be required for 1,500 feet of a double-track railroad? *Ans.* 1,200 ties.

10. If 6 tickets for 200 miles' travel cost \$36, what is the cost of 9 tickets for 300 miles at the same rate?

11. If, in a cotton field, where the rows are four feet apart, a man in 2 days can plough 8 acres, 2 furrows to the row, in how many days can he plough 16 acres, 3 furrows to the row?

12. If, in 4-foot rows, a man in 3 days can plough 12 acres, 2 furrows to the row, how many days will he require in 5-foot rows to plough 18 acres, 3 furrows to the row? *Ans.* $5\frac{5}{8}$ da.

13. If the corn for 30 horses for 8 days cost \$45, when corn is selling at \$.75 per bushel, what should be the cost of the corn to feed 45 horses for 4 days, when corn is selling at \$.60 per bushel?

14. If the interest on \$1 for 1 year is \$.06, what is the interest at the same rate on \$12 for $2\frac{1}{2}$ years? *Ans.* \$1.80.

15. If the interest on \$12 for $2\frac{1}{2}$ years is \$1.80, what sum of money will yield \$2.50 in 1 year at the same rate?

16. A man is paid by the hour; if, working 3 hours per day for 4 days, he earns \$4.80, how much will he earn in 5 days working 5 hours per day? *Ans.* \$10.

17. If 14 acres, yielding $22\frac{1}{2}$ bushels per acre, supply a farmer with corn for 8 months, how much per acre must be the yield of 21 acres to supply him for 12 months?

18. If 21 acres, yielding $22\frac{1}{2}$ bushels per acre, supply a farmer with corn for one year, how many acres, yielding $37\frac{1}{2}$ bushels per acre, will supply him for 6 months?

COMPARISON OF VALUES.

191. Comparison of Values is the process of finding a given quantity's value, expressed in a different measure of value.

The process is sometimes called **Conjoined Proportion**.

PROBLEMS.

1. Given: 4 bu. corn is worth 7 bu. oats; 14 bu. oats is worth 5 bu. wheat; how many bushels of corn are equal in value to 1 bushel of wheat?

COMPARISON.

7 bu. oats is worth 4 bu. corn.
14 bu. oats is worth $\frac{14}{7} \times 4$ bu. corn.
5 bu. wheat is also worth $\frac{14}{7} \times 4$ bu. corn.
1 bu. wheat is worth $\frac{1}{5} \times \frac{14}{7} \times 4$ bu. corn.

PROCESS.

$$\frac{1}{5} \times \frac{\overset{2}{\cancel{14}}}{\cancel{7}} \times 4 \text{ bu. corn} = \frac{8}{5} \text{ bu. corn} = 1\frac{3}{5} \text{ bu. corn.}$$

1 bu. wheat is worth $1\frac{3}{5}$ bu. corn.

2. If 5 bu. potatoes is worth 6 bu. oats, and 12 bu. oats is worth $\frac{1}{2}$ ton of hay, how much hay is 1 bushel of potatoes worth?

3. If 2 oranges are worth 3 apples, and 4 apples are worth 6 peaches, and 18 peaches are worth 2 melons, what part of a melon is 1 orange worth?

4. If 2 days' work by A is worth 3 days' work by B, and 6 days' work by B is worth 7 days' work by C, what part of a day should A work to equal the value of a day's work by C?

Ans. $\frac{4}{7}$ day.

PARTITIVE PROPORTION.

192. The division of a number or a quantity into proportional parts is called **Partitive Proportion.**

PROBLEMS.

1. Divide $500 into two such parts that one part will be to the other part as 5 to 3.

COMPARISON.

If one part is to the other part as 5 to 3, the whole is to the smaller part as 8 to 3, and the smaller part is $\frac{3}{8}$ of the whole.

$\frac{3}{8}$ of $500 = $\frac{1}{8}$ of $1,500 = $187.50.
The larger part is $500 − $187.50 = $312.50.

2. Divide $1,000 into two such parts that one part will be to the other part as 12 to 13.

3. Divide 360 acres of land into two such parts that one part will be to the other part as 19 to 5. *Ans.* 285 A.; 75 A.

4. If two partners gain in trade $1,000, what should be the profit of each, one having contributed $9 of the capital to every $5 that the other contributed?

5. Divide $1,500 into three parts that are to each other as 1, 2, and 2. *Ans.* $300; $600; $600.

6. Divide $1,600 into three parts that are to each other as 2, 3, and 3.

7. Divide $1,800 into three parts that are to each other as 2, 3, and 4.

8. Two partners having a capital of $10,000, gained $2,000. A's part of the capital was $7,500; what should be his profit?

$10,000 : $7,500 : : $2,000 : A's profit.

9. B & C's capital was $16,800; B's part was $7,200. They lost $2,800; find C's part of the loss. *Ans.* $1,600.

10. A, B, and C have a capital of $15,000; A's part was $4,000, and B's $5,000; they gained $3,000; find C's share of the profit.

11. A invested $6,000, and B $4,000; at the end of a year the profit was $2,000, but A had for 2 months of the year withdrawn his share of the capital; what should be A's share of the profit?

The capital for the year was only $9,000; A withdrew $6,000 for 2 months, lessening the year's capital by $1,000.

12. In a partnership in business A had an investment of $6,000 for 4 months, B $4,000 for 5 months, and C $2,000 for 6 months; at the end of 6 months the business showed a loss of $98; how much did each partner lose?

Comparison.

The unit, or standard of comparison, is 1 month.

Regard the investment for each succeeding month as a separate and distinct investment.

A four times invested $6,000 for 1 month = $24,000 for 1 month.
B five times invested $4,000 for 1 month = $20,000 for 1 month.
C six times invested $2,000 for 1 month = $12,000 for 1 month.
Their total investments for 1 month = $56,000

The proportional losses are found by processes previously given.

13. A, B, and C are partners. A's investment, $2,000, is used for 2 years; B's investment, $3,000, is used for 18 months; C's investment, $3,000, is used for 20 months; divide the profit, $3,240, proportionally.

Ans. A $960; B $1,080; C $1,200.

14. Three men bought a plantation for which $15,000 was paid: A furnished $8,000, B $4,000, and C $3,000; how much did each receive one year afterward when they sold the plantation for $21,000?

15. A's investment is $\frac{3}{8}$ of the whole; B's investment is $\frac{1}{4}$ of the whole; C's investment is $3,000. A's money is in use the whole time; B's money is in use $\frac{3}{4}$ of the time; C's money is in use $\frac{7}{8}$ of the time; divide the profit, $2,850, proportionally.

16. A's investment is $\frac{3}{8}$ of the whole; B's investment is $\frac{1}{4}$ of the whole; the whole investment is $3,600. A's money is in use 12 months; B's money is in use $\frac{3}{8}$ as long as A's; C's money is in use $\frac{7}{8}$ as long as B's; divide the profit, $730, proportionally.

REVIEW PROBLEMS.

1. If $ $\frac{3}{4}$ will pay for $\frac{3}{10}$ bushel of wheat, for what part of a bushel of wheat will $ $\frac{3}{10}$ pay?

2. How many rails, each 30 feet long, are there in 10 miles of a double-track railroad?

3. If the interest on $1 for one year is $.08, what is the interest on $10 for 2 years? *Ans.* $1.60.

4. If $\frac{3}{8}$ of a number is 4,840, what is $\frac{3}{4}$ of the number?

5. The difference in weight between 10 pounds of feathers and 10 pounds of gold equals how many Troy grains?

6. A field of 30 acres yields $26\frac{1}{2}$ bushels of grain per acre; find the amount paid for the total yield at the rate of $4 for every 5 bushels. *Ans.* $636.

7. Find the difference of the product of $62\frac{1}{2}$ times 38, and $38\frac{1}{2}$ times 62.

8. At 3 cents a mile for what distance is a railway ticket that cost $10.50?

9. When it is 7 o'clock A.M. at Paris, 2° east longitude, what is the time at New Orleans, 90° west longitude?
Ans. 12:52 A.M. by N. O. time.

10. If a ship sail at the rate of 352 yards per minute, how many miles would she sail in 24 hours?

11. Find the reciprocal of the ratio of $\frac{3}{8} : \frac{3}{4}$.

12. A man purchased a horse, and sold him for $150; the amount he paid was to the amount he received as 1 to $1\frac{1}{14}$; how much did he pay for the horse?

13. If a number be divided by 3, the quotient will be the reciprocal of the ratio of 1 : 3; find the number.

14. How many turkeys @ $1.60 apiece would be equal in value to 48 chickens @ $ $\frac{1}{5}$ apiece?

15. A ploughman who makes one furrow at a time, is ploughing in rows that are 200 yards long; how many furrows must he make to walk 18 miles? *Ans.* $158\frac{2}{5}$ furrows.

16. Put 1,452 bushels of oats into an equal number of $2\frac{1}{2}$-bushel and 3-bushel sacks; how many sacks are there of each size? *Ans.* 264.

17. What does a man pay for 2 years' use of $100 borrowed money, the rate of interest being $.07 on the dollar per year? *Ans.* $14.

18. Two ships pass each other at sea: one goes $12\frac{1}{2}$ miles per hour, the other $14\frac{1}{2}$ miles per hour; in what time will they be $3\frac{1}{2}$ miles apart? *Ans.* $7\frac{7}{9}$ minutes.

19. If beef sells at $8\frac{1}{2}$ pounds to the dollar, how much can be bought for $4.40?

20. A man builds a fence round a square garden 200 feet to the side; he uses palings 3 inches wide, and nails them 2 inches apart; how many palings are in the fence?

21. If 500 lb. cotton-seed yield $8\frac{3}{4}$ gallons of oil, how many quarts of oil can be produced from 3 tons of cotton-seed?

22. A man borrowed $700, promising to pay in 6 months, and to pay for the use of the money $.03 on every dollar borrowed; at the end of 9 months he pays the debt with interest for the whole time. How much does he pay? *Ans.* $731.50.

23. Two trains pass each other on a double-track railway; in 3 minutes they are 6,600 yards apart; one runs at the rate of 40 miles per hour; find the speed per hour of the other train. *Ans.* 35 miles.

24. A man sells a farm for $2,200, gaining 10 cents on each dollar that he paid for the farm; find the amount that he paid for the farm. *Ans.* $2,000.

25. Three men catch 144 trout, and share them according to the number of persons in the family of each. In A's family there are 4 persons, in B's 5, and in C's 7; how many trout does each man take home?

26. If 40 hoes cost $20, and 100 axes cost $75, how many axes cost as much as 24 hoes?

27. A man borrowed \$500 for 2 years, and paid, at the end of the time, the money borrowed and \$.14 besides on each dollar borrowed; how much did he pay in all? *Ans.* \$570.

28. A train running 25 miles per hour was delayed $1\frac{1}{4}$ hr. by a wash-out; it then increased its speed $\frac{1}{5}$, and reached B just on time; find the distance from B to the point where the wash-out took place. *Ans.* $187\frac{1}{2}$ miles.

29. At the rate of 3 miles per hour, a raft floats past the landing at 9 A.M.; the down-going steamer, at the rate of 15 miles per hour, passes the landing at 5 P.M. What o'clock is it when the steamer overtakes the raft? *Ans.* 7 P.M.

30. A man sends by telegraph a night message of 18 words; the rate for a day message is 60 cents for the first 10 words, and 3 cents on each word over 10; the night rate is $\frac{2}{3}$ of the day rate; what does the man pay?

31. A man invested \$2,000 in a house and lot, which he afterwards sold for \$2,200; how many cents did he gain on each dollar invested? *Ans.* 10 cents.

32. A rents to B a field of 42 acres; they agree that the field shall be planted in corn, and that the rent shall be $\frac{1}{4}$ of the crop of corn; the yield is $27\frac{1}{2}$ bushels per acre; corn is selling at \$.65 per bushel; find the value of the rent.

33. A farmer uses for all his stock $4\frac{3}{8}$ bushels of corn per day; he uses for meal $1\frac{5}{8}$ bushels of corn per day; of his crop from $62\frac{1}{2}$ acres he sells 170 bushels of corn, and still has a year's supply of new corn; find the yield per acre of the $62\frac{1}{2}$ acres.

34. At \$.01$\frac{1}{2}$ per mile for hauling 100 pounds of freight, what does a wagoner charge for hauling 5,000 pounds 20 miles?

35. A, B, and C engage in trade. A's investment is \$500, B's \$600, and C's \$700. A's money is in use 7 months, B's 6 months, and C's 5 months; how shall they share a profit of \$265?

36. A boat goes up the river at a rate of 9 miles per hour; she comes down at the rate of 13 miles per hour. Allowing 2 hr. 30 min. for stops, in what time will she go up the river 65 miles and return?

37. From D to C the distance is 300 miles; a train starts from D at 7 A.M., and arrives at C at 7 P.M.; another train starts from C at 8:30 A.M., and arrives at D at 8:30 P.M. The rate of speed of both trains being constant, at what hour do they meet? *Ans.* 1:45 P.M.

38. A man pays \$28 interest for 4 years' use of \$100 borrowed money; find the interest for one year on \$1.

Ans. \$.07.

39. A cotton-merchant bought 40 bales of low-middling cotton, averaging 461 pounds to the bale, @ \$.07$\frac{1}{4}$, and 30 bales of middling cotton @ \$.07$\frac{1}{2}$; he paid \$2,369.65 for the 70 bales; find the average weight of the 30 bales. *Ans.* 459 pounds.

40. A farmer has 4 stacks of fodder in a ten-acre field; 3 of the stacks average 250 bundles to the stack; in the other stack there are 290 bundles; find the weight of the whole at 1$\frac{7}{8}$ pounds to the bundle.

41. Find the cost of ploughing 16$\frac{1}{2}$ acres at the rate of \$.02 per square rod.

42. Find the cost of painting a ceiling 36 feet by 24 feet @ \$1.25 per square yard.

43. A man owes \$12,000; failing in business he has only \$8,000 with which to pay; how much is paid on the dollar?

Ans. \$.66$\frac{2}{3}$.

How much is paid to a creditor whose approved claim is \$560? *Ans.* \$373.33.

44. If $\frac{2}{5}$ of $\frac{3}{7}$ of a number is 420, what is $\frac{2}{3}$ of $\frac{5}{7}$ of the number?

45. A farmer's hoe-hands can hoe 11 acres of cotton per day; his plough-hands can plough 15$\frac{1}{2}$ acres per day; the hoes are 13$\frac{1}{2}$ acres ahead of the ploughs; in what time will the ploughs overtake the hoes?

46. A man pays \$35 for one year's use of \$500; find the interest on \$1. *Ans.* \$.07.

ADDITIONAL REVIEW PROBLEMS.

Page 139. Solve by *analysis* instead of *proportion*.

FOR ORAL WORK.

Review — Induction.

1. What is a unit? What is a ratio?

2. What is the ratio of $\frac{1}{3}$ to $\frac{2}{3}$?

3. What is a reciprocal? What is the reciprocal ratio of $\frac{1}{3}$ to $\frac{2}{3}$?

4. What are Aliquot Parts of numbers?

5. Give five of the aliquot parts of $1.

6. What is a couplet? An antecedent? A consequent?

7. When are four numbers said to form a proportion?

8. When the first three terms of a proportion are given, how is the fourth term found?

9. The ratio is 2; the third term is 7; what is the fourth term?

10. The ratio is $\frac{1}{3}$; the fourth term is 9; what is the third term?

11. The product of what two terms equals the product of what other two terms?

12. When are numbers said to form a compound proportion?

13. What is the ratio of 1 cent to 1 dollar?

14. What is the ratio of 1 nickel to 1 dollar?

15. Of what number is 6 six hundredths?

16. Of what number is 9 nine hundredths?

17. A year is how many hundredths of a century?

18. A man rode 100 miles for $3; what rate did he pay per hundred miles? What is the ratio of 3 cents to $3?

19. How many hundredths of a dollar is $.25?

20. A man borrowed $100 for a year, and paid $7 for the use of the money; how many cents did he pay for the use of $1?

21. What is the ratio of 7 dollars to 100 dollars?

22. If money is loaned for a year at the rate of $7 on the hundred dollars, what is the rate on $1?

23. If it costs $1.50 to ship 200 pounds of freight, what is the rate per hundred?

24. What is the ratio of 12 cents to $1?

PERCENTAGE.

DEFINITIONS AND PRINCIPLES.

193. The general name **Percentage** is applied to operations in each of which the given or required **Ratio** is a fraction whose denominator is 100.

194. In Percentage the ratio is called the **Rate.**

The words per cent mean by the hundred:
6 per cent means 6 : 100, or $\frac{6}{100}$ to 1.

195. The sign of Percentage (%) is read *per cent.*

6 % is a contracted form of $6 \times \frac{1}{100}$; it is written decimally, .06, or as a common fraction $\frac{6}{100}$, and in words *six per cent.*

196. In Percentage two numbers are given, and a third number is required.

(i) The number 1,000 and the rate 6 %, are given; it is required to find 6 %, or $\frac{6}{100}$, of 1,000. $\frac{6}{100} \times 1{,}000 = 60$.

(ii) The numbers 60 and 1,000 are given; it is required to find how many hundredths of 1,000 are in 60. $60 : 1{,}000 = \frac{6}{100}$, or 6 %.

(iii) The rate 6 %, or $\frac{6}{100}$, and the number 60 are given; it is required to find the number, of which 60 is $\frac{6}{100}$. If 60 is $\frac{6}{100}$ of the number, the number is $\frac{100}{6}$ of $60 = 1{,}000$.

197. The number or quantity of which a number of hundredths is given or required, is called the **Base.**

198. The number or quantity which is a given or required number of hundredths of the **Base** is called the **Percentage.**

199. The sum of the **Base** and the **Percentage** is called the **Amount.** When the **Percentage** is subtracted from the **Base,** the remainder is called the **Difference.**

200. The Rate per hundred units of the Base is called the **Rate Per Cent.**

Thus; $6 : 100 = \frac{6}{100}$ to 1; the *rate* is $\frac{6}{100}$, or .06, or 6%.
The *rate per cent*, or rate per hundred, is 6.

EXERCISES.

1. Express 5% as a common fraction. *Ans.* $\frac{5}{100}$.
2. Express 6% as a decimal fraction. *Ans.* .06.
3. What per cent is $\frac{7}{100}$? *Ans.* 7%.
4. Express 12% both as a common and a decimal fraction.
5. Express 9% both as a common and a decimal fraction.
6. Express 5% both as a common and a decimal fraction.
7. What per cent of a number is $\frac{3}{4}$ of the number?
8. What per cent of a number is $\frac{3}{10}$ of the number?
9. What per cent of a number is $\frac{7}{8}$ of the number? *Ans.* $87\frac{1}{2}\%$.
10. What per cent of a number is $\frac{3}{8}$ of the number? *Ans.* $37\frac{1}{2}\%$.

To find the Percentage.

201. To find the percentage of a number is to find a given number of hundredths of the number. (See **196.** i.)

PROBLEMS.

1. A man paid $150 for a horse, which he afterward sold at a profit of 16%; what did he gain?

PROCESSES.

ANALYSIS.

100% of cost = $150.
1% of cost = $1.50.
16% of cost = 16 × $1.50 = $24.
The gain = $24.

$$\begin{array}{r} \$150 \\ .16 \\ \hline \$24.00 \end{array}$$

$\frac{16}{100} \times \$150 = \$24.$

2. Find 22% of a farm consisting of 350 acres.
3. Find 33% of the population of a city containing 75,000 people.
4. Find $7\frac{1}{2}\%$ of a crop amounting to 36 bales of cotton averaging 450 pounds to the bale.

5. Find $2\frac{1}{4}\%$ of the area of a state which contains 265,780 square miles.

6. Find $8\frac{1}{2}\%$ of the cost of 1,000 bushels of wheat @ \$.85 per bushel.

7. Find $\frac{5}{8}\%$ of the cost of picking 7,200 pounds of seed-cotton at \$.55 per hundred.

8. A farmer buys a field which adds 19% to his original farm of 300 acres; of how many acres does his farm consist?

PROCESSES.

(1.)	(2.)
300 Acres.	300 Acres.
.19	1.19
57.00	2700
300	3300
357 Acres.	357.00 Acres.

In either process the base is added to the percentage.

9. A merchant sold his stock of goods for 96% of the cost; the cost was \$3,800; how much did he lose?

$$100\% - 96\% = 4\%;\ .04 \times \$3800 = \$152.$$

10. U. S. coin is made of 90% pure metal, and 10% alloy; how many grains of gold are in 200 five-dollar coins, each weighing 129 grains?

11. Find the difference between $2\frac{3}{4}\%$ of \$5,000 and $3\frac{1}{4}\%$ of \$6,000.

12. A man's farm is in 3 fields; in the first field there is 32% of the farm, in the second, 40%. The farm consists of 390 acres; how many acres in the third field?

13. A man earns \$75 per month, and spends each week 22% of his month's wages; counting 4 weeks to the month, what does he spend each month?

14. How many quarts are $2\frac{1}{2}\%$ of 1,500 bushels?

15. How many pounds are $3\frac{1}{3}\%$ of 1,500 tons?

16. How many minutes are $6\frac{1}{4}\%$ of 24 hours?

17. How many yards are $\frac{1}{8}\%$ of 5 miles? *Ans.* 11 yards.

18. How many cents are $8\frac{1}{8}\%$ of \$1,200?

19. How many ounces are $12\frac{1}{2}\%$ of 2 tons?
20. How many pints are $16\frac{2}{3}\%$ of 900 gallons?
21. How many seconds are $\frac{1}{6}\%$ of 1 week?
22. How many rods are $\frac{1}{8}\%$ of 400 miles?
23. How many square feet are $33\frac{1}{3}\%$ of $3\frac{1}{8}$ acres?
24. How many square yards are 75% of 1 square mile?
25. How many cubic inches are $16\frac{2}{3}\%$ of 6 cubic yards?
26. How many Troy grains are $\frac{1}{2}\%$ of 4 Avoirdupois pounds?
27. How many farthings are $66\frac{2}{3}\%$ of £2?
28. How many bushels are $87\frac{1}{2}\%$ of 1,600 quarts?
29. How many gallons are $\frac{7}{8}\%$ of 2,400 quarts?
30. How many miles are $2\frac{1}{2}\%$ of 352,000 yards?
31. How many square rods are $93\frac{3}{4}\%$ of 1 square mile?
32. How many days are $62\frac{1}{2}\%$ of 16 weeks?
33. How many minutes are $\frac{5}{16}\%$ of 1 day?
34. Find $12\frac{1}{2}\%$ of the decimal .12344. *Ans.* ——
35. Find $37\frac{1}{2}\%$ of the decimal .89064. *Ans.* .33399.
36. Find $31\frac{1}{4}\%$ of the decimal .16016.
37. Find $33\frac{1}{3}\%$ of the decimal .8976.
38. Find $87\frac{1}{2}\%$ of the number 123.28. *Ans.* 107.87.

202. The base and rate being given, to find the percentage: *Multiply the base by the rate.*

To find the amount:

Multiply the base by 1 plus the rate.

To find the difference:

Multiply the base by 1 minus the rate.

Concise expressions, called *formulas*, are frequently employed, in order to show the relation of any one of the terms to the others. Let A represent the amount, B the base, D the difference, P the percentage, and R the rate.

The product of the base by the rate equals the percentage.

FORMULAS.

For the percentage,	$R \times B = P$
For the amount,	$(1 + R) \times B = A$
For the difference,	$(1 - R) \times B = D$

To find the Rate.

203. To find the rate is to find the ratio of the percentage to the base.

PROBLEMS.

1. A man paid $150 for a horse, which he afterward sold at a profit of $24. What per cent did he gain?

ANALYSIS.

$\$150 = 100\%$ of cost.
Gain of $\$1 = \frac{100}{150}\%$ of cost.
Gain of $\$24 = 24 \times \frac{100}{150}\%$ of cost.
$24 \times \frac{100}{150}\%$ of cost $= 16\%$ of cost.
Gain $= 16\%$ of cost.

COMPARISON.

Profit on $\$150 = \$24$.
Profit on $\$1 = \frac{1}{150}$ of $\$24$.
$\frac{1}{150}$ of $\$24 = \$.16$.
Rate is $\$.16$ to $\$1 = .16$, or 16%.

PROCESS.

$\frac{\$24}{\$150} = .16.$

AS A RATIO.

$\$24 : \$150 = \frac{16}{100}$, or 16%.

2. A miller takes for toll $\frac{1}{7}$ of the corn he grinds; what per cent of the corn does he take?

3. A farmer bought land at $10 per acre, and sold it at $12 per acre; what per cent did he gain?

PROCESSES.

(1.)

$$\begin{array}{r} \$12 \\ \underline{10} \\ \$10)\underline{\$2.00} \\ .20 \end{array}$$

(2.)

$$\begin{array}{rl} \$10)\underline{\$12.00} & \\ 1.20 & = 1 + \text{rate}. \\ \underline{1.} & \\ .20 & ;\ \text{rate} = .20 \text{ to } 1, \text{ or } 20\%. \end{array}$$

COMPARISON (2).

For $10 invested he receives $10 + $2 *gain.*
For $1 invested he receives $1 + $.20 *gain.*
The rate of gain is 20 cents to 1 dollar.
As a ratio; $\$.20 : \$1 = \frac{20}{100}$, or 20%.

4. A merchant bought sugar at 5 cents a pound, and sold it for 6 cents a pound; what per cent did he gain?

5. A bale of cotton was bought for $42.50, and was afterward sold for $44.05; what per cent was gained?

6. A farmer sowed 48 bushels of wheat, and harvested 1,488 bushels; find the per cent increase. *Ans.* 3,000%.

7. If $66\frac{2}{3}\%$ of a number is $66\frac{2}{3}$, what is the number?

8. A trader made a profit of 25% by selling sheep at \$5 a head; what did he pay per head for the sheep?

ANALYSIS.

He paid 100% of cost.
He received 125% of cost.
125% of cost = \$5.
1% of cost = $\frac{1}{125}$ of \$5.
$\frac{1}{125}$ of \$5 = \$.04.
100% of cost = \$4.

COMPARISON.

Selling price = $\frac{5}{4}$ cost.
$\frac{4}{5}$ of selling price = cost.
$\frac{4}{5}$ of \$5 = \$4.

By Proportion.

125% : 100% : : \$5 : —.

9. A sold a horse to B for \$108 and gained 20%; what did A pay for the horse?

10. A grocer sold sugar for \$106, and gained 6 per cent; what did he pay for the sugar?

11. A drover sold horses at \$90 a head and lost 10%; what did he pay per head for the horses?

ANALYSIS.

He paid 100% of cost.
He lost 10% of cost.
He received 90% of cost.
90% of cost = \$90.
1% of cost = \$1.
100% of cost = \$100.

COMPARISON.

Selling price = $\frac{9}{10}$ cost.
$\frac{10}{9}$ of the selling price = cost.
$\frac{10}{9} \times \$90 = \$100$.

By Proportion.

90% : 100% : : \$90 : —.

12. B sold his land for \$24 per acre, at a loss of 25%; what had he paid per acre?

13. Find the quantity of which $3\frac{1}{3}$ acres is $33\frac{1}{3}\%$.

14. Find the quantity of which 15 quarts is $2\frac{1}{2}\%$.
Ans. 150 gallons.

15. Find the quantity of which 1 square mile is $62\frac{1}{2}\%$.

16. Find the quantity of which 1 square rod is $2\frac{1}{2}\%$.

17. Find the quantity of which 3 pecks is 75%.

18. Find the quantity of which 116 pounds is $33\frac{1}{3}\%$.

19. Find the quantity of which 250 bushels is $93\frac{3}{4}\%$.

20. Find the quantity of which \$6.25 is $\frac{1}{4}\%$. *Ans.* \$2,500.

21. Find the quantity of which \$18.75 is $37\frac{1}{2}\%$.

22. Find the quantity of which \$31.25 is $\frac{5}{8}\%$.

23. Find the quantity of which \$10 is $18\frac{3}{4}\%$.

24. Find the quantity of which $1.25 is $16\frac{2}{3}\%$.
25. Find the quantity of which $3.75 is $3\frac{1}{8}\%$.
26. Find the number of which $33\frac{1}{3}$ is $33\frac{1}{3}\%$.
27. Find the number of which $66\frac{2}{3}$ is $16\frac{2}{3}\%$.
28. Find the number of which $93\frac{3}{4}$ is $\frac{15}{16}\%$.
29. Find the number of which $22\frac{1}{2}$ is $11\frac{1}{4}\%$.
30. Find the number of which $52\frac{1}{2}$ is $17\frac{1}{2}\%$.

206. The percentage and the rate given, to find the base:
Divide the percentage by the rate.

The amount and the rate given, to find the base:
Divide the amount by 1 plus the rate.

The difference and the rate given, to find the base:
Divide the difference by 1 minus the rate.

FORMULAS.

$$\mathrm{P} \div \mathrm{R} = \mathrm{B} \qquad \mathrm{A} \div (1 + \mathrm{R}) = \mathrm{B} \qquad \mathrm{D} \div (1 - \mathrm{R}) = \mathrm{B}$$

PROFIT AND LOSS.

207. The principles of percentage, as already given, apply to all operations for ascertaining the profit or the loss in business transactions.

In profit and loss the cost, or amount invested, is the base.

PROBLEMS.

1. A farm was bought for $6,400, and was afterwards sold at a gain of 6%; what was the profit?

2. In order to gain 5%, for what price per bushel must a dealer sell wheat that cost him $.80 per bushel?

3. For what price per barrel does a dealer, who loses 8%, sell flour that cost him $4.25 per barrel?

4. In order to gain 15%, for what price per yard must a merchant sell silk that cost him $2 per yard?

5. A man paid $80 for a wagon, and sold it at a loss of 25%; how much did he get for it?

6. A man bought a house for $2,500, and sold it at a profit of $200; what per cent did he gain?

7. A trader bought a lot of sheep for $550, and sold them for $600; what per cent did he gain?

8. A dealer bought a carriage for $360, and afterward sold it for $320; what per cent did he lose?

9. A dealer gains $40 in selling a lot, making a profit of 20% on his investment; what did he pay for the lot?

10. A merchant received $450 for goods sold at $12\frac{1}{2}\%$ profit; what did the goods cost him?

11. A merchant receives $300 for goods sold at $6\frac{1}{4}\%$ loss; what did the goods cost him?

12. If a butcher buys beef at $.06 per pound, at what price must he sell it to gain $66\frac{2}{3}\%$?

13. A contractor lost $2\frac{1}{2}\%$ on a house, the erection of which cost him $3,100; how much money did he lose?

14. A merchant bought coffee @ $$.18\frac{3}{4}$ per pound, and sold it at a profit of $33\frac{1}{3}\%$; find the selling price.

15. What per cent is gained in buying cotton @ $.08 per pound, and selling it @ $$.08\frac{1}{2}$? *Ans.* $6\frac{1}{4}\%$.

16. What per cent is lost in purchasing a farm for $3,260, and selling it for $3,000?

17. What per cent is lost in buying wheat @ $.90 per bushel, and selling it @ $.85 per bushel? *Ans.* $5\frac{5}{9}\%$.

18. A grocer sold eggs at $$.27\frac{1}{2}$ per dozen, and lost $8\frac{1}{3}\%$; find the cost per dozen.

19. A dealer began business with $4,500; the first year he cleared 18%, which he added to his capital; what was his capital at the beginning of the second year?

20. A trader bought 150 head of cattle @ $22 per head; 6 of them were lost, but he sold the others at such price that he gained $9\frac{1}{11}\%$ on his purchase; at what price did he sell?

21. A merchant gained $.04 per yard on sheeting which he sold at 25% profit; what had he paid for the sheeting?

22. A bought two lots, one costing as much as the other; he sold one for $1,200, and gained $33\frac{1}{3}\%$; he sold the other at a profit of 50%; how much did he gain on the two lots?

Ans. $750.

23. W made two equal investments; he gained by one of them 20%, and lost by the other 25%; the money received from both was \$4,875; what had he invested? *Ans.* \$5,000.

24. If butter is bought @ \$.27½ per pound, and is sold at $27\frac{3}{11}$% profit, what is the selling price?

25. What per cent is gained in buying 300 barrels of flour at an entire cost of \$1,350, and selling it at \$4.75 per barrel?

26. What per cent is saved by purchasing a barrel of sugar containing 250 pounds @ \$.05½ per pound, instead of making 10 purchases, of 25 pounds of sugar each, @ \$.06 per pound?

27. A dealer paid \$30 for 12 boxes of lemons, and sold them @ \$3.50 per box; what per cent did he gain?

28. A cotton-merchant lost $1\frac{3}{7}$% on a purchase made @ \$.08¾ per pound; for what price did he sell? *Ans.* \$.08⅝.

29. What per cent is $\frac{25}{100}$ of 25%? *Ans.* 6¼%.

30. A dealer buys a case of shoes containing 60 pairs for \$84; at what price per pair must he sell in order to gain 25%?

31. A trader paid \$70 for a horse, and in selling him gained $\frac{1}{7}$ of the cost; what per cent was gained?

32. A dealer bought goods and sold them at a profit of ⅙ of the cost; what per cent was gained?

33. A grocer bought eggs at 15 cents per dozen, and sold them to a huckster at a profit of 33⅓%; the huckster sold them at a profit of 25%; find the huckster's selling price.

Ans. 25¢ per dozen.

34. A stationer who pays \$1.20 per ream for paper, sells it by the quire at a profit of 66⅔%; find his price per quire.

35. A grocer bought salt at 16⅔% less than the market price, and sold it at 20% more than the market price; what per cent did he gain? *Ans.* 44%.

36. An agent bought cotton-seed @ 12¢ per bushel of 32 pounds, the market price being \$9 per ton; what per cent was saved?

37. If 40 gross of buttons cost a dealer \$72, at what price per dozen must he sell them in order to gain 66⅔%?

Ans. 25¢ per dozen.

38. If the selling price of goods is $\frac{5}{6}$ of the cost, what per cent is lost?

39. A merchant pays $450 for 100 barrels of flour; the freight costs him 45¢ per barrel, and the drayage 5¢ per barrel; at what price must he sell to gain 10%?

40. A retailer sells potatoes @ $.40 a peck and gains 60%; what did he pay per bushel?

41. If the selling price of goods is $\frac{7}{6}$ of the cost, what per cent is gained?

42. If the cost of goods is $\frac{5}{6}$ of the selling price, what per cent is gained?

43. A merchant bought 700 bushels of wheat @ $.70 per bushel, and sold the whole for $500; what per cent did he gain?

44. A ranchman lost in a norther $16\frac{2}{3}$% of his herd of cattle, and had 2,430 head remaining; how many did he lose?

45. A square mile of land cost the purchaser $1,920; he afterward sold it by the acre at a profit of $133\frac{1}{3}$%; at what price per acre did he sell? *Ans.* $7 per acre.

46. What per cent is gained in buying wool at $.$16\frac{1}{2}$ per pound and selling it @ $.$18\frac{3}{4}$ per pound?

47. A man bought two sections of land at $2 per acre; ten years afterward he sold the whole for $12,000; what per cent did he gain? *Ans.* $368\frac{3}{4}$%.

48. A train's regular rate of speed is 30 miles per hour; if it increases this rate to 35 miles per hour, by what per cent is its rate increased?

49. In 1883 a state legislature passed an act requiring railroads to reduce the passenger-rate from 5¢ per mile to 3¢ per mile; what per cent reduction was required?

50. Receipts from the sales of tickets last year amounted to $1,000,000; passenger-rates are reduced this year 40%; travel is increased 80%; the increased expense to the railroads is $30,000; find the gain to the railroads.

51. Mr. Jones deposited $1,000 in bank; the first week he drew out $\frac{1}{4}$ of it, the next week 20% of it, and the next week 15% of it; how much did he then have in bank? *Ans.* $400.

52. Mr. Rogers had a deposit of \$1,600 in bank; the first week he drew out 20% of it; the next week he drew out 25% of the remainder, and the next week he drew out the balance; how much did he draw the third week? *Ans.* \$960.

TRADE DISCOUNT.

208. A reduction of the price of any article offered for sale is called a **Discount**.

From their printed, or list, price manufacturers and jobbers give discounts to the retailer in order that he may sell for a profit at the printed price.

The discounts are reckoned at so many per cent; frequently more than one discount is given.

Thus "20 and 5" means 20% less than 100%, 80%, and then 5% less than 80%, 76%.

The cost of goods at a discount of "20 and 5" is 76% of the list or printed price.

PROBLEMS.

1. A dealer purchased a bill of goods, the invoice being for \$400 with 20% off; find the cost. *Ans.* \$320.

2. An invoice was \$350, 25% off; find the cost of the goods.

3. An invoice was \$500 discounted 20% and 10%; find the cost of the goods. *Ans.* \$360.

4. Find the difference between an invoice of \$132, with 25% discount, and an invoice of \$132, discounted 20% and 5%. *Ans.* \$1.32.

5. What is the difference on an invoice of \$340, between 40% direct discount, and discounts of 25% and 15%?

6. A dealer buys a book, list price \$1.00, at a discount of 20%; he sells the book for \$1.00; what per cent is his profit?

7. Find the cost of a bill of goods, at list price \$580, with discounts of 20%, 10%, and 5%.

8. The list price is \$375; discounts, 20%, 15%, and 5%; find the cost.

9. The list price is \$570; discounts 40% and 10%; find the cost.

10. Find the difference between a direct discount of 20%, and discounts of 15% and 5%. *Ans.* 20% is $\frac{3}{4}$% greater.

11. A man paid $190 for goods at discounts of 20% and 5%; find the list price of the goods.

ANALYSIS.

1st Dis. = 20% of 100% = 20%.	76% of list price = $190.
2d Dis. = 5% of 80% = 4%.	1% of list price = $\frac{1}{76}$ of $190 = $2.50.
Both discounts = 24%.	100% of list price = 100 × $2.50 = $250.

12. A dealer paid $288 for goods at 20% and 10% off; find the list price of the goods. *Ans.* $400.

13. Find the difference between a direct discount of 40% and discounts of 20% and 20%.

14. A dealer paid $320 for goods at 20% and 20% off; find the list price.

COMMISSION.

209. Agents, Commission Merchants, Factors, and Brokers, are names applied to those who transact business for others, either in buying or selling; their compensation is a percentage of the amount spent or received, and is called **Commission.**

(A Broker usually sells or buys only by sample, and is not in possession of the goods sold; his percentage is called brokerage.)

210. The one who sends to a commission merchant goods to be sold is called the *shipper* or *consignor.* The one to whom the goods are shipped, or consigned, is called the *consignee;* the goods are called a *consignment.* The money due to the consignor after all expenses of shipping, selling, etc., have been paid is called the **Net Proceeds.**

211. The Principles of Percentage, as already developed, apply to Commission.

PROBLEMS.

1. A real estate agent sells for A a house and lot; the amount paid is $3,000; the agent charges a commission of $2\frac{1}{2}$% for selling; how much does A receive? *Ans.* $2,925.

2. A commission merchant sells a consignment of goods for $540.50; after deducting 2% for commission, $7.50 for freight and drayage, and $1.00 for insurance, how much does he pay to the shipper?

3. A travelling salesman who receives a percentage of 5% instead of a salary, sells during the year goods to the amount of \$75,000; deducting \$150 per month for expenses, what is his net income? *Ans.* \$1,950.

4. A commission merchant sells a consignment of oats for \$1,728; he pays \$15.50 for freight, etc., and charges $1\frac{3}{4}\%$ commission; find the net proceeds.

5. On \$7,000 received an agent charges $1\frac{1}{2}\%$, and invests the remainder in unimproved land at \$5 per acre; how much land does he buy? *Ans.* 1,379 acres.

6. An agent who charges 2% on money actually invested, receives \$3,978 to be invested in town lots; find the actual investment.

ANALYSIS.

$$
\begin{aligned}
100\% \text{ of investment} + 2\% \text{ of investment} &= \$3{,}978.\\
102\% \text{ of investment} &= \$3{,}978.\\
1\% \text{ of investment} &= \tfrac{1}{102} \text{ of } \$3{,}978.\\
100\% \text{ of investment} &= 100 \times \tfrac{1}{102} \text{ of } \$3{,}978 = \$3{,}900.
\end{aligned}
$$

7. Find the sum invested when an agent first deducts $2\frac{1}{2}\%$ commission on investment, the sum received being \$3,690.

8. An agent charges $1\frac{3}{4}\%$ of the amount invested; he receives \$956.45; how much is his commission?

ANALYSIS.

$$
\begin{aligned}
101\tfrac{3}{4}\% \text{ of investment} &= \$956.45.\\
1\% \text{ of investment} &= \tfrac{4}{407} \text{ of } \$956.45.\\
1\tfrac{3}{4}\% \text{ of investment} &= \tfrac{7}{4} \times \tfrac{4}{407} \text{ of } \$956.45 = \$16.45.
\end{aligned}
$$

9. At 2% of the actual investment, how much is an agent's commission when the investment and his commission amount to \$3,000?

10. An agent who charges commission only on what he invests, receives \$3,672, and invests \$3,600; what per cent does he charge?

ANALYSIS.

$$
\begin{aligned}
\text{Commission on } \$3{,}600 &= \$72.\\
\text{Commission on } \$1 &= \tfrac{1}{3600} \text{ of } \$72 = \$.02.\\
\text{The rate of } 2¢ \text{ to } \$1 &= 2\%.
\end{aligned}
$$

11. A commission merchant sells on consignment 1,700 bushels of oats @ \$.42 per bushel; his commission is \$35.70; what per cent does he charge? *Ans.* 5%.

12. B sent an agent \$1,958.40 to invest at discretion; the agent's charge was 2% of the amount invested; after deducting his commission, he invested in corn at \$.64 per bushel; how many bushels did he buy?

TAXES.

212. Sums of money collected from the people for the support of the government of a Nation, State, County, or City, are called **Taxes.**

213. Fixed property, such as lands and houses, is called **Real Estate.** Other property, such as money, furniture, cattle, etc., is called **Personal Property.**

A government officer, called an assessor, estimates the value of the real estate and personal property of the individual; the tax is a certain per cent of the assessed value of property.

214. In many States each man of voting age, unless exempted by law, must pay a capitation tax, or **Poll Tax,** which means a tax on the head, or on the person himself.

215. The principles of Percentage apply to Taxes.

In many cases, as the tax rate is a very small per cent, it is rated at so many mills on the dollar. A **Mill** is a tenth of a cent; the tax of 1 mill on the dollar is a tax of $\frac{1}{10}$ of 1 per cent.

PROBLEMS.

1. The real estate of a town is valued at \$750,000; what is the tax at the rate of $\frac{3}{4}$%?

2. A man pays $1\frac{1}{8}$% on real estate assessed at \$3,500, and personal property valued at \$1,500, and pays a poll tax of \$1.50; find the amount he pays.

3. Find the total revenue from a State tax of 15 mills on the dollar, the assessed valuation of the real estate being \$587,337,550, the assessed valuation of the personal property being \$269,189,050, there being 409,993 polls at \$1 per head, and 85% of the total tax being net revenue.

Ans. \$11,269,208.20.

4. A town levies a special property tax for school purposes; the real estate is valued at $1,700,000; the personal property at $300,000; upon this entire property it is required to raise $10,000; at what rate per cent is the tax levied?

$$\$10,000 = \tfrac{1}{200} \text{ of } \$2,000,000.$$
$$\tfrac{1}{200} \text{ of } \$1 = 5 \text{ mills.}$$

5. Find the assessed value of the property of a city whose tax rate is 6 mills on the dollar, and whose entire property tax amounts to $21,336.

$$\tfrac{6}{10} \text{ of } 1\,\% \text{ of assessed value} = \$21,336.$$
$$100\,\% \text{ of assessed value} = \text{what amount?}$$

6. A man's poll tax, $1.25, is $\frac{1}{3}$ of his whole tax; upon how much property does he pay, the rate being 1%?

Ans. $250.

7. A city requires $135,000 and levies a tax on property; what amount must be collected in order to raise the required sum and to pay the collector 2% of the amount collected?

8. If it cost 2% to collect, and 5% of the tax assessed be non-collectible, what amount must be levied in order to raise $13,965?

$$2\,\% \text{ of } 95\,\% \text{ of the levy} = \text{what per cent of the levy?}$$
$$93\tfrac{1}{10}\,\% \text{ of the levy} = \text{what sum?}$$
$$100\,\% \text{ of the levy} = \text{what sum?}$$

9. If it costs 2% to collect, and 4% of the tax is non-collectible, what is the amount of revenue from a levy of $8,500?

Ans. $7,996.80.

10. What is the assessed valuation of property, upon which a tax of $91 is raised at the rate of 4 mills on the dollar?

11. A school district votes a tax of 5 mills on the dollar; the cost of collection is $12; if the teacher receives $80 per month, and the property taxed is valued at $122,400, for how many months will the receipts pay the teacher's salary?

Ans. 7½ months.

12. A county raises a property tax of $36,000 by a rate of $1\frac{1}{8}\%$; $\frac{1}{8}$ of the amount is raised upon personal property; find the value of the real estate in the county. *Ans.* $2,800,000.

DUTIES.

216. Taxes levied by the National Government upon imports are called **Duties** or **Customs.**

The duties are collected by Custom House officers; an avoidance of the payment of duties is the crime called smuggling.

217. Duties fixed at a per cent of the cost of the imports are called **Ad Valorem Duties.**

Duties fixed at a certain sum for each pound, gallon, yard or other unit of measure, are called **Specific Duties.**

In the collection of specific duties, no tax is laid upon the weight of the box, barrel, or other covering of the goods, and a deduction is made because of waste in some cases.

218. Allowance, or deduction, made for the weight of coverings is called tare. Allowance for waste is called breakage.

Breakage applies only to bottles containing certain liquids, named in the tariff laws.

In computing the amount of ad valorem duties a fraction less than a half-dollar in the invoice is rejected; if in the invoice there is a fraction as great as a half-dollar, it is reckoned as a whole dollar. So, in specific duties, a fraction of a pound, gallon, etc., is rejected if it is less than $\frac{1}{2}$, and is considered 1 if it is $\frac{1}{2}$ or more. The entire base is the cost of the goods plus the expenses previous to final shipment.

PROBLEMS.

1. A pays duties on 250 boxes of lemons, invoice and expenses amounting to $625; find the amount he pays, there being a specific duty of 13¢ per box, and an ad valorem duty of 30% of cost. *Ans.* $220.

2. At what price per box must A sell his lemons, after paying $5 drayage, in order to make a profit of 25%?

3. B pays duty on 150 watches; price in U. S. money $12 each; expenses of shipping to steamer $11.50; duty 25%; drayage on arrival 50 cents; what does each watch cost?

Ans. $15.10.

4. What per cent does B gain in selling his watches at $20 each?

$15.10 : $4.90 : : 100% of cost : —.

5. C paid duties on 144 dozen table-knives; cost, including shipping, \$1 per dozen; duties, 30% ad valorem, and 10¢ per dozen specific; at what price per dozen must he sell to gain 25%? *Ans.* \$1.75.

6. W paid duty on two cases of tinware, weighing 950 pounds; duty 2.2¢ per pound, tare 90 pounds; what amount did he pay? *Ans.* \$18.92.

7. D paid a duty of \$55 less 5% for breakage; what amount did he pay?

8. C paid duties on 72 dozen pocket-knives; cost, including shipping, \$3.50 per dozen; duties, \$2 per dozen specific, and 50% ad valorem; what did each knife cost him?

INSURANCE.

219. **Insurance of Property** is security against loss by fire, etc.

220. The company which assumes a risk of loss, and secures the owner, is called the **Underwriter**, or **Insurer**.

221. The written contract between the owner and the insurer is called a **Policy**.

222. The sum paid for the policy is called the **Premium**.

The premium is a percentage of the amount insured.

The rate varies greatly according to the supposed risk, and according to the length of time for which the property is insured. The cost for insuring a house against fire for a year is more than half as great as the cost of insuring it for two years. If loss occurs, the loss is estimated, or adjusted, and the full amount of the loss is paid, if it is within the amount insured. Companies insure buildings, etc., for only a part of their value; such commodities as cotton, etc., whose cash value is always known, may be insured for their full value. When the amount of insurance is very small, insurance agents sometimes charge for writing the policy.

(The principles of percentage do not specially apply to problems in Life Insurance which provides that a sum of money be paid to the insured at a future time, or to his heirs when he dies.)

PROBLEMS.

1. A has his dwelling insured for \$3,000 for a year; the rate is $1\frac{1}{2}$%; find the premium. *Ans.* \$45.

2. B paid $750 for a year's policy, covering $60,000 on his store; find the rate. *Ans.* $1\frac{1}{4}\%$.

3. C paid $60.75 for a year's policy, the rate being $1\frac{1}{8}\%$; find the amount of the policy. *Ans.* $5,400.

4. A stock of goods valued at $75,600 was insured for $\frac{5}{8}$ of its value, at $\frac{7}{8}\%$; find the premium.

5. A building and its contents are valued at $6,600; for what amount is a policy made that covers a premium of $1\frac{1}{2}$ per cent and $\frac{2}{3}$ of the property?

$\frac{2}{3}$ of $6,600 = $4,400.
$98\frac{1}{2}\%$ of amount = $4,400.
100% of amount = what sum?

6. A dwelling and contents valued at $6,000 are insured for a year for $\frac{3}{5}$ of their value at $1\frac{1}{2}\%$; the dwelling burns and nothing is saved; find the owner's loss including premium.

7. A shipment of grain was insured for $5,640; the premium paid was $28.20; find the rate.

8. A merchant insures his stock of goods for $\frac{2}{3}$ of its value at $1\frac{1}{8}\%$; the store burns; his loss, being the premium paid and $\frac{1}{3}$ of the value of the goods, is $2,556.25; what was the value of the stock of goods? *Ans.* $7,500.

9. For insuring $\frac{3}{4}$ of the value of a public building worth $125,000, the cost was $937.50; find the rate.

10. A company insured for $4,500 a stock of goods at $1\frac{3}{4}\%$; the stock burns; find the company's loss. *Ans.* $4,421.25.

11. A house valued at $4,800 is insured for $\frac{5}{8}$ of its value at a cost of $90; find the rate.

12. A man insured his furniture for 6 months for $750; he paid $1 for the policy, and $\frac{3}{4}\%$ premium; find the amount paid.

13. A year's insurance of $1,500 is made on furniture at $1\frac{1}{4}\%$; after 6 months the owner moves into another house where, the risk being greater, $\frac{1}{8}\%$ more has to be paid; find the total cost.

14. Find the annual premium to be paid upon a life insurance policy for $5,000, the rate of premium being $19.75 per $1,000. *Ans.* $98.75.

INTEREST.

DEFINITIONS AND PRINCIPLES.

223. Compensation for the use of money is called **Interest.**

Whenever the payment of a debt is postponed, the debtor uses the creditor's money.

For the use of a horse, hire is paid; for the use of houses and lands, rent is paid; for the use of money, interest is paid.

224. The sum of money used is called the **Principal,** or the principal debt.

225. The **Rate** of **Interest** is the rate by which the interest is computed.

The rate is the ratio of the interest to the principal.

The rate is the rate per annum if not otherwise stipulated.

226. The **Time** is the period during which the principal bears interest.

227. The **Amount** is the sum of the **Principal** and the **Interest.**

228. **Legal Interest** is interest at a rate fixed by law, and applies to cases where no rate is agreed upon by borrower and lender.

The legal rate varies in different countries and states, and may vary in the same country or state at different times. Some states permit borrower and lender to agree upon a higher rate than the legal rate; this greater rate is called the conventional rate.

Usury is interest at a greater rate than the maximum rate authorized by law; the penalty for exacting usury is, generally, the forfeiture of the interest.

To find the Interest.

229. To find the interest of a sum of money is to find how much should be paid for the use of a given principal at a given rate for a given period of time.

PROBLEMS.

1. A borrows from B \$500 for 3 years at 6% interest. What must A pay for the use of B's money?

ANALYSIS.

Interest of \$500 for 1 yr. at 1% = .01 of \$500 = \$5.
Interest of \$500 for 1 yr. at 6% = 6 × \$5 = \$30.
Interest of \$500 for 3 yr. at 6% = 3 × \$30 = \$90.

2. What amount does B receive? (See problem 1.)

\$500 + \$90 = \$590 = Amount.

3. Compute the interest of \$364 for $2\frac{1}{2}$ years at 8%.

ANALYTICAL PROCESS.

$$\begin{array}{lr}
\text{Interest of \$364 for 1 yr. at 1\%} = & \$3.64 \\
 & 8 \\ \hline
\text{Interest of \$364 for 1 yr. at 8\%} = & \$29.12 \\
 & 2\frac{1}{2} \\ \hline
 & 1456 \\
 & 5824 \\ \hline
\text{Interest of \$364 for } 2\frac{1}{2} \text{ yr. at 8\%} = & \$72.80
\end{array}$$

4. Find the interest of \$575 for $3\frac{1}{3}$ years at 9%.

5. Find the interest of \$1,240 for $3\frac{1}{2}$ years at 12%.

6. Find the interest of \$2,350 for 2 years at $4\frac{1}{2}$%.

7. Find the interest of \$375.20 for $1\frac{1}{2}$ years at 8%.

8. Find the amount of \$675.25 for 5 years at $7\frac{1}{2}$%.

ANALYSIS.

For 1 yr. at 1% the interest of \$675.25 is $\frac{1}{100}$ of \$675.25.

.01 of \$675.25 = \$6.7525.

At $7\frac{1}{2}$% the interest is $7\frac{1}{2}$ × \$6.7525 = \$50.644.

For 5 yr. the interest is 5 × \$50.644 = \$253.22.

The amount is \$675.25 + \$253.22 = \$928.47.

PROCESS.

$$\begin{array}{lr}
\text{The interest of} & \\
\text{\$675.25 for 1 yr. at 1\%} = & \$6.7525 \\
 & 7\frac{1}{2} \\ \hline
 & 337625 \\
 & 472675 \\ \hline
\text{\$675.25 for 1 yr. at } 7\frac{1}{2}\text{\%} = & \$50.644 \\
 & 5 \\ \hline
\text{\$675.25 for 5 yr. at } 7\frac{1}{2}\text{\%} = & \$253.22 \\
 & 675.25 \\ \hline
\text{Amount} = & \$928.47
\end{array}$$

(In adding the partial products in the second step of the foregoing process, \$.00075 is considered 1 mill; the final result is sufficiently accurate, being but 2 mills greater than that obtained by employing the exact, but prolonged, process.)

Find the amount:

9. Of $340.50 for 2 years at 8%.

10. Of $450.25 for 3 years at 7%.

11. Of $1,025.75 for $2\frac{1}{2}$ years at 6%. *Ans.* $1,179.61.

12. Of $1,675.35 for $3\frac{1}{3}$ year at 9%. *Ans.* $2,177.95.

13. Of $2,236.87 for 4 years at $8\frac{1}{2}$%. *Ans.* $2,997.40.

14. Find the interest of $675.40 from Jan. 1, 1891, to Jan. 1, 1893, at 12%. *Ans.* $162.096.

15. Find the amount of $840.50 from June 15, 1890, to Dec. 15, 1892, at 10%. *Ans.* $1,050.625.

16. Find the amount of $680.10 from Feb. 18, 1890, to Aug. 18, 1893, at 8%. *Ans.* $870.528.

230. To find the interest on any principal for 1 year at 1%:
Find 1% of the principal.

To find the interest on any principal for 1 year at any rate:
Multiply the interest at 1% by the rate per cent.

To find the interest for any number of years:
Multiply the interest for 1 year by the **number** *of years.*

To find the amount:
Add the principal to the interest.

Let P represent the principal, I the interest, R the rate per cent, and T the ratio of the time to 1 year.

FORMULA. $I = T \times R \times 1\%$ of P

231. To find the Interest for a Number of Months.

PROBLEMS.

1. Find the interest of $240 for 5 mo. at 10%.

ANALYSIS.

Interest of $240 for 1 yr. at 1% = $2.40.
Interest of $240 for 1 yr. at 10% is 10 × $2.40 = $24.
Interest of $240 for 1 mo. at 10% is $\frac{1}{12}$ × $24 = $2.
Interest of $240 for 5 mo. at 10% is 5 × $2 = $10.

2. Find the interest of $2,400 for 10 months at 7%.

3. Find the interest of $3,000 for 3 months at 9%.

4. Find the interest of $300 for $1\frac{1}{2}$ months at 6%.

5. Find the interest of \$450 for 3 yr. 5 mo. at 8%.

ANALYSIS.

Interest of \$450 for 1 yr. at 1% = \$4.50.
Interest of \$450 for 1 yr. at 8% is 8 × \$4.50 = \$36
Interest of \$450 for 1 mo. at 8% is $\frac{1}{12}$ of \$36 = \$3
Interest of \$450 for 3 yr. at 8% is 3 × \$36 = \$108
Interest of \$450 for 5 mo. at 8% is 5 × \$3 = 15
Interest of \$450 for 3 yr. 5 mo. at 8% is \$123

PROCESS.

1% of \$450 = \$4.50
8
12)\$36.00 × 3 = \$108 Int. 3 yr. at 8%.
\$3.00 × 5 = 15 Int. 5 mo. at 8%.
\$123 Int. 3 yr. 5 mo. at 8%.

Find the interest of:

6. \$385 for 7 months at 8%. *Ans.* \$17.96.

7. \$796.50 for 11 months at 9%. *Ans.* \$65.71.

8. \$829.25 for 2 yr. 5 mo. at 10%. *Ans.* \$200.40.

9. \$5243 for 4 yr. 8 mo. at 12%. *Ans.* \$2936.08.

Find the amount of:

10. \$573.25 for 1 yr. $4\frac{1}{2}$ mo. at 6%. *Ans.* \$620.54.

11. \$1,280 for 3 yr. 7 mo. at $4\frac{1}{2}$%. *Ans.* \$1,486.40.

12. \$824.46 for 2 yr. $8\frac{1}{3}$ mo. at 5%. *Ans.* \$935.53.

13. \$1,525.43 for 2 yr. $7\frac{2}{3}$ mo. at $8\frac{1}{2}$%. *Ans.* \$1867.59.

14. Find the interest of \$825 from May 5, 1892, to Mar. 5, 1893, at 2% per month.

15. Find the amount of \$1,640 from June 10, 1893, to Nov. 20, 1893, at 8%.

(Each day is $\frac{1}{30}$ of a month; 15 da. = $\frac{1}{2}$ mo.; 10 da. = $\frac{1}{3}$ mo., etc.)

16. Find the amount of \$1,000.50 from Mar. 15, 1891, to July 30, 1892, at 8%.

17. Find the amount of \$729.71 from May 23, 1889, to Nov. 13, 1892, at 1% per month.

18. Find the interest of \$1,343.76 from Nov. 11, 1890, to June 21, 1893, at 10%.

19. Find the amount of \$726.75 from June 4, 1891, to July 14, 1893, at 8%.

20. Find the interest of $1,800 from Nov. 6, 1891, to Apr. 16, 1893, at $7\frac{1}{2}\%$.

21. Find the amount of $2,500 from July 20, 1892, to June 10, 1893, at $6\frac{1}{2}\%$.

22. Find the amount of $3,600 from Oct. 15, 1891, to Nov. 30, 1892, at $7\frac{1}{2}\%$.

232. To find the interest for 1 month:

Take 1 twelfth of the interest for 1 year.

To find the interest for any number of months:

Multiply the interest for 1 month by the **number** *of months.*

To find the interest for any number of years and months:

Multiply the interest for 1 year by the **number** *of years.*

Multiply the interest for 1 month by the **number** *of months.*

Add the two products.

Or,

Reduce the months to a fraction of a year and multiply the interest for 1 year by the total **number** *of years;* or, *Reduce the years to months, and multiply the interest for 1 month by the total* **number** *of months.*

FORMULA. (T = ratio of time to 1 month.)

$$I = \tfrac{1}{12}\,T \times R \times 1\% \text{ of } P$$

233. To find the Interest for Days.

PROBLEMS.

1. Find the interest of $720 for 7 days at 10%.

ANALYSIS.

Interest of $720 for 1 yr. at 1%		= $7.20.
Interest of $720 for 1 yr. at 10%	= 10 × $7.20	= $72.
Interest of $720 for 1 mo. at 10%	= $\frac{1}{12}$ of $72	= $6.
Interest of $720 for 1 da. at 10%	= $\frac{1}{30}$ of $6	= $.20.
Interest of $720 for 7 da. at 10%	= 7 × $.20	= $1.40.

2. Find the interest of $900 for 11 days at 8%.

3. Find the interest of $1,440 for 19 days at 10%.

4. Find the interest of $540 for 27 days at 6%.

5. Find the interest of $1,080 for 16 days at 5%.

6. Find the amount of $1,000 for 14 days at 6%.

7. Find the amount of \$300 for 22 days at $7\frac{1}{2}\%$.

8. Find the amount of \$500 for 28 days at $6\frac{1}{2}\%$.

9. Find the interest of \$1,375 for 3 yr. 5 mo. 19 da. at 8%.

ANALYSIS.

The interest of \$1,375 for
1 yr. at 1% = \$13.75.
1 yr. at 8% = 8 × \$13.75 = \$110; for 3 yr. = 3 × \$110 = \$330.
1 mo. at 8% = $\frac{1}{12}$ of \$110 = \$9.167; for 5 mo. = 5 × \$9.167 = \$ 45.83
1 da. at 8% = $\frac{1}{30}$ of \$9.167 = \$.306; for 19 da. = 19 × \$.306 = \$ 5.81
Interest of \$1,375 for 3 yr. 5 mo. 19 da. at 8% is \$381.64

PROCESS.

.01 of \$1,375 = \$13.75 Int. 1 yr. 1%.

$$
\begin{array}{r}
\$13.75 \\
8 \\
\hline
12)\$110.00 \times 3 = \$330 \text{ Int. 3 yr.} \\
30)\$9.167 \times 5 = \$\ 45.83 \text{ Int. 5 mo.} \\
\$.306 \times 19 = \$\ \ 5.81 \text{ Int. 19 da.} \\
19 \quad \$381.64 \text{ Int. 3 yr. 5 mo.} \\
\hline
2754 \\
306 \\
\hline
\$5.814
\end{array}
$$

10. Find the amount of \$1,256 for 1 yr. 7 mo. 13 da. at 9%.

11. Find the amount of \$890 for 11 mo. 23 da. at 10%.

12. Find the interest of \$675.50 for 2 yr. 27 da. at 9%.

13. Find the amount of \$1,385 for 4 yr. 8 mo. at 5%.

14. Find the amount of \$1,063.15 for 9 mo. 17 da. at $4\frac{1}{2}\%$.

15. Find the amount of \$983.65 for 3 yr. 26 da. at $7\frac{1}{2}\%$.

16. Find the interest of \$17.25 for 21 days at 8%.

17. Find the interest of \$450 for 6 mo. at $1\frac{1}{2}\%$ per month.

18. Find the interest of \$800 for 8 yr. 4 mo. at 9%.

19. Mr. Moore lends his neighbor \$147.35; at the end of 105 days he received his money with interest at 7%; how much did he receive?

20. Find the amount of \$229.15 from Mar. 20, 1892, to Jan. 13, 1893, at $8\frac{1}{2}\%$.

21. Find the amount of \$341.75 from July 27, 1891, to Mar. 20, 1893, at 11%.

22. Find the amount of \$3,625.80 from Oct. 13, 1892, to Nov. 10, 1892, at $1\frac{1}{4}\%$ per month.

23. Jan. 2, 1893, A borrowed from B $500 at 8%; he paid in full May 20, 1893; what sum did B receive?

24. A farm was sold for $4,500; two thirds of the price was paid in cash; the balance was paid in 9 mo. 15 da., with interest at 7%; find the interest.

25. Mr. Wilson loaned his neighbor $350 for 3 mo. 20 da., at the rate of 1% per month; what amount was paid?

26. Mr. Jones bought a house and lot for $3,600; he paid $\frac{1}{2}$ cash; the balance was paid at the end of 2 yr. and 3 mo., with interest at 12%; find the interest.

27. Mr. F. owes $475, with interest since April 13, 1893, at 6%; if he pays in full to-day, how much will he pay?

28. Which yields the more: $5,600 invested in a farm rented at $500 per year, or $5,600 drawing interest at 9%?

234. To find the interest at any rate for 1 day:

Take $\frac{1}{30}$ of the interest for 1 month, or $\frac{1}{360}$ of the interest for 1 year.

To find the interest for any number of days:

Multiply the interest for 1 day by the given **number** *of days.*

To find the interest for any number of months and days:

Multiply the interest for 1 month by the given **number** *of months; multiply the interest for 1 day by the given* **number** *of days; add the two products.*

To find the interest for any number of years and days:

Multiply the interest for 1 year by the given **number** *of years; multiply the interest for 1 day by the given* **number** *of days; add the two products.*

To find the interest for any number of years, months, and days:

Multiply the interest of 1 year by the given **number** *of years; multiply the interest of 1 month by the given* **number** *of months; multiply the interest of 1 day by the given* **number** *of days; add the three products.*

FORMULA. (T = ratio of time to 1 day.)

$$I = \frac{1}{360} T \times R \times 1\% \text{ of } P.$$

235. The Use of the Formulas.

PROBLEMS.

1. Find the interest of $125 for 3 yr. at 4%.

PROCESS.

(See formula, page 173.)

Substitute $\begin{cases} \$125 \text{ for P.} \\ 4 \text{ for R.} \\ 3 \text{ for T.} \end{cases}$ $I = 3 \times 4 \times 1\% \text{ of } \$125 = \$15$

2. Find the interest of $500 for 6 yr. at 6%.
3. Find the interest of $4,500 for 5 yr. at 5%.
4. Find the interest of $950 for 3 yr. at 7%.
5. Find the interest of $250 for 2 yr. 4 mo. at 9%.

PROCESS.

(See formula, page 175.)

Substitute $\begin{cases} \$250 \text{ for P.} \\ 9 \text{ for R.} \\ \frac{28}{12} \text{ for T.} \end{cases}$ $I = \dfrac{\overset{7}{\cancel{28}}}{\underset{4}{\cancel{12}}} \times \overset{3}{\cancel{9}} \times 1\% \text{ of } \$250 = \$52.50$

6. Find the interest of $360 for 1 yr. 8 mo. at 10%.
7. Find the interest of $480 for 2 yr. 6 mo. at 9%.
8. Find the interest of $450 for 3 yr. 9 mo. at 8%.
9. Find the interest of $720 for 1 yr. 6 mo. at 7%.
10. Find the interest of $450 for 1 yr. 2 mo. 12 da. at 8%.

PROCESS.

(See formula, page 177.)

Substitute $\begin{cases} \$450 \text{ for P.} \\ 8 \text{ for R.} \\ \frac{432}{360} \text{ for T.} \end{cases}$ $I = \dfrac{432}{\underset{45}{\cancel{360}}} \times \cancel{8} \times 1\% \text{ of } \overset{\$10}{\cancel{\$450}} = \$43.20$

11. Find the interest of $960 for 2 yr. 8 mo. 20 da. at 9%.
12. Find the interest of $375 for 4 mo. 5 da. at 5%.
13. Find the interest of $750 for 1 mo. 15 da. at 9%.
14. Find the interest of $1,875 for 1 yr. 1 mo. 15 da. at 8%.
15. Find the interest of $3,750 for 2 yr. 2 mo. 5 da. at 9%.

REVIEW WORK. By the formulas: Problems on page 174.

(The slight variance in some of the answers is attributable to the rejection or acceptance of the fourth decimal figure.)

236. Twelve Per Cent Method.

PROBLEMS.

1. Find the interest of \$500 for 1 yr. 8 mo. at 12%.

ANALYSIS.

12 % per year = 1 % per month.
1 yr. 8 mo. = 20 mo.
Interest of \$500 for 1 mo. at 1 % per month = \$5.
Interest of \$500 for 20 mo. at 1 % per month = 20 × \$5 = \$100.

2. Find the interest of \$300 for 2 yr. 1 mo. 10 da. at 9%.

ANALYSIS.

9 % per year = $\frac{3}{4}$ % per month.
2 yr. 1 mo. 10 da. = $25\frac{1}{3}$ mo.
Interest of \$300 for 1 mo. at 1 % per month = \$3.
Interest of \$300 for $25\frac{1}{3}$ mo. at 1 % per month = $25\frac{1}{3}$ × \$3 = \$76.
Interest of \$300 for $25\frac{1}{3}$ mo. at $\frac{3}{4}$ % per month = $\frac{3}{4}$ of \$76 = \$57.

Find by the twelve per cent method:

3. The interest of \$400 for 3 yr. 6 mo. 10 da. at 9%.
4. The interest of \$500 for 2 yr. 8 mo. 25 da. at 8%.
5. The interest of \$450 for 1 yr. 5 mo. 15 da. at 7%.
6. The interest of \$3,650 for 1 yr. 7 mo. 20 da. at 6%.
7. The interest of \$750 for 2 yr. 15 da. at 6 %.
8. The interest of \$1,936.50 for 1 yr. 2 mo. 18 da. at 9%.
9. The interest of \$350.75 for 9 mo. 21 da. at 6%.
10. The amount of \$756 for 3 yr. 1 mo. 15 da. at 7%.
11. The amount of \$955.50 for 2 yr. 6 mo. 10 da. at $7\frac{1}{2}$%.
12. The amount of \$1,250 for 1 yr. 3 mo. 21 da. at $6\frac{1}{2}$%.

237. To find the interest for 1 month at 12%:

Find 1% of the principal.

To find the interest for any time at 12%:

Reduce the time to months: multiply the interest for 1 month by the **number** *of months.*

To find the interest at any rate for any time by the 12% method:

Multiply the interest at 12% by the ratio of the given rate to 12%.

REVIEW WORK. By the 12 % method: Problems on p. 173.

238. Six Per Cent Method.

PROBLEMS.

1. Find the interest of $1 for 2 yr. 4 mo. 24 da. at 6%.

ANALYSIS.

At 6% the interest

Of $1 for 1 yr. = 6¢ ($.06).

Of $1 for 1 mo. = $\frac{1}{12}$ of 6¢ = $\frac{1}{2}$¢ ($.005), or 5 mills.

Of $1 for 1 da. = $\frac{1}{30}$ of 5 mills = $\frac{1}{6}$ mill ($.000$\frac{1}{6}$).

Hence

The interest of $1 for 2 yr. at 6%	= 2 × $.06	= $.12
The interest of $1 for 4 mo. at 6%	= 4 × $.005	= $.02
The interest of $1 for 24 da. at 6%	= 24 × $.000$\frac{1}{6}$	= $.004
The interest of $1 at 6% for 2 yr. 4 mo. 24 da.		= $.144

2. Find the interest of $1 for 3 yr. 7 mo. 19 da. at 6%.
3. Find the interest of $1 for 4 yr. 9 mo. 27 da. at 6%.
4. Find the interest of $1 for 3 yr. 11 mo. 5 da. at 6%.
5. Find the interest of $800 for 2 yr. 3 mo. 9 da. at 6%.

PROCESS.

Interest of $1 at 6%.

For 2 yr. =	$.12	Interest of $1 =	$.1365
For 3 mo. =	.015		800
For 9 da. =	.0015	Interest of $800 =	$109.2000
	$.1365		

6. Find the interest of $750 for 2 yr. 7 mo. 16 da. at 6%.
7. Find the interest of $945.50 for 3 yr. 5 mo. 7 da. at 6%.
8. Find the interest of $875 for 2 yr. 1 mo. 9 da. at 6%.

By the six per cent method:

9. Find the interest of $675 for 3 yr. 8 mo. 18 da. at 8%.

ANALYSIS.

Interest of $1 for 3 yr. 8 mo. 18 da. at 6% = $.223.

Interest of $675 for 3 yr. 8 mo. 18 da. at 6% = 675 × $.223 = $150.525.

Interest of $675 for 3 yr. 8 mo. 18 da. at 8% = $\frac{8}{6}$ of $150.525 = $200.70.

Find by the six per cent method:

10. The interest of $955.50 for 2 yr. 20 da. at 7%.
11. The interest of $847.25 for 1 yr. 8 mo. 7 da. at 8%.
12. The interest of $301.75 for 1 yr. 6 mo. 19 da. at 9%.
13. The amount of $756.75 for 2 yr. 2 mo. 5 da. at 7$\frac{1}{2}$%.

14. The interest of \$187.90 for 3 yr. 2 mo. 17 da. at 8%.

15. The amount of \$945.35 for 2 yr. 7 mo. 6 da. at $7\frac{1}{2}$%.

16. The amount of \$325.70 for 1 yr. 11 mo. 28 da. at 5%.

239. To find the interest at 6%:

For the interest of \$1 any number of years:

Multiply 6 cents by the **number** *of years.*

For the interest of \$1 any number of months:

Multiply $\frac{1}{2}$ cent by the **number** *of months.*

For the interest of \$1 any number of days:

Multiply $\frac{1}{6}$ mill by the **number** *of days.*

For the interest of any principal:

Multiply the interest of \$1 by the **number** *of dollars contained in the principal.*

By the 6 per cent method to find the interest at any per cent:

Multiply the interest at 6% by the ratio of the given rate to 6%.

240. The Sixty-Day Six Per Cent Method.

PROBLEMS.

1. Find the interest of \$575 for 1 yr. 4 mo. 12 da. at 6%.

ANALYSIS.

6% for 12 mo. = 1% for 60 da. or 2 mo.
1% for 60 da. = $\frac{1}{10}$% for 6 da.

Interest of \$575 for 2 mo. at 6% = \$5.75.
Interest of \$575 for 16 mo. at 6% = 8 × \$5.75 = \$46.
Interest of \$575 for 12 da. at 6% = $\frac{1}{5}$ of \$5.75 = \$ 1.15
Interest of \$575 for 1 yr. 4 mo. 12 da. at 6% = \$47.15

2. Find the interest of \$1,200 for 1 yr. 5 mo. 25 da. at 7%.

ANALYTICAL PROCESS.

The interest at
6% for 60 da. = 1% of \$1,200 = \$12; for 17 mo. = $8\frac{1}{2}$ × \$12 = \$102
6% for 6 da. = $\frac{1}{10}$ of \$12 = \$1.20; for 25 da. = $4\frac{1}{6}$ × \$1.20 = \$ 5
Interest of \$1,200 for 1 yr. 5 mo. 25 da. at 6% = \$107
Interest of \$1,200 for 1 yr. 5 mo. 25 da. at 7% = $1\frac{1}{6}$ × \$107 = \$124.833.

3. Find the interest of \$1,000 for 1 yr. 6 mo. 9 da. at 7% by the sixty-day method.

4. Find the interest of \$1,250 for 2 yr. 3 mo. 15 da. at 7% by the sixty-day method.

Find by the sixty-day method:

5. The amount of $115.75 for 2 yr. 1 mo. 6 da. at 7%.
6. The amount of $235.25 for 1 yr. 9 mo. 18 da. at 7½%.
7. The amount of $750 for 3 yr. 3 mo. 7 da. at 8%.
8. The amount of $315.60 for 1 yr. 6 mo. at 9%.
9. The interest of $715 for 1 yr. 7 mo. 11 da. at 8%
10. The interest of $550 for 2 yr. 2 mo. 17 da. at 5%.

241. To find the interest at 6% for 60 days:

Find 1% of the principal.

To find the interest at 6% for any time:

For years and months: *Multiply the interest for 60 days by the ratio of the years and months to 2 months.*

For days: *Find the interest by aliquot parts.*

To find the interest at any rate per cent:

Multiply the interest at 6% by the ratio of the given rate to 6%.

242. Method by Aliquot Parts.

PROBLEMS.

1. Find the interest of $640 for 2 yr. 10 mo. 27 da. at 9%.

ANALYTICAL PROCESS.

The interest of $640 for		1 yr. at 1%	= $ 6.40
		1 yr. at 9% is 9 × $ 6.40	= $ 57.60
2 yr.		2 yr. at 9% is 2 × $57.60	= $115.20
10 mo.	6 mo. =	½ yr. at 9% is ½ of $57.60	= $ 28.80
	3 mo. = ½ of	6 mo. at 9% is ½ of $28.80	= $ 14.40
	1 mo. = ⅓ of	3 mo. at 9% is ⅓ of $14.40	= $ 4.80
27 da.	15 da. = ½ of	1 mo. at 9% is ½ of $ 4.80	= $ 2.40
	10 da. = ⅓ of	1 mo. at 9% is ⅓ of $ 4.80	= $ 1.60
	2 da. = ⅕ of	10 da. at 9% is ⅕ of $ 1.60	= $ .32
Interest of $640 for 2 yr. 10 mo. 27 da. at 9%			= $167.52

Find by aliquot parts:

2. The interest of $825 for 3 yr. 11 mo. 25 da. at 6%.
3. The interest of $650 for 2 yr. 7 mo. 16 da. at 7%.
4. The interest of $750 for 1 yr. 5 mo. 5 da. at 7%.
5. The interest of $925 for 1 yr. 9 mo. 9 da. at 8%.
6. The interest of $816.50 for 2 yr. 2 mo. 7 da. at 9%.

7. The amount of $1,350 for 1 yr. 8 mo. 16 da. at 7%.

8. The amount of $750.25 for 2 yr. 6 mo. at 7½%.

9. The interest of $980.50 for 1 yr. 9 mo. 15 da. at 7½%.

EXACT INTEREST.

243. To compute interest for 365 days, instead of 360 days, to the year is called computation of **Exact Interest.**

PROBLEMS.

1. Find the interest at 6% of $840 borrowed Jan. 1, 1892, and paid March 16, 1893.

PROCESS.

From Jan. 1, 1892 to Jan. 1, 1893, is . . .	1 yr.
From Jan. 1, 1893 to Feb. 1, 1893, is	31 da.
From Feb. 1, 1893 to Mar. 1, 1893, is	28 da.
From Mar. 1, 1893 to Mar. 16, 1893, is . . .	15 da.
	1 yr. 74 da.

$$
\begin{array}{rr}
1\% \text{ of } \$840 = & \$8.40 \\
 & 6 \\ \hline
\text{Interest for 1 yr.} = & 50.40 \\
 & 1\tfrac{74}{365} \\ \hline
 & 50.40 \\
 & 10.21 \\ \hline
\text{Interest 1 yr. 74 da.} = & \$60.61
\end{array}
$$

2. Find the exact interest of $900 at 7% from May 3, 1892, to June 6, 1893.

3. Find the exact interest of $750 at 8% from Nov. 7, 1892, to May 19, 1893.

4. Find the exact interest of $650.50 at 8% from June 9, 1891 to Aug. 7, 1892.

5. Find the exact interest of $75 at 6% for 93 days.

244. To find exact interest for a number of days:

Multiply the interest for 1 year by the ratio of the days to 365 days.

REVIEW WORK: *by different methods.*

Pages 176, 177, probs. 19 to 28; p. 178, probs. 11 to 15; p. 179, probs. 3 to 12; p. 180, probs. 10 to 13; p. 182, probs. 5 to 10.

(The work in review, outlined above, is intended to furnish the student with sufficient experience to enable him to decide upon the most expeditious method of working a given problem in interest.)

PROMISSORY NOTES.

245. A written promise to pay a specified sum of money, with or without interest, is called a **Note,** or a Promissory Note.

246. The sum specified is called the **Face** of the Note.

The sum promised is written in figures at the top or bottom of the note, and in words in the body of the note.

247. The person promising, by his signature, to make the payment is called the **Maker** of the note.

248. The person to whom payment is promised is called the **Payee.**

The note is said to be made *in favor of* the payee.

The person who owns the note, whether he be the payee, or one to whom the note has been sold, is called the **Holder.**

249. A person who becomes security for the payment of a note made by another, does so by writing his name across the back of the note. In doing this he is said to **Indorse** the note, and he is called the **Indorser.**

In lending money to a man who gives his note for payment, it is customary for the lender to demand security, and very frequently more than one indorser is demanded. Each indorser is liable (see **273**) for the amount of the note unless otherwise stipulated in his indorsement; the stipulation commonly used is the phrase "without recourse."

250. The time when the note becomes legally due is called the **Maturity** of the note.

In most states a note becomes legally due 3 days later than the date of payment specified in the note. Suit for the collection of the note cannot be begun before the expiration of the 3 extra days, called "days of grace." The days of grace, however, are ignored in interest calculations, except in transactions with banks. When the days of grace are not allowed, the note matures at the expiration of the time specified.

A note may draw interest from its date, or from any later specified time. When no interest is specified, the note begins to draw interest at maturity.

Compute by the most expeditious method the amount of each of the following

NOTES.

1. $275. DALLAS, TEXAS, Mar. 16, 1893.

Six months after date I promise to pay *Albert J. Hunter*, or order, *Two Hundred Seventy-five and* $\frac{00}{100}$ Dollars, with interest at 10%, for value received. MORGAN W. RAMSEY.

What will be due at maturity? *Ans.* Amount $ ——.

2. $347.35. MEMPHIS, TENN., Oct. 21, 1891.

On demand I promise to pay *H. W. Owens*, or order, *Three Hundred Forty-seven and* $\frac{35}{100}$ Dollars, with interest at 6%, for value received. JAMES R. CONWAY.

This note was paid in full on May 14, 1892. Find the amount paid. *Ans.* $359.10.

3. $1,724.00. MOBILE, ALA., Aug. 10, 1892.

One day after date I promise to pay *R. W. Montgomery*, or order, *One Thousand Seven Hundred Twenty-four and* $\frac{00}{100}$ Dollars, with interest at 7% from date, for value received.

JOHN H. REESE.

Mr. Montgomery sold this note on Jan. 3, 1893 to J. W. Scott. Whose name was written on the back of the note? If Mr. Scott paid the full face value of the note, with interest at 7%, how much did he pay?

Answers. Mr. ——'s name; $——.

Mr. Reese paid Mr. Scott in full on April 20, 1893. How much did he pay? *Ans.* $ ——.

4. $1,068.50. COLUMBIA, S.C., March 4, 1892.

Two years after date we, jointly and severally, promise to pay *W. E. Morrison*, or order, *One Thousand Sixty-eight and* $\frac{50}{100}$ Dollars, with interest at 8% for value received.

R. B. LEWIS.
G. K. SOMERS.

Find the amount due at maturity.

From the following data write notes bearing interest from date, and find by the most expeditious methods the amounts due at maturity.

	DATE.	PAYEE.	FACE.	RATE.	MAKER.	MATURITY.
5.	May 24, 1890	R. Brown	$732.15	6%	D. Brooks	Aug. 7, 1890.
6.	Mar. 17, 1891	B. J. Dean	$1,095	7%	R. F. Hood	Apr. 24, 1892.
7.	Dec. 15, 1891	W. L. Lee	$984.35	8%	S. P. Ayers	Jan. 7, 1893.
8.	July 14, 1892	H. J. Bowe	$864.40	8%	J. Thomas	Feb. 10, 1893.
9.	Dec. 8, 1891	L. W. Ray	$745.25	5%	J. J. Lewis	Mar. 10, 1893.
10.	July 14, 1892	F. Hughes	$1,090	4%	Bell Bros.	Apr. 7, 1893.

To find the Rate.

251. The Principal, Interest, and Time being given, to find the rate is to find what per cent of the principal is the interest for 1 year.

PROBLEMS.

1. At what rate will $800 produce $192 interest in 3 years?

ANALYSIS.

At the required rate the

Interest of $800 for 3 yr. = $192.
Interest of $800 for 1 yr. = $\frac{1}{3}$ of $192 = $64.
Interest of $1 for 1 yr. = $\frac{1}{800}$ of $64 = $.08.

The ratio of $.08 to $1 is .08; hence the rate is 8%.

2. At what rate will $400 produce $90 in $2\frac{1}{2}$ years?
3. At what rate will $600 produce $56 interest in $2\frac{1}{3}$ years?
4. At what rate will $900 amount to $1,170 in 5 years?
5. At what rate will $725 produce $$83.37\frac{1}{2}$ in 2 yr. 3 mo. 18 da.?

ANALYTICAL PROCESS.

2 yr. 3 mo. 18 da. = 2.3 yr., or $\frac{23}{10}$ yr.

At the required rate

The interest of $725 for $\frac{23}{10}$ yr. = $83.375.
The interest of $725 for $\frac{1}{10}$ yr. = $\frac{1}{23}$ of $83.375.
The interest of $725 for 1 yr. = $\frac{10}{23}$ of $83.375.
The interest of $1 for 1 yr. = $\frac{1}{725}$ of $\frac{10}{23}$ of $83.375.

$\frac{1}{725} \times \frac{10}{23} \times \$83.375 = \$.05$.

The ratio of $.05 to $1 is .05; hence the rate is 5%.

6. At what rate will $625.50 amount to $702.645 in 1 yr. 6 mo. 15 da.?

ANALYSIS BY 1%.

It is evident that the required rate contains 1% as many times as the given interest contains the interest at 1%.

The interest of $625.50 for 1 yr. 6 mo. 15 da. at 1% is, as shown in the process, $9.6431¼. $77.145, the interest at the required rate, contains $9.6431¼, the interest at 1%, 8 times.

8 times 1% = 8% = required rate.

PROCESS.

$702.645 Amt.
625.50 Prin.
$77.145 Int.

$625.50 Prin.
.01
$6.2550 Int. 1 yr. 1%.
3.1275 Int. 6 mo. 1%.
.2606¼ Int. 15 da. 1%.
$9.6431¼ = Int. 1%.

8
$9.643125)$77.145 entire int.
77.145
8 times 1% = 8%.

It is necessary in employing this process to obtain an exact result in every operation.

7. At what rate will $1,080 produce $75.39 interest in 11 mo. 29 da.? *Ans.* 7%.

8. At what rate will $825 amount to $1028.225 in 2 yr. 19 da.?

9. At what rate will $375 amount to $379.4375 in 2 mo. 11 da.? *Ans.* 6%.

10. On Feb. 4, 1891, W. K. Hamilton made his note in favor of T. C. Howard for $720, bearing interest from date. On April 19, 1893, the amount due was $879; what was the rate of interest? *Ans.* 10%.

11. July 10, 1891, a merchant bought goods on 60 days' (2 mo.) time; he did not pay, however, until Dec. 24, 1891, when he paid $1,473.28, the account having drawn interest since it became due. The invoice was for $1,440; what rate of interest did the merchant pay? *Ans.* 8%.

252. To find the rate:

Divide the entire interest by the interest at 1%; multiply 1% by the quotient.

FORMULA.

$$R = \frac{I}{T \times 1\% \text{ of } P}$$

To find the Principal.

253. To find the Principal is to find what sum will yield a given interest in a given time at a given rate.

PROBLEMS.

1. What principal will yield \$81 in 1 yr. 6 mo. at 9% ?

ANALYSIS.

The interest for 1 yr. 6 mo. = \$81.
The interest for 6 mo. = $\frac{1}{3}$ of \$81 = \$27.
The interest for 1 yr. = 2 × \$27 = \$54.
9% of the principal = \$54.
1% of the principal = $\frac{1}{9}$ of \$54 = \$6.
100% of the principal = 100 × \$6 = \$600.
\$600 is the principal required.

2. What principal will yield \$125 in 2 yr. 6 mo. at 8% ?

3. What principal will yield \$126 in 2 yr. 4 mo. at 8% ?

4. What principal will yield \$130 in 3 yr. 3 mo. at 5% ?

5. What principal will gain \$140 in 1 yr. 8 mo. at 7% ?

6. What principal will produce \$39.05 in 2 yr. 3 mo. 15 da. at 8% ?

ANALYSIS BY \$1.

It is evident that the required principal contains \$1 as many times as the given interest contains the interest of \$1.

The interest of \$1 at 8% for 2 yr. 3 mo. 15 da. is \$.18$\frac{1}{3}$.

\$39.05, the interest of the entire principal, contains \$.18$\frac{1}{3}$, the interest of \$1, 213 times.

213 times \$1 = \$213.

\$213 is the principal required.

PROCESS 1.

The interest of \$1 at 8% for 2 yr. 3 mo. 15 da. = \$.18$\frac{1}{3}$.
\$39.05 ÷ \$.18$\frac{1}{3}$ = 213.
213 × \$1 = \$213.

PROCESS 2.

2 yr. 3 mo. 15 da. = $2\frac{7}{24}$ yr. = $\frac{55}{24}$ yr.
Int. for $\frac{55}{24}$ yr. = \$39.05.
Int. for $\frac{1}{24}$ yr. = $\frac{1}{55}$ × \$39.05 = \$.71.
Int. for 1 yr. = 24 × \$.71 = \$17.04.
8% of Prin. = \$17.04.
1% of Prin. = $\frac{1}{8}$ of \$17.04 = \$2.13.
100% of Prin. = 100 × \$2.13 = \$213.

What principal will produce:

7. \$184.80 in 3 yr. 8 mo. at 6% ? *Ans.* \$840.

8. \$87 in 9 mo. 20 da. at 10% ? *Ans.* ——

9. \$475.65 in 5 yr. 12 da. at 7% ? *Ans.* \$1,350.

10. $450 in 3 yr. 6 mo. 15 da. at 8% ? *Ans.* ——

11. $75.39 in 11 mo. 29 da. at 7% ? *Ans.* $1,080.

12. $123 in 3 yr. 5 mo. at 8% ? *Ans.* $450.

254. To find the Principal:

Divide the given interest by the interest of $1 for the given time at the given rate; multiply $1 by the quotient.

FORMULAS.

$$1\% \text{ of } P = \frac{I}{T \times R} \qquad P = \frac{100 \times I}{T \times R}$$

To find the Time.

255. To find the time is to find the time required for a given principal to produce a given interest at a given rate.

PROBLEMS.

1. In what time will $600 produce $210 at 7% ?

ANALYSIS.

The interest on $600 at 7% for 1 yr. is $42.

$42 = interest for 1 yr.

$1 = interest for $\frac{1}{42}$ yr.

$210 = interest for $\frac{210}{42}$ yr. = 5 yr.

2. In what time will $420 produce $84 at 5% ? *Ans.* 4 yr.

3. In what time will $800 amount to $1200 at 10% ?

4. In what time will $500 at 4% produce $15 ? *Ans.* 9 mo.

5. In what time will $1,500 at 3% amount to $1,950 ?

6. In what time will $624 at 7% produce $76.44 ?

ANALYSIS.

$624 at 7% will produce in 1 yr. $43.68.

The time required for $624 at 7% to produce $76.44 is as many times 1 year as $76.44 contains $43.68.

$76.44 contains $43.68, $1\frac{3}{4}$ times; $1\frac{3}{4} \times 1$ yr. = 1 yr. 9 mo.

7. In what time at $4\frac{1}{2}$% will $1,115.36 amount to $1,240.838 ? *Ans.* 2 yr. 6 mo.

8. In how many months will $1,250 at 10% gain 114.58\frac{1}{3}$? *Ans.* 11 mo.

9. In how many days will $4,500 at 9% amount to $4,529.25 ? *Ans.* 26 da.

† **10.** On May 13, 1890, a man gave his note due 6 months after date for \$750, with interest at 12% after maturity; when he paid the note it amounted to \$907.50; find the day of the payment. *Ans.* Aug. 13, 1892.

11. In a state where the legal rate is 8%, a man bought a bill of goods amounting on 3 months' time to \$1,500; when he paid the debt it amounted to \$1,780; how long was it after the bill was made until he paid the debt? *Ans.* 2 yr. 7 mo.

256. To find the time:

Divide the given interest by the interest on the given principal at the given rate for any unit of time, as 1 year, or 1 month, or 1 day, as the case may require. Multiply the unit of time by the quotient.

FORMULAS.

$$\text{(Years) } T = \frac{I}{R \times 1\% \text{ of } P}$$

$$\text{(Months) } T = \frac{12 \times I}{R \times 1\% \text{ of } P}$$

$$\text{(Days) } T = \frac{360 \times I}{R \times 1\% \text{ of } P}$$

MISCELLANEOUS PROBLEMS.

1. At what rate will \$744 produce \$135.57⅛ in 2 yr. 3 mo. 10 da.? *Ans.* 8%.

2. The amount of a note was \$919.014; the principal was \$801; how long had it been bearing interest, the rate being 6%? *Ans.* 2 yr. 5 mo. 14 da.

3. A note for \$607.35 dated June 19, 1890, bearing interest at 4%, was paid Mar. 11, 1893; find the amount.

4. What sum of money loaned at 6% will gain \$167.25 in 3 yr. 8 mo. 18 da.?

5. In what time will \$850.50, loaned at 10%, gain \$272.16? *Ans.* 3 yr. 2 mo. 12 da.

6. At what rate will \$631.24 produce \$47.343 in 7 mo. 15 da.?

7. What sum at 9% will amount to \$1,174.355 in 3 yr. 6 mo. 18 da.? *Ans.* \$890.

8. Find the interest of a note for \$3,010 for 2 yr. 7 mo. 7 da. at 8%. *Ans.* \$626.75.

9. At what rate will \$480 in 3 yr. 1 mo. 24 da. amount to \$585.84? *Ans.* 7%.

10. In what time will \$480 at 9% amount to \$561.60?
Ans. 1 yr. 10 mo. 20 da.

11. A man borrowed \$200 at 6% on Dec. 10, 1890; when he paid the debt it amounted to \$225; find the day of payment.
Ans. Jan. 10, 1893.

12. A sum of money loaned at 5% amounted to \$900 in 2 yr. 6 mo.; what sum was loaned?

13. Which will amount to more, \$350 at 10% for 2 years, or \$250 at 10% for 3 years?

14. A borrowed \$400 for 3 years, and paid in full \$472; B borrowed \$500 for 2 years, and paid in full \$555; what rate of interest did each pay?

15. At 5% a sum of money amounts in a certain time to \$550; at 5% in half the time it amounts to \$525; at what rate will it amount to \$620 in 2 yr.? *Ans.* 12%.

16. In 2 years a sum of money at a certain rate amounts to \$432; in 2 years at twice the rate it amounts to \$464; in what time will it amount to \$475 at 10%?
Ans. 1 yr. 10 mo. 15 da.

17. A borrowed \$700 at 6%, and paid in full \$724.50. B borrowed \$660 at 5%, and paid in full \$679.25; for what length of time did each keep the money borrowed?

18. A note bearing interest at 7% was made Jan. 2, 1893; the full payment, \$428.82, was made April 20, 1893; find the face of the note. *Ans.* \$420.

19. A note for \$420 was made July 6, 1892, and paid in full Jan. 2, 1893; the amount paid was \$432.32; find the rate. *Ans.* 6%.

20. B borrowed at 9% \$6,000, with which capital he started a business; his living having cost $\frac{3}{8}$ of his profits, at the end of a year he paid the interest, and found that his capital was \$6,960; what per cent had he gained in business? *Ans.* $41\frac{3}{5}$%

COMPOUND INTEREST.

257. When interest is charged for a single period of time, however long or short, such interest is called **Simple Interest.**

In simple interest the interest grows at a constant rate; if the money is borrowed for 2 years at 10% per annum, the interest is 20% for the 2 years; the principal undergoes no change until the end of the period.

258. When interest is charged for a period of time at the end of which the interest is added to the principal, and for another period of time interest is charged on the increased principal, such interest is called **Compound Interest.**

In compound interest 10% per annum for 2 years amounts to 21% of the principal; because for the second year, there is a gain of 10% of 10% of the principal, which equals 1 per cent of the principal.

The period in compound interest may be a year, a half-year, or any length of time. When no time is specified, a year is understood.

PROBLEMS.

1. Find the amount and the interest of $500 at compound interest for 3 years at 10%.

ANALYSIS.

The interest of $500 for 1 yr. at 10% = $50.
The amount of $500 for 1 yr. at 10% = $50 + $500 = $550.
The interest of $550 for 1 yr. at 10% = $55.
Compound amount of $500 for 2 yr. at 10% = $55 + $550 = $605.
The interest of $605 for 1 yr. at 10% = $60.50.
Compound amount of $500 for 3 yr. at 10% = $60.50 + $605 = $665.50.
Compound interest of $500 for 3 yr. at 10% = $665.50 – $500 = $165.50.

2. Find the difference between the simple interest and the compound interest of $700 for 4 years at 10%. *Ans.* $44.87.

3. Find the difference between the simple interest and the compound interest of $650 for 3 yr. at 6%. *Ans.* $7.16.

4. Find the difference of the simple interest and the compound interest of $825 for 3 yr. at 8%.

5. Find the difference of the simple interest and the compound interest of $1,000 for 4 yr. at 7%.

6. Find the compound interest of $1,500 for 5 yr. at 9%.

7. From the table, p. 194, compute the compound interest of $980 for 2 yr. 6 mo. at 10%.

PROCESS.

At 10%,
Amount of $1 for 2 yr. = $1.21
Interest of $1.21 for 6 mo. = .0605
Amount of $1 for 2 yr. 6 mo. = $1.2705
Interest of $1 for 2 yr. 6 mo. = $ .2705
Interest of $980 for 2 yr. 6 mo. = 980 × $.2705 = $265.09.

8. Compute the compound interest of $250 at 9% for 6 yr. 4 mo.

9. Compute the compound interest of $375.25 at 8% for 5 yr. 9 mo.

10. Find the compound interest of $1,325 for 3 yr. 2 mo. 15 da. at 6%.

11. Find the compound interest of $2,000 for 2 yr. 6 mo. at 4% for each half-year.

12. Find the compound interest of $1,800 for 2 yr. 9 mo. at 4% for each half-year.

13. Find the compound interest of $1,200 for 1 yr. 6 mo. at 3% for each 3 months.

14. Find the compound interest of $2,000 for 3 yr. 9 mo. at 10% per year.

15. Find the compound interest of $2,500 for 4 yr. 8 mo. at 9% per year.

16. Find the compound interest of $3,675 for 3 yr. 5 mo. at 8% per year.

259. To find compound interest:

Find, as in simple interest, the amount of the principal for the first period; on this amount as a new principal find the amount for the second period; continue this operation for each succeeding period; from the last amount subtract the original principal.

TABLE.

Showing the amount of $1 at compound interest from 1 yr. to 20 yr.

Yr.	2½ per cent.	3 per cent.	3½ per cent.	4 per cent.	5 per cent.	6 per cent.
1	1.025	1.03	1.035	1.04	1.05	1.06
2	1.050625	1.0609	1.071225	1.0816	1.1025	1.1236
3	1.076891	1.092727	1.108718	1.124864	1.157625	1.191016
4	1.103813	1.125509	1.147523	1.169859	1.215506	1.262477
5	1.131408	1.159274	1.187686	1.216653	1.276282	1.338226
6	1.159693	1.194052	1.229255	1.265319	1.340096	1.418519
7	1.188686	1.229874	1.272279	1.315932	1.4071	1.50363
8	1.218403	1.26677	1.316809	1.368569	1.477455	1.593848
9	1.248863	1.304773	1.362897	1.423312	1.551328	1.689479
10	1.280085	1.343916	1.410599	1.480244	1.628895	1.790848
11	1.312087	1.384234	1.45997	1.539454	1.710339	1.898299
12	1.344889	1.425761	1.511069	1.601032	1.795856	2.012197
13	1.378511	1.468534	1.563956	1.665074	1.885649	2.132928
14	1.412974	1.51259	1.618695	1.731676	1.979932	2.260904
15	1.448298	1.557967	1.675349	1.800944	2.078928	2.396558
16	1.484506	1.604706	1.733986	1.872981	2.182875	2.540352
17	1.521618	1.652848	1.794676	1.947901	2.292018	2.692773
18	1.559659	1.702433	1.857489	2.025817	2.406619	2.854339
19	1.59865	1.753506	1.922501	2.106849	2.52695	3.0256
20	1.638616	1.806111	1.989789	2.191123	2.653298	3.207136

Yr.	7 per cent.	8 per cent.	9 per cent.	10 per cent.	11 per cent.	12 per cent.
1	1.07	1.08	1.09	1.10	1.11	1.12
2	1.1449	1.1664	1.1881	1.21	1.2321	1.2544
3	1.225043	1.259712	1.295029	1.331	1.367631	1.404908
4	1.310796	1.360489	1.411582	1.4641	1.51807	1.573519
5	1.402552	1.469328	1.538624	1.61051	1.685058	1.762342
6	1.50073	1.586874	1.6771	1.771561	1.870414	1.973822
7	1.605781	1.713824	1.828039	1.948717	2.07616	2.210681
8	1.718186	1.85093	1.992563	2.143589	2.304537	2.475963
9	1.838459	1.999005	2.171893	2.357948	2.558036	2.773078
10	1.967151	2.158925	2.367364	2.593742	2.83942	3.105848
11	2.104852	2.331639	2.580426	2.853117	3.151757	3.478549
12	2.252192	2.51817	2.812665	3.138428	3.49845	3.895975
13	2.409845	2.719624	3.065805	3.452271	3.883279	4.363492
14	2.578534	2.937194	3.341727	3.797498	4.31044	4.887111
15	2.759031	3.172169	3.642482	4.177248	4.784588	5.473565
16	2.952164	3.425943	3.970306	4.594973	5.310893	6.130392
17	3.158815	3.700018	4.327633	5.05447	5.895091	6.86604
18	3.379932	3.996019	4.71712	5.559917	6.543551	7.689964
19	3.616527	4.315701	5.141661	6.115909	7.263342	8.61276
20	3.869684	4.660957	5.604411	6.7275	8.062309	9.646291

ANNUAL INTEREST.

260. When it is stipulated in a note that the interest is payable annually, the note is said to bear **Annual Interest.**

The interest may also be payable semi-annually, or quarterly, or at the end of any period stipulated in a note.

The laws of some of the states permit upon annual interest the rate of interest which the principal bears; some other states provide that unpaid annual interest shall bear interest at the legal rate; in the problems to follow, unless the legal rate is given, the unpaid interest is assumed to bear interest at the principal's rate.

Annual interest differs from compound interest.

In compound interest the interest of the interest is computed for each succeeding stipulated period.

In annual interest the interest of the interest is computed for only that one period of time which begins on the day when the interest becomes due and ends on the day of settlement.

PROBLEMS.

1. A note was given May 10, 1890, for \$300, with interest payable annually at 10%; the note became due May 10, 1892; no interest had been paid; find the amount and the interest of the note.

EXPLANATION.

The annual interest is the same as the compound interest, because in either case there is only 1 period for which the interest of the interest is computed.

PROCESS.

Int. \$300, 1 yr. 10% = \$30; Amt. \$330.
Int. \$330, 1 yr. 10% = \$33; Amt. \$363.
\$363 − \$300 = \$63, Int.

2.

\$1,200. SAN ANTONIO, TEXAS, July 9, 1890.

Four years after date, for value received, I promise to pay *J. W. Peterson*, or order, *Twelve Hundred and* $\frac{00}{100}$ Dollars, with interest from date at 10% per annum, payable annually.

CHARLES S. MORGAN.

Mr. Morgan prefers to pay the whole interest when the note becomes due; find the amount of the note when due.

ANALYSIS.

The interest of $1,200 for 1 yr. at 10% is $120.

The interest of the principal for the first year is..............	$120
(The interest, unpaid, bears interest for the 2d, 3d, and 4th years.)	
The interest of $120 for 3 yr. at 10% is......................	36
The interest of the principal for the second year is...........	120
(The interest, unpaid, bears interest for the 3d and 4th years.)	
The interest of $120 for 2 yr. at 10% is......................	24
The interest of the principal for the third year is.............	120
(The interest, unpaid, bears interest for the 4th year.)	
The interest of $120 for 1 yr. at 10% is......................	12
The interest of the principal for the fourth year is...........	120
The total interest to be paid is...........	$552

$1,200 + $552 = $1,752, Amount.

SHORT PROCESS.

1% of $1,200 = $12.
10% of $1,200 = 10 × $12 = $120 Int. on prin. 1 yr.
4
$480 Int. on prin. 4 yr.

1% of $120 = $1.20.

10% of $120 =	$12	Int. on annual int. for 1 yr.
	6	
	$72	Int. on annual int. for 6 yr.
	480	Int. on prin. 4 yr.
	1,200	Principal.
	$1,752	Amount.

One year's interest of the unpaid annual interest for 1 yr. is $12.

To multiply $12 at once by 6 gives a product equal to the combined products of 3 × $12, 2 × $12, and 1 × $12.

3. Find the amount of $1,200 at compound interest for 4 years at 10%; find the difference of the amount at compound interest and the amount at annual interest. *Ans.* $4.92.

4. On a note for $840 with interest at 9% due annually, nothing was paid till 3 years after date; what was then the amount? *Ans.* $1,087.21.

5. B buys a farm for \$5,000, and pays half cash; for the balance he gives his note due in 5 years, with interest payable annually at 8% per annum; he pays nothing until the note becomes due; find the amount of the note.

6. A real estate dealer makes a note of \$7,500, due in 2 years, with interest at 10% per annum, payable semi-annually; he fails to pay the interest as it becomes due; what is the amount of the note at maturity? *Ans.* \$9,112.50.

7. D makes a note of \$1,380 for 2 yr. 6 mo. at 12% per annum, the interest payable quarterly; he pays no interest until the note becomes due; find the amount. *Ans.* \$1,849.89.

8. In a state whose legal rate is 6% H makes a note of \$1,250 for 3 yr. 3 mo., with interest at 8%, payable semi-annually; he pays no interest; find the amount due at maturity. *Ans.* \$1,602.

261. To find annual or periodical interest:

Find the interest of the principal for the given rate and time.

Find the interest on each unpaid periodical interest.

Add the several sums.

PARTIAL PAYMENTS.

262. A **Partial Payment** is a payment of part of a debt.

263. The **United States Rule** is the method most generally used in computations in interest involving partial payments. The United States Rule was first announced by Chancellor Kent of New York; it is the rule adopted by the United States Courts, and by the Courts of nearly all the states.

The United States Rule is based upon these principles, to wit:

Interest must not bear interest.

A payment made must not bear interest in favor of him who pays; therefore,

A partial payment must be applied first to the payment of the interest of a debt, and if part of the payment pays the interest, the remainder must be applied to the payment of part of the principal.

1. A makes in favor of B a note of $500, due in 2 years, with interest at 6%; 1 year afterward A pays $200 on the note; find the amount of the note at maturity.

ANALYSIS.

Interest of $500 for 1 yr. at 6% = $30.
Partial payment is $200; $30 *pays the interest.*
$200 − $30 = $170; $170 *pays part of principal.*
Second principal = $500 − $170 = $330.
Interest of $330 for 1 yr. at 6% = $ 19.80.
Amount at maturity = $330 + $19.80 = $349.80.

2. C makes in favor of D a note of $800, due in 1 yr. 6 mo., with interest at 8%; 1 year afterward C pays $300 on the note; find the amount of the note at maturity. *Ans.* $586.56.

3. B makes in favor of D a note of $800, due 1 yr. 10 mo., with interest at 8%; 1 year afterward B makes a payment of $50; 6 months still later B makes a payment of $250; find the amount of the note at maturity.

ANALYSIS.

Interest of $800 for 1 yr. at 8% =	$64	
First partial payment =	$50	
The first partial payment leaves	$14	*unpaid interest.*
Interest of $800 for 6 mo. at 8% =	$32	
The debt is now $800 +	$46	*unpaid interest.*
The second partial payment =	$250;	$46 *pays the interest.*
	$204	*pays part of the principal.*

The remaining principal is $800 − $204 = $596
Interest of $596 for 4 mo. at 8% = $15.893
Amount due at maturity = $611.893

4. A note for $600 bearing interest at 10% has two indorsed payments; the first is for $100, paid 1 year from the date of the note; the second is for $225, paid 2 years from the date of the note; 6 months after the second payment is made, the note is paid in full; find the amount of the last payment. *Ans.* $410.55.

5. A note of $1,000, with interest at 10%, has indorsed upon it a payment of $75, one year after date, and a payment of $500, 6 months later; find the amount paid in full 2 yr. 6 mo. from date of note. *Ans.* $632.50.

6.

$1,250. RALEIGH, N.C., Jan. 20, 1890.

On demand I promise to pay *John B. Handy*, or order, *One Thousand Two Hundred Fifty and* $\frac{00}{100}$ Dollars, with interest at 10% per annum from date, for value received.

C. F. HENDERSON.

Indorsements: Dec. 10, 1890, received $350; Sept. 21, 1891, received $75; Jan. 1, 1892, received $23.75; July 16, 1892, received $750; find balance due Sept. 2, 1892.

Short process showing each succeeding result (to two decimal places only).

For 10 mo. 20 da. at 10% the interest of	$1,250.00	= $111.10
The first payment is $350 =	$ 238.90	+ $111.10
The amount now due is	$1,011.10	+ 00.00 Int.
For 9 mo. 11 da. at 10% the interest of	$1,011.10	= $ 78.92
The second payment is	000.00	+ $ 75.00
The amount now due is	$1,011.10	+ $ 3.92 Int.
For 3 mo. 10 da. at 10% the interest of	$1,011.10	= $ 28.09
The amount now due is	$1,011.10	+ $ 32.01 Int.
The third payment is	00.00	+ $ 23.75
The amount now due is	$1,011.10	+ $ 8.26 Int.
For 6 mo. 15 da. at 10% the interest of	$1,011.10	= $ 54.77
The amount now due is	$1,011.10	+ $ 63.03 Int.
The fourth payment is $750 =	$ 686.97	+ $ 63.03
The amount now due is	$ 324.13	+ 0.00 Int.
For 1 mo. 16 da. at 10% the interest of	$ 324.13	= $ 4.14
Balance due Sept. 2, 1892, is	$ 324.13	+ $ 4.14 = $328.27.

7.

$975. INDIANAPOLIS, Mar. 18, 1890.

One day after date, for value received, we jointly or severally promise to pay *R. B. Hall*, or order, *Nine Hundred Seventy-Five* Dollars, with interest at 8% per annum from date.

R. R. BLAIR,
B. C. MOORE.

Indorsements: Jan. 3, 1891, $325; May 17, 1891, $240; find amount due Jan. 1, 1892.

8. $644.40. TOPEKA, KAN., Mar. 13, 1890.

On or before Dec. 1, 1892, I promise to pay *Joseph P. Edwards*, or order, *Six Hundred Forty-Four and* $\frac{40}{100}$ Dollars, with interest at 7% per annum from date.

For value received. J. K. HUNT.

Indorsements: Dec. 28, 1890, $300; April 18, 1892, $15.25; June 13, 1892, $260.50; find amount due Dec. 1, 1892.

Ans. $147.84.

9. $1,600. KNOXVILLE, TENN., Nov. 5, 1890.

Thirty days after date, for value received, I promise to pay *John R. Reynolds*, or order, *One Thousand Six Hundred* Dollars, with interest at 9% per annum from date.

R. S. DONALD.

Indorsements: Nov. 5, 1891, $400; May 5, 1892, $500. July 1, 1892 the note was paid in full; find the amount then paid.

264. The following, in the words of its author, is the

UNITED STATES RULE.

1.

"*The rule for casting interest when partial payments have been made is to apply the payment, in the first place, to the discharge of the interest then due.*

2.

"*If the payment exceeds the interest, the surplus goes toward discharging the principal, and the subsequent interest is to be computed on the balance of the principal remaining due.*

3.

"*If the payment be less than the interest, the surplus of interest must not be taken to augment the principal, but the interest continues on the former principal until the period when the payments, taken together, exceed the interest due, and then the surplus is to be applied toward discharging the principal, and the interest is to be computed on the balance as aforesaid.*"

THE MERCHANTS' RULE.

265. The method of computing interest in Partial Payments known as the **Merchants' Rule** is frequently employed by mutual consent. It is used in cases where the settlement of notes or accounts is made within a year.

266. The Merchants' Rule is based upon the following equitable principle:

The borrower should receive the interest of all he pays, and pay the interest of all he borrows.

PROBLEMS.

1. A note of $850 was made Jan. 18, 1892, with interest at 6%. On the note were the following indorsements of payments received: April 18, 1892, $200; July 18, 1892, $250; Sept. 18, 1892, $200; find amount due on the note Jan. 18, 1893.

SHORT PROCESS.

Int. of $850 for 1 yr. at 6% = $51
  Amt. of $850 for 1 yr. = $850 + $51 = $901, Amount of principal.
Int. of $200 for 9 mo. at 6% = $9
  Amt. of $200 for 9 mo. = $200 + $9 = $209
Int. of $250 for 6 mo. at 6% = $7.50
  Amt. of $250 for 6 mo. = $250 + $7.50 = $257.50
Amt. of $200 for 4 mo. at 6% = $200 + $4 = $204

$670.50 Amt. of Pay'ts.
Difference of Amt. of Prin. and Amt. of Pay'ts = $230.50 Amt. due.

2. A note was given May 3, 1891, for $1,060 with 8% interest. Indorsement of payments received: July 18, 1891, $275; Oct. 23, 1891, $366; Dec. 12, 1891, $180; Feb. 15, 1892, $225; find the amount due May 3, 1892. *Ans.* $56.39.

3. In a state whose legal rate is 8% a customer owes a merchant $1,250 due on Oct. 15, 1890; after the account became due the following payments were made: Jan. 1, 1891, $500; March 16, 1891, $425; June 10, 1891, $75; Sept. 1, 1891, $250; find the amount due Oct. 15, 1891.

267. To compute interest on partial payments by the Merchants' Rule:

Add to the principal its interest for the whole time. Add to each payment its interest for the time from the day of payment till the day of settlement. Subtract from the amount of the principal the sum of the amounts of the payments.

PERIODICAL INTEREST WITH PARTIAL PAYMENTS.

268. The laws of some of the states permit interest upon unpaid interest.

In some cases a contract or custom decides that when a partial payment of a debt does not liquidate the interest due the payment shall be deducted at once from the amount due, and that the remaining principal shall continue to draw interest at the rate named in the note or other contract.

PROBLEMS.

1. On June 16, 1890, W. G. Farmer made a note in favor of R. B. Parsons for $1,200, due in 2 yr., with interest at 12%, payable semi-annually. On this note Mr. F. paid $400 on Nov. 16, 1890; $300 on June 16, 1891, and $100 on Dec. 1, 1891; find the amount due on the note at maturity.

PROCESS.

Interest of $1,200 till 1st payment, $60; amount =	$1,260
First payment, 5 mo. from date of note =	$ 400
The new principal for one month =	$ 860
Interest of $860 till end of 1st period, 1 mo. =	$ 8.60
Principal for second period, 6 mo. =	$ 868.60
Interest of $868.60 for second period, 6 mo. =	$ 52.12
Amount at end of second period, 6 mo. =	$ 920.72
Payment at end of second period =	$ 300
Principal till third payment, 5½ mo. =	$ 620.72
Interest of $620.72 till third payment, 5½ mo. =	$ 34.14
Amount at time of third payment =	$ 654.86
Third payment, 5½ mo. from second payment =	$ 100
Principal till end of third period, ½ mo. =	$ 554.86
Interest of $554.86 till end of third period, ½ mo. =	$ 2.77
Principal for last period of 6 mo. =	$ 557.63
Int. of $557.63 for 6 mo. =	$ 33.46
Amount due at maturity =	$ 591.09

2. A note for $1,200, with interest at 8%, payable semi-annually, was given July 10, 1889; payments were made as follows: Jan. 10, 1890, $450; July 10, 1890, $635.15; find amount due Jan. 10, 1891. *Ans.* $202.56.

3. Mr. Brice gave his note on Oct. 11, 1890, for $4,000, with interest at 10%, payable semi-annually; on this note Mr. B. paid $125 April 11, 1891; $210.75 Oct. 11, 1891; and $175 Jan. 1, 1892; find amount due April 11, 1892. *Ans.* $4,094.05.

4. Mr. Goode gave his note Nov. 13, 1890, for $125, with interest at 10%, payable semi-annually; on this note Mr. G. paid $6.25 May 13, 1891; $11.25 Nov. 13, 1891; $66 May 13, 1892; $15.60 June 19, 1892, and Nov. 13, 1892, paid in full; find the total sum of Mr. Goode's payments.

Ans. $145.90.

BANK DISCOUNT.

269. The sum of money which a bank lends upon a note is called the **Proceeds** of the note.

In lending money banks require from the borrower a written obligation to pay more than he receives; they also commonly require that the note be indorsed by one or more persons as security for payment.

270. The face of the note corresponds to the Base in Percentage.

Bank Discount is computed at a per cent of the Face.

271. The difference between the **Face** of the note and the **Proceeds** is called the **Bank Discount.**

The Bank Discount corresponds to the Percentage.

272. The maturity of a note discounted at a bank is 3 days later than the time of payment written in the note.

Banks allow and charge for the 3 Days of Grace. A borrower, who makes his note of $100 for 60 days is charged a sum equal to the interest of the $100 for 63 days, and receives the proceeds or difference; on the 63d day he pays the $100; if he wishes to use the money longer the bank may demand a new note upon which the bank will charge discount; but if the borrower can neither give a new note nor pay the one that is due, the note is protested.

273. A **Protest** is a written declaration, in legal form, by a Notary public, that payment of the note is refused. Notice of a protest is sent to each indorser liable for the amount of the note. (See **249**.)

To find Bank Discount.

274. To find the Bank Discount is to find a sum equal to the interest at a given rate upon the Face of the Note for a specified time.

PROBLEMS.

1. Find the bank discount and proceeds of a 60-day note for \$300 at 10%.

ANALYSIS.

Interest of \$300 for 1 yr. at 10% = \$30.
Interest of \$300 for 2 mo. at 10% = $\frac{1}{6}$ of \$30 = \$5.
Interest of \$300 for 3 da. at 10% = $\frac{1}{20}$ of \$5 = \$ .25
Interest of \$300 for 63 da. at 10% = \$5.25 Bank Discount.
\$300 — \$5.25 = \$294.75, Proceeds.

2. Find the bank discount and proceeds of a 90-day note for \$200 at 12%.

3. Find the bank discount and proceeds of a 60-day note for \$2,000 discounted at 6% 30 days after its date.

Ans. Dis. \$11.

4. Find the day of maturity, the bank discount, and the proceeds of a 90-day note for \$2,904, made Jan. 6, 1893, and discounted the same day at 8%.

The note is to be paid in bank 93 da. after date.
There remain in Jan. 25 da.
There are in Feb. 28 da.
There are in Mar. 31 da.
93 da. − 84 da. = 9 days for April.
The note should be paid in bank April 8, 1893.

Notes written on Sunday are not legal: when the maturity of a note falls on Sunday, or on a legal holiday, the legal maturity of the note falls on the preceding day. April 9, 1893, was Sunday.

Write notes from the following data: find for each the date of maturity, the bank discount, and the proceeds.

	Date.	Face.	Time.	Rate.
5.	June 9, 1892	$1,045	30 da.	10%.
6.	July 5, 1892	$875.50	60 da.	9%.
7.	Mar. 3, 1893	$1,260.15	45 da.	8%.
8.	June 29, 1891	$1,080.45	90 da.	12%.

9.

$875. LITTLE ROCK, ARK., May 29, 1892.

One year after date, for value received, I promise to pay T. L. Chism, or order, *Eight Hundred Seventy-Five and* $\frac{00}{100}$ Dollars, with interest at 6% per annum from date.

W. B. JACKSON.

Mr. Chism keeps this note until Feb. 21, 1893, when a bank discounts it at 8%; find the maturity, the bank discount, and the proceeds.

The amount at maturity is $875 + 6% interest of $875 for 1 yr. 3 da. The discount is the discount of the amount at maturity, at 8%, for the time between the date of discount and the time of maturity.

10. A note of $525.75, with interest at 10%, due 6 months after date, was made Aug. 10, 1892. Dec. 1, 1892, it was discounted at bank at 10%; find the bank discount.

Write notes from the following data: find the maturity, discount, and proceeds of each.

	Date.	Face.	Int. Rate.	Time.	Day Dis.	Dis. Rate.
11.	May 12, 1890	$1,260	4%	9 mo.	Dec. 20, 1890	6%.
12.	Apr. 13, 1892	$317.20	10%	1 yr.	Feb. 1, 1893	12%.
13.	Aug. 30, 1892	$1,473.75	8%	4 mo.	Nov. 20, 1892	8%.
14.	Dec. 23, 1892	$2,496	5%	6 mo.	Mar. 3, 1893	9%.
15.	Feb. 1, 1893	$118.80	7%	60 da.	Feb. 8, 1893	8%.

275. To find Bank Discount:

At the rate of bank discount compute the simple interest of the amount of the note at maturity, from the date of discount till the date of maturity.

To find the Base of Bank Discount.

276. To find the Base of Bank Discount is to find the amount, or face, of a note discounted at bank.

PROBLEMS.

1. For what sum is a 30-day note made when a bank discounts it at 12% by lending the maker $989?

ANALYSIS.

Interest of $1 for 33 da. at 12% = $.011.
Proceeds of $1 = $1 − $.011 = $.989.
If $.989 be produced from $1, $989 is produced from as many times $1 as $989 contains $.989 = 1,000.
1,000 times $1 = $1,000.
The note is made for $1,000.

COMPARISON.

$ $\frac{989}{1000}$ is produced from $1.
$1 is produced from $ $\frac{1000}{989}$.
$989 is produced from 989 × $ $\frac{1000}{989}$.
989 × $ $\frac{1000}{989}$ = $1,000.

BY PROPORTION.

$.989 : $1 :: $989 : ——.

2. For what sum is a 60-day note made, when a bank discounting it at 8%, lends the maker $725? *Ans.* $735.29.

3. Find the face of a 90-day note, for which the maker receives $375, bank discount 10%.

4. A merchant borrows $2,000 for 60 days; bank discount 8%; find the face of the note.

5. A bank discounts, at 8%, a note due in 45 days, and lends the maker $1,250; find the face of the note.

6. Find the face of a 60-day note for which a bank, discounting at 10%, lends the maker $3,500.

277. To find the Face of a Note discounted at Bank:

Divide the proceeds given by the proceeds of $1 for the given rate and time; multiply $1 by the quotient.

TRUE DISCOUNT.

278. The **Present Worth** of a debt is that sum of money, which, if loaned, will produce the amount of the debt at maturity.

Thus, if the current rate of interest be 6%, $100 is worth as much as a note for $106 payable in 1 year, without interest.

279. The difference between the amount of a debt and its present worth is called True Discount.

Thus, a man who holds a note for $106 due, without interest, in 1 year, would be willing to deduct $6 from the face of the note and accept $100 cash to-day, if the current rate of interest is 6%.

In operations of True Discount, 3 days' grace is not considered.

PROBLEMS.

1. Find the present worth and the true discount of a debt of $228 due in 2 years, the current rate of interest being 7%.

ANALYSIS.

Amt. of $1 at 7% for 2 yr. = $1.14.

If $1.14 due in 2 yr. is now worth $1, $228 due in 2 yr. is now worth as many times $1 as $228 is times $1.14 = 200 times. 200 × $1 = $200, present worth.

$228 − $200 = $28, true discount.

COMPARISON.

114¢ = amount from 100¢.

1¢ = amount from $\frac{100}{114}$¢.

$228 = amount from 22,800 × $\frac{100}{114}$¢.

22,800 × $\frac{100}{114}$¢ = $200.

BY PROPORTION.

$1.14 : $1 :: $228 : ——.

Find the present worth and true discount of:

2. $105 due in 1 year; current rate, 5%.

3. $208 due in 2 years; current rate, 4%.

4. $309 due in 3 years; current rate, 3%.

5. Find the present worth and true discount of $875 due in 2 yr. 3 mo. 18 da., the current rate being 10%.

6. Find the true discount of $684.50 due in 8 mo. 20 da., the current rate being 9%. *Ans.* $41.77+.

7. Find the present worth and true discount of $1,235 due in 1 yr. 7 mo. 19 da., current rate being 8%.

8. Find the present worth of a note for $1,000 due in 1 year, with interest at 8%, current rate being 10%.

9. B buys a house for $7,250 cash; he sells it at once to C for $8,000 half cash, half in 1 year without interest; find B's gain, the current rate being 8%.

10. W offers H a farm for $8,000 cash, or for $9,000 a third cash, a third in 1 year, and a third in 2 years, the deferred payments without interest; the current rate being 10%, find the difference of the two offers.

11. Find the difference between the bank discount and the true discount, each at 10%, from a note for $654 due in 90 days.

12. Find the present worth of a note for $500, due the 16th of next August, current rate 9%.

13. Find the true discount of $756.30 due in 6 mo. 15 da., the current rate being 8%.

280. To find the Present Worth of a Note:

Divide the amount of the note by the amount of $1 for the given time at the current rate; multiply $1 by the quotient.

To find the True Discount:

Subtract the present worth from the amount.

DRAFTS AND BILLS.

281. An order, written to a bank by an individual or a firm, for the immediate payment of money is called a **Check.**

282. A written order to an individual or a firm for the payment of money is called a **Draft.**

A written order, or draft, especially when made by one bank upon another bank, is sometimes called a **Bill of Exchange.**

A, in Memphis, wishes $1,000. He knows that Smith & Co. in New York will honor his draft. He draws at sight on Smith & Co. for $1,000, and asks a Memphis bank to cash it. If satisfied that the draft is valid, either because A is well known or because his draft is indorsed, the bank pays A $1,000 if the exchange is at par. The Memphis bank sends the draft to a New York bank which collects $1,000 from Smith & Co. The New York bank now has $1,000 of the Memphis bank's money, and the Memphis bank will sell to Memphis merchants New York exchange for $1,000 to balance accounts. Memphis merchants will wish to pay money in New York, and will buy from the Memphis bank bills of exchange on New York, rather than send money, because the bill is drawn "in favor of" the person whom they wish to pay. The New York bank will not pay the amount of the bill unless it is indorsed by the person (payee) in whose favor it is drawn. Thus the bill is sent by mail without risk; if lost, it is duplicated without expense.

Merchants frequently make use of banks in the collection of debts by drawing drafts on customers. If the drafts are not paid, they may be protested.

283. A **Sight Draft** is an order for immediate payment.

284. A **Time Draft** is an order to pay at a future time.

Banks commonly sell Bills and buy Drafts. A bank may pay for a draft less or more than the amount of its face, or it may pay a sum equal to the amount of its face.

285. When a draft or bill is sold for less than the amount of its face the exchange is made at a **Discount**.

When a draft or bill is sold for a sum equal to the amount of its face the exchange is made at **Par**, or equality.

When a draft or bill is sold for more than the amount of its face the exchange is made at a **Premium**.

286. An **Acceptance** of a draft binds the "drawee" or person upon whom the draft is drawn.

The "drawee" signifies acceptance by writing across the front of the draft the word "accepted," with the amount and date if he so chooses, above his signature. Very frequently sight drafts are paid when presented.

The principles of Percentage apply to Drafts.

PROBLEMS.

1. A buys New York exchange for \$500 at $\frac{1}{4}\%$ premium; find the cost of the bill. *Ans.* \$501.25.

2. Find the proceeds ot a sight draft for \$888.80 which a bank discounts at $\frac{1}{8}$ of 1%. *Ans.* \$887.69.

3. A man pays \$401 for New York exchange at $\frac{1}{4}\%$ premium; find the face of the bill. *Ans.* \$400.

4. B in New York buys exchange on Meridian at $\frac{1}{8}\%$ discount; the face of the bill is \$480; how much does he pay? *Ans.* \$479.40.

5. *First National Bank.*

\$750.00. BIRMINGHAM, ALA., Mar. 22, 1892.

At sight pay to *Hiram Henry & Co.*, or order, *Seven Hundred Fifty* $\frac{00}{100}$ Dollars, and charge to account of

T. O. SMITH, *Cashier.*

To Southern National Bank, New York.

Find the amount paid for this draft at $\frac{3}{8}\%$ premium.

6.

$540.00. NEW ORLEANS, Jan. 16, 1893.

At sight pay to the order of *myself,* ----------------------
Five Hundred Forty $\frac{00}{100}$--------------------------------Dollars,
value received, and charge to account of

H. R. LOOMIS.

To H. B. Claflin & Co., New York.

Mr. Loomis indorses this draft, and the bank pays for it at $\frac{1}{4}\%$ discount; find the proceeds. *Ans.* $538.65.

7.

City National Bank.

$2,500.00. DALLAS, TEXAS, Sept. 10, 1892.

At sixty days sight pay to *C. D. Orr,* or order, -------------
Two Thousand Five Hundred $\frac{00}{100}$----------------------Dollars,
and charge to account of

E. M. REARDON, *Cashier.*

To the Planters' and Mechanics' Bank, Philadelphia.

What amount does Mr. Orr pay for this draft, the current rate of interest being 6%, and the premium on the bill $\frac{1}{4}\%$?

SHORT ANALYSIS.

Bank discount of $2,500	for 60 da. at 6%		= $25.
Bank discount of $2,500	for 3 da.	= $\frac{1}{20}$ of $25	= $ 1.25.
Bank discount of $2,500	for 63 da.		= $26.25.
$2,500	− $26.25	= $2,473.75, Proceeds.	
Exchange on $2,500	at 1%	= $25.	
Exchange on $2,500	at $\frac{1}{4}$ of 1%	= $6.25.	
Cost of draft = $2,473.75 + $6.25		= $2,480.	

In the foregoing there are two transactions: first, as the Dallas bank has the use of Mr. Orr's money for 63 da. it allows Mr. Orr bank discount at the current rate of interest; in effect the bank pays Mr. Orr $26.25 for the use of $2,500 for 63 da.; second, the bank charges $\frac{1}{4}$ of 1% for the transaction in exchange.

8. B pays $1,977 for a bill of exchange due in 60 days; the current rate of interest is 8%; the exchange is made at $\frac{1}{4}$ of 1% premium; find the face of the bill.

SHORT ANALYSIS.

Bank discount of $1 at 8% for 63 da. = $.014.
Cost at par of each $1 of the bill = $1 − $.014 = $.986.
Cost of exchange of each $1 of the bill at $\frac{1}{4}$ of 1% = $.0025.
Cost of each $1 of the bill = $.9885.

If $.9885 will pay for a draft for $1, $1,977 will pay for a draft that is as many times $1 as $1,977 is times $.9885 = 2,000 times.

2,000 × $1 = $2,000.

9. Prove problem 8.

Ans. Proceeds, $1,972; Exchange, $5.

10. A draft due 90 days after sight cost $945.75; the current rate of interest was 6%; the exchange was made at a discount of $\frac{1}{4}$ of 1%; find the face of the draft.

11. Prove problem 10.

12. Exchange for $2,500 cost a merchant $2,506.25; find the rate of exchange. *Ans.* $\frac{1}{4}$% premium.

13. A merchant in Meridian owes a note of $1,500 with 6 months' interest at 6%; he pays with New York exchange, which he buys at a premium of $\frac{1}{4}$%; find the cost of the bill.

Ans. $1,548.86.

14. A 90-day bill of exchange for $1,000 was bought at a premium of $\frac{1}{4}$ of 1%; the current rate of interest was 10%; find the cost of the bill. *Ans.* $976.66.

15. A commission merchant sells a consignment of goods for $1,560; he deducts $2\frac{1}{2}$% as commission, and with the remainder buys sight exchange at $\frac{1}{8}$% premium; find the face of the bill of exchange.

16. An agent sells for B 160 acres of land at $22.50 per acre, and charges as commission 5% of the first $500, 3% of the next two amounts of $500 each, and 2% of the remainder; after deducting his commission he buys a sight draft in favor of B; find the face of the draft, exchange being at $\frac{1}{8}$% premium. *Ans.* $3,498.62.

STOCKS AND BONDS.

287. The capital of a company or a corporation is called **Stock.**

288. A **Share** of stock is the unit of a company's capital, or money and other effects with which it transacts business.

A large sum is necessary for a great enterprise. Men, combining their energies and their means, form a Stock Company. Each man pays for a number of shares. The face, or par, value of a share is usually $100; if the business prospers, a share may become of greater value; if the business is not profitable, a share decreases in value.

289. Profit divided among the stockholders is called a **Dividend.** Undivided profit is called a **Surplus.**

A dividend is commonly a percentage of the stock.

290. A loss is met by an **Assessment**, each member or stockholder being required to pay his part, which is a percentage of the face, or par, value of the stock which he holds.

291. A **Certificate of Stock** is an instrument of writing showing that the holder is the owner of a specified number of shares of the stock.

292. A **Bond** is the written obligation of a government or a corporation to pay a certain sum at a specified time.

A government or a corporation, in order to raise money, issues bonds bearing interest; these bonds are bought by those who seek safe investments for their money. The government provides by taxation for the interest accruing, and for the redemption of the bonds at maturity.

A **Coupon Bond** recites that the corporation, or goverment, issuing it, is indebted to the holder of the bond for the money which its face indicates, and for interest on the bond at a certain per cent, the interest payable at certain periods. Thus, a bond for $100 issued by the City of Houston on the 1st day of July, 1876, for 30 years, with 6% interest, payable semi-annually, recites these facts and provides, by printing them on

the same sheet with the bond, 60 coupons each of the value of $3 at maturity, each thus representing the interest of the bond for 6 months. These coupons are numbered from 1 to 60 successively; they mature alternately on Jan. 1 and July 1, until the last, which matures when the bond itself matures on July 1, 1906. As each coupon matures it is cut off from the sheet and presented for payment.

293. Stocks are at par when they sell for their face value, and at a premium or a discount when they sell respectively for more or less than their face value.

294. A **Stock Broker** is an agent who buys and sells stocks and bonds for others; his commission, which is computed at a per cent of the face value of the stocks or bonds bought or sold, is called Brokerage.

(In the following problems a share or bond unqualified is of the face value of $100.)

PROBLEMS.

1. R owns 10 shares of the stock of the Eutaw Company, which pays an annual dividend of 20%; find R's income from his investment. *Ans.* $200.

2. W owns 12 shares of the stock of a company which pays an average semi-annual dividend of 18% per annum; what income does W derive from his stock?

Ans. Semi-annual income, $108.

3. B owns 20 shares of the stock of a company, which being unsuccessful levies an assessment of 8%; what must B pay?

Ans. $160.

4. On June 1, 1891, L bought 25 shares of a mining company's stock. June 30, 1891, there was an assessment of 15%; Dec. 31, 1891, there was a dividend of 20%; June 30, 1892, there was a dividend of 25%; Dec. 31, 1892, there was an assessment of 10%; find L's income from his stock for the time. *Ans.* $500.

5. A holder of bank stock received $750 as his part of a 5% dividend; how many shares did he own?

6. A dealer in real estate owns 75 shares of the stock of a town company which declares a dividend of 25%; how much does he receive?

(The market values of bonds and stocks are frequently quoted without employing the dollar-sign. In the following problem 117 indicates that $117 was paid for each $100 of the face of the bond.)

7. A broker buys for H 25 bonds at 117, and charges for brokerage $\frac{1}{4}$ of 1%, or 25 cents on each bond; find the total cost.

$117 + $.25 = cost of each bond.
25 × $117.25 = what amount?

8. A holds 175 shares of factory stock, and receives $1,225 dividend; at what per cent was the dividend declared?

Ans. 7%.

9. Find the cost of 15 Central Pacific Railroad bonds at $108\frac{1}{2}$, brokerage $\frac{1}{4}$%.

10. How much does the owner receive from the sale of 20 shares I. C. R. R. stock at 103, brokerage $\frac{1}{4}$%? *Ans.* $2,055.

11. With brokerage at $\frac{1}{4}$%, a number of bonds at 108 cost $4,330; how many bonds were bought?

Price of 1 bond = $108.
Brokerage = $ .25.
Cost of 1 bond = $108.25.
$4,330 = cost of how many bonds?

12. With brokerage at $\frac{1}{8}$%, how many U. S. $1,000 bonds at 117 can be bought for $26,938.75? *Ans.* 23.

13. A speculator bought 75 shares of stock at 82, and sold them at 83, brokerage being $\frac{1}{8}$% on each transaction; what was the speculator's profit? *Ans.* $56.25.

14. For 3 years the Tennessee Mining Company declared an annual dividend of 15%; find the number of shares owned by Charles Logan who received $9,000.

15. If 125 shares of stock cost $9,781.25, and the brokerage is $\frac{1}{4}$%, what is the market value of each share?

16. A broker bought 125 bonds, and sold 130 shares of stock; find his brokerage at $\frac{1}{8}$%. *Ans.* $31.87.

17. A man buys 25 shares of stock at 76; at what price must he sell to gain $250, brokerage on each transaction being $\frac{1}{4}$%?

To gain on 25 shares $250 he must gain on 1 share $\frac{1}{25}$ of $250 = $10.
To buy and sell 1 share costs $76 + 25¢ + 25¢ = $76.50.
To gain $10, a share that costs $76.50 must be sold for $86.50.

18. A man sells 15 U. S. 1000-dollar bonds at 127; he had paid his broker $\frac{1}{4}$% for buying, and now pays $\frac{1}{8}$% for selling; his profit is $600; at what price did he buy?
Ans. 122$\frac{5}{8}$, or $1,226.25 for each bond.

19. Brokerage being $\frac{1}{4}$% for each transaction, at what price does a man, who loses $500, sell 125 shares of stock bought at 67$\frac{1}{4}$? *Ans.* $63.75 per share.

20. Find the cost of 100 shares of stock at 74$\frac{3}{4}$, brokerage being $\frac{1}{4}$%.

21. If stock bought at $75, including brokerage, yields an annual dividend of 6%, at what rate of interest should $75 be loaned to equal the profit on the stock?

Investment of $75 yields 6% of $100.
6% of $100 = $6.
If $75 yields $6, $1 yields $\frac{1}{75}$ of $6 = $.08.
The interest should be $.08 on the dollar. The rate should be 8%.

22. A man buys stock at 124$\frac{7}{8}$, and pays brokerage $\frac{1}{8}$%; the stock yields an annual dividend of 5%; what rate of interest would equal this profit? *Ans.* 4%.

23. State bonds bearing 6% annually are quoted at 66$\frac{5}{12}$; brokerage is $\frac{1}{4}$; what per cent on the investment would these bonds pay? *Ans.* 9%.

24. How many 4% bonds will give an annual income of $1,236?

25. How much must be invested in 8% bonds at 74, brokerage $\frac{1}{4}$%, to yield an income of $4,440? *Ans.* $41,208.75.

26. Solve problem 21 by proportion.

27. From problem 25 find the per cent of the investment paid by the bonds.

28. To yield 8% of the investment, at what price must 5% bonds be purchased, brokerage being $\frac{1}{4}$% ?

> **A 5% bond yields $5 interest.**
> **8% of a sum of money is $5.**
> **What is the whole sum?**

29. A university invests $142,720 in 6% state bonds at $63\frac{7}{8}$, brokerage being $\frac{1}{8}$%; how many 1000-dollar bonds are bought? What per cent is made on the investment?

30. With brokerage at $\frac{1}{8}$%, what price is paid for 6% stocks that yield 4% of the investment? *Ans.* $$149\frac{7}{8}$.

31. What is the price of 5% stocks that yield a profit equal to that of 6% bonds bought at 96?

32. What per cent of the investment is yielded by 5% bonds, bought at 62, including brokerage?

33. If brokerage is $\frac{1}{8}$%, how much must be invested in 6% municipal bonds at $102\frac{7}{8}$, to yield an income of $4,500?

Ans. $77,250.

34. What per cent of the investment do 6% bonds yield when bought at 8% discount, including brokerage?

35. What per cent of the investment do 6% bonds yield when bought at 8% premium, including brokerage?

36. Which is the more profitable, to loan money at 6% interest or to buy 4% bonds at $67\frac{1}{4}$, including brokerage?

37. A has $40,800; he invests it in 4% state bonds costing $63\frac{1}{2}$, brokerage being $\frac{1}{4}$%; find the income.

38. What per cent of the investment is yielded by 6% bonds, bought at $16\frac{5}{8}$% discount, including brokerage?

39. A minor owned a ranch from which he received an annual rent of $3,000; the annual expense for taxes, repairs, etc., was $500. His guardian applied to the Probate Court for an order to sell the ranch for $50,000, and invest the money in 4% state bonds for which $62\frac{3}{8}$ and brokerage of $\frac{1}{8}$% must be paid; why should the order be granted?

Ans. Income would be increased $700.

REVIEW WORK. — All of Partitive Proportion.

EQUATION OF PAYMENTS.

295. Equation of Payments is a process for finding a single date for the just payment of several sums due on different dates.

(Equation of Payments depends upon custom or upon mutual agreement).

296. The date at which the payment of the total amount would justly liquidate the several debts is called the **Equated Time** or the **Average Time.**

297. The time to elapse between a given date and the equated time is called the **Average Term of Credit.**

The Term of Credit for each sum is the time to elapse between a given date and the day the sum becomes due.

In finding the equated time for the payment of an account which consists of several items of account made at different dates, any date may be assumed as the standard of comparison by which to reckon the equated time.

298. The date assumed for comparison is called the **Focal Date.**

PROBLEMS.

1. A customer owes a merchant $1,500 for goods bought to-day at prices that include interest. He agrees to pay $600 in 1 month, $400 in 2 months, $500 in 3 months. He pays the $1,500 at one payment; how long after the purchase?

COMPARISON.

If he should pay now the merchant would gain

The interest of $600 for 1 mo. = Int. of $1 for 600 mo.
The interest of $400 for 2 mo. = Int. of $1 for 800 mo.
The interest of $500 for 3 mo. = Int. of $1 for 1,500 mo.

If he should pay $1,500 now, the merchant would gain the int. of $1 for 2,900 mo., or the interest of $1,500 for $\frac{1}{1500}$ of 2,900 mo.

$\frac{1}{1500}$ of 2,900 mo. = $1\frac{14}{15}$ mo. or 1 mo. 28 da.

If the payment be made now, the merchant would gain and the customer would lose the interest of $1,500 for 1 mo. 28 da.; therefore the payment should be made 1 mo. 28 da. after the purchase.

2. A merchant buys goods for which he agrees to pay \$2,400; terms: \$600, 1 month; \$300, 2 months; \$900, 3 months; \$600, 4 months; find the average term of credit.

Ans. $2\frac{5}{8}$ months.

3. A merchant buys goods on Jan. 2, 1893, for which he agrees to pay as follows: \$400 cash; \$600 in 1 month; \$600 in 2 months; and \$600 in 4 months; find the equated time.

COMPARISON.

Int. of \$ 400 for 0 mo. = Int. of \$1 for 0 mo.
Int. of \$ 600 for 1 mo. = Int. of \$1 for 600 mo.
Int. of \$ 600 for 2 mo. = Int. of \$1 for 1,200 mo.
Int. of \$ 600 for 4 mo. = Int. of \$1 for 2,400 mo.
Int. of \$2,200 for – mo. = Int. of \$1 for 4,200 mo.

By proportion:

\$2,200 : \$1 : : 4,200 mo. : – mo.

4. From problem 3 find the date of payment.

EXPLANATION.

The equated time is $\frac{1}{2200}$ of 4,200 mo. = $1\frac{10}{11}$ mo.
$\frac{10}{11}$ mo. = 30 × $\frac{10}{11}$ da. = $\frac{300}{11}$ da. = $27\frac{3}{11}$ da.
The equated time is 1 mo. $27\frac{3}{11}$ da.

1893	1	2
	1	$27\frac{3}{11}$ da.
1893	2	$29\frac{3}{11}$ da.

Adding 1 mo. $27\frac{3}{11}$ da. to 1 mo. 2 da., the result is 2 mo. $29\frac{3}{11}$ da.; $29\frac{3}{11}$ da. is completed on the 30th day. There is no 30th day of Feb.; the date of payment is Mar. 2, 1893.

5. On. Jan. 15, 1890, a man sold his farm, the terms being \$4,800, $\frac{1}{4}$ cash, $\frac{1}{4}$ in 1 year, and the balance in 2 years; find the average term of credit and the average time of payment.

Ans. 1 yr. 3 mo.; April 15, 1891.

6. A owes B \$500, due in 6 months; at the end of 3 months A pays \$250; when should he pay the balance?

EXPLANATION.

(1) A claims from B the use of \$500 for 6 mo.
(2) After 3 mo. A claims the use of \$500 for 3 mo., and
(3) The interest of \$250 for 3 mo.
(4) A's claim now equals the interest of \$750 for 3 mo.
(5) To balance the interest of \$750 for 3 mo., A holds for 9 mo. the \$250 still due.

7. C bought goods from D as follows: Mar. 10, $200 on 60 days' credit; Mar. 31, $300 on 90 days' credit; April 20, $500 on 60 days' credit; find the equated time of payment.

EXPLANATION.

$200 is due 60 da. after Mar. 10; due May 9.
$300 is due 90 da. after Mar. 31; due June 29.
$500 is due 60 da. after April 20; due June 19.

On May 9, C claims the use of:

$200 from May 9 to May 9, 0 da. = use of $1 for 0 da.
$300 from May 9 to June 29, 51 da. = use of $1 for 15,300 da.
$500 from May 9 to June 19, 41 da. = use of $1 for 20,500 da.

$1,000 : $1 : : 35,800 da. : 35.8 da.

The equated time is the 36th day after May 9, which is June 14.

8. W bought goods from a merchant as follows: May 1, $100 on 30 days' credit; June 1, $200 on 60 days' credit; July 1, $200 on 60 days' credit; take for the focal date the day on which the last payment becomes due, and find the equated time.

EXPLANATION.

$100 is due 30 da. after May 1; due May 31.
$200 is due 60 da. after June 1; due July 31.
$200 is due 60 da. after July 1; due Aug. 30.

On Aug. 30, the merchant could claim the interest of:

$200 from Aug. 30 to Aug. 30, 0 da. = Int. of $1 for 0 da.
$200 from July 31 to Aug. 30, 30 da. = Int. of $1 for 6,000 da.
$100 from May 31 to Aug. 30, 91 da. = Int. of $1 for 9,100 da.

$500 : $1 : : 15,100 da. : $30\frac{1}{5}$ da.

The equated time is 30 da. before Aug. 30, or July 31.

(In these problems half a day or a fraction greater than half a day should be considered a day; less than half a day should not be considered.)

9. Solve problem 7 by assuming as the focal date the day on which the last payment becomes due.

10. Solve problem 8 by assuming as the focal date the day on which the first payment becomes due.

11. Mr. Wilson bought goods as follows:

$300 on 2 months' credit, $500 on 3 months' credit, $500 on 4 months' credit; find the average term of credit.

12. P. J. Willis & Bro. sold a customer a bill of goods; he agreed to pay $100 in 30 days, $200 in 60 days, $300 in 90 days; find the average term of credit.

13. On Feb. 23, 1893, W. D. Cleveland & Co. sold goods to a country merchant, who agreed to pay as follows:

$250 in 30 days, $300 in 60 days, $500 in 90 days.

If the merchant should wish to pay the whole amount at one payment, when should the payment be made?

Ans. May 1, 1893.

AVERAGING ACCOUNTS.

299. Averaging Accounts is the process for finding the Equated Time for paying the Balance of an account.

PROBLEMS.

1. A. & Co. buy goods from B. & Co. as follows:

May 1, $350, on 4 months' credit. (Due Sept. 1.)
Aug. 1, $500, on 3 months' credit. (Due Nov. 1.)

. . . .

(2 months before $350 is due) July 1, A. & Co. pay $200.
(1 month before $500 is due) Oct. 1, A. & Co. pay $250.
Find the equated time for the balance of the account.

Explanation.

A. & Co.'s Interest Credits on Nov. 1.
$200 paid 2 mo. before due; $200 for 2 mo. = $1 for 400 mo.
$250 paid 1 mo. before due; $250 for 1 mo. = $1 for 250 mo.
Total in favor of A. & Co., $1 for 650 mo.

B. & Co.'s Interest Credit (unpaid part of first item).
$350 − $200 = $150 for 2 mo. $150 for 2 mo. = $1 for 300 mo.
Interest difference in favor of A. & Co., $1 for 350 mo.
A. & Co. owe B. & Co. $850 − $450 = $400.
B. & Co. owe A. & Co. interest of $1 for 350 mo.
Interest of $1 for 350 mo. = Interest of $400 for $\frac{1}{400}$ of 350 mo.
$\frac{1}{400}$ of 350 mo. = 26 da. Nov. 1, plus 26 da. = Nov. 27.

2. A merchant buys goods as follows:

Jan. 2, $500 on 60 days; Feb. 1, $500 on 30 days; Mar. 3, $300 on 60 days. Feb. 1, he pays $250; Mar. 3, he pays $500; find amount due May 2, at 6% on unpaid balances.

EXPLANATION.

Let the payment of $500 on Mar. 3 balance one item of $500 due Mar. 3.
Item of Feb. 1 gives seller interest credit of:
$500 from Mar. 3 (due) to May 2, 60 da. = $1 for 30,000 da.
$300 from May 2 (due) to May 2, 0 da. = $1 for 0 da.
Seller's interest credit = $1 for 30,000 da.

Payment of Feb. 1 gives buyer interest credit of:
$250 from Feb. 1 to May 2, 90 da. = $1 for 22,500 da.

Balance of interest in favor of seller = $1 for 7,500 da.
The buyer owes $500 + $300 − $250 = $550.
The buyer owes interest of $1 for 7,500 da. at 6%.
Interest = 7,500 times $\frac{1}{6}$ mill = 1,250 mills = $1.25.
$550 + $1.25 = $551.25, balance due.

3. Solve problem 2 by the Merchants' Rule. *Ans.* $551.25.

4.

Dr. R. S. DUNCAN. *Cr.*

1892				1892			
Apr.	1	To mdse 60 da.	$450	June	15	By cash	$500
May	15	" " 60 da.	600	July	1	" "	300
June	30	" " 90 da.	750	Aug.	15	" "	600

Find the equated time for paying the balance of the foregoing account. *Ans.* Oct. 13, 1892.

5. At 10% interest what balance would Mr. Duncan owe on Jan. 13, 1893, if payment were not sooner made?

Ans. $410.

6. Solve problem 4 by the Merchants' Rule, supposing that settlement is made Oct. 13, 1892, and that the rate of interest is 10%.

REVIEW PROBLEMS.

1. A ranchman lost $5\frac{1}{2}\%$ of his cattle and had 1,890 head remaining; how many had he at first?

2. B owns a farm $\frac{4}{7}$ of whose value, estimated at \$22.50 per acre, is \$1,428.55$\frac{4}{7}$; find the number of acres in the farm.

3. A man invests \$2,500 in a house and lot which he lets for \$250 per year; he pays \$62.50 per year for taxes, insurance, and repairs; what per cent does he gain?

4. Find the day and the hour when $18\frac{3}{4}\%$ of the calendar year 1892 ended. *Ans.* March 9, 3 P.M.

5. X spends each month $6\frac{1}{4}\%$ of his year's salary; at the end of July he has spent \$769.50; find the amount of his salary.

6. Find the difference between the interest of \$2,575 for 6 months at 6%, and the true discount at 6% from a note of \$2,575 due in 6 months without interest.

7. The three points A, B, and C are in a direct line, with B in the intermediate position: from A to B is $10\frac{1}{2}$ rods; from B to C is $37\frac{1}{2}\%$ of the distance from A to C; find the distance from A to C. *Ans.* 16 rd. 4 yd. 1 ft. $2\frac{2}{5}$ in.

8. Find the difference between the present worth of a note of \$2,520 due in 3 months without interest, current rate being 8%, and the proceeds of a note of \$2,520 for 90 days, discounted at bank at 8%. *Ans.* \$2.67.

9. The area of Mr. Jones's farm is to the area of Mr. Brown's farm as $6\frac{1}{2}$ to $5\frac{3}{4}$: Mr. Brown's farm consists of 598 acres; find the number of acres in Mr. Jones's farm.

10. In a proportion whose ratio is $11\frac{2}{5}$ the first term is \$.60 and the last term is 240 bushels of corn; find the other two terms.

11. Based upon problem 10 write another problem whose answer will be \$144.

12. To equal $6\frac{1}{4}\%$ of a number, at what rate must 5% of the number be increased?

13. At what rate of interest must $ 328 be loaned to amount to $ 357.11 in 1 yr. 3 mo.?

14. A cistern whose capacity is 577,500 cubic inches is full of water; how long will the water last if each day's use requires $33\frac{1}{3}\%$ of 48 gallons?

15. A note of $ 600, dated July 2, 1892, is due in 1 year with 6% interest from date; find its present worth January 2, 1893, the current rate of interest being 10%. *Ans.* $ 605.71.

16. B is a point half-way between A to the east and C to the west: when the sun rises at A at 7 o'clock it rises at C at 8:30; find the number of degrees of longitude between B and C.

17. What principal will amount to $ 881.25 in 1 yr. 9 mo. at 10%?

18. In Fahrenheit's thermometer the boiling point is 212°, and the freezing point is 32°; in the Centigrade thermometer the boiling point is 100°, and the freezing point is 0°; when the mercury in a Fahrenheit stands at 68° at what degree should it stand in a Centigrade? *Ans.* 20°.

19. A merchant buys goods amounting in the invoice to $ 750; the terms are 4 months, or 5% off for cash; how much does he save by paying cash, the current rate being 9%?

Ans. $ 15.66.

20. Find the amount, at compound interest, of $ 41.75, at 8%, for 5 yr. 6 mo., payable semi-annually.

21. Of his week's wages, a man pays $\frac{1}{3}$ for board, $\frac{1}{4}$ for clothing, and $\frac{1}{6}$ for other expenses, and has $ 2.60; find the amount of his week's wages.

22. A merchant holds two notes made by a customer: one is for $ 375, due Dec. 16, 1892, the other is for $ 450, due Feb. 1, 1893; how much should he receive in settlement of both on Oct. 1, 1892, current rate of interest being 9%?

23. Of his month's salary a man pays $\frac{1}{4}$ for rent, and $\frac{1}{3}$ for clothing: the sum spent for clothing is $ 10 more than that paid for rent; find the amount of his month's salary.

24. In what time will $ 500 amount to $ 625, at 10% per annum?

25. An agent received $31.50 as his commission for purchasing goods amounting to $1,260; what per cent did he charge? *Ans.* $2\frac{1}{2}\%$.

26. At what per cent is insurance effected for $3,000, when the premium is $45?

27. If 45% of a corn crop amounts to 3,555 bushels, what per cent of the crop is 1,000 bushels?

28. What sum must be collected in order to build a schoolhouse to cost $10,000, and pay 2% for collection?

29. A man sold his farm, which he had previously rented each year for a sum equal to 8% of the amount received from the sale; he lends the proceeds of the sale at 6%, and finds his income reduced $150; how much did he receive for the farm?

30. A church was insured for $12,000 in one company at $\frac{7}{8}\%$, and for $9,500 in another company at 1%; find the cost of the total insurance.

31. B, C, and D formed a partnership; B contributed $7,000 for 12 months, C $8,000 for 11 months, D $9,000 for 10 months; divide proportionally a profit of $3,930.

32. May 31 a merchant bought a bill of goods as follows: $500, cash; $825, 3 months' credit; $675, 4 months' credit; find the equated time of payment.

33. A merchant marks his goods to sell at an advance of 25%, and he sells a hat for $2.25 and allows the customer a discount of 10% from his marked price; what did the merchant pay for the hat?

34. A steamer makes her regular daily run in 6 hr. 10 min.; equipped with a new engine she makes the same run in 5 hr. 30 min.; what per cent has her speed been increased? *Ans.* $12\frac{4}{33}\%$.

35. If the interest of a sum of money for 1 yr. 6 mo. 3 da. is $45.90, what is the interest of the same sum at the same rate for 1 yr. 6 da.?

ADDITIONAL REVIEW WORK, pages 38 and 39.

INVOLUTION.

300. A **power** of a number is the number itself or the product of factors each of which is the number itself (see **42**).

Every number is the first power of itself.

The second and third powers have the special names *square* and *cube* respectively.

301. The number used to indicate the power is called the **exponent.**

A small figure is placed above and to the right of a number whose power is to be expressed or obtained:

6^2 indicates the square of 6; it may also indicate that the second power of 6 is to be obtained.

$3^4 = 3 \times 3 \times 3 \times 3 = 81$; 8^5 signifies the fifth power of 8, or that 8 is to be raised to the fifth power · the fifth power of 8 is the product of 5 factors each of which is 8.

$(\frac{1}{2})^2$ signifies $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

302. The process of raising a number to a required power is called **Involution.**

EXERCISES.

1. Raise the number 3 to the second power.

2. Find the product of the square of 4 and the cube of 3.

Find the values of the following expressions:

3.	4^5	**11.**	$(5\frac{1}{2})^4$	**19.**	$(16\frac{2}{3})^3$
4.	5^4	**12.**	1.5^4	**20.**	$(37\frac{1}{2})^4$
5.	16^3	**13.**	2.75^2	**21.**	$.3^3$
6.	3^6	**14.**	16.1^2	**22.**	$(.16\frac{2}{3})^2$
7.	2^{12}	**15.**	$(\frac{2}{3})^3$	**23.**	$.14^2$
8.	3^8	**16.**	$(.05\frac{1}{4})^2$	**24.**	1.5^3
9.	$(1\frac{1}{2})^3$	**17.**	$(\frac{4}{5})^3$	**25.**	$.15^3$
10.	$(2\frac{3}{4})^3$	**18.**	16.8^3	**26.**	15^3

27. Find the difference between the second power of 1, and the second power of 10.

28. Find the difference between the second power of 10, and the second power of 100.

29. Find the difference between the second power of 100, and the second power of 1,000.

30. Write the square of the least integral number that can be written with one figure; with two figures; with three figures; with four figures.

31. Find the difference between the third power of 1, and the third power of 10.

32. Find the difference between the third power of 10, and the third power of 100.

33. Find the difference between the third power of 100, and the third power of 1,000.

34. Write the cube of the least integral number that can be written with two figures; with three figures.

TABLE OF THE SQUARES OF THE NUMBERS 1 TO 25.

$1^2 = 1$	$6^2 = 36$	$11^2 = 121$	$16^2 = 256$	$21^2 = 441$
$2^2 = 4$	$7^2 = 49$	$12^2 = 144$	$17^2 = 289$	$22^2 = 484$
$3^2 = 9$	$8^2 = 64$	$13^2 = 169$	$18^2 = 324$	$23^2 = 529$
$4^2 = 16$	$9^2 = 81$	$14^2 = 196$	$19^2 = 361$	$24^2 = 576$
$5^2 = 25$	$10^2 = 100$	$15^2 = 225$	$20^2 = 400$	$25^2 = 625$

TABLE OF THE CUBES OF THE NUMBERS 1 TO 25.

$1^3 = 1$	$6^3 = 216$	$11^3 = 1,331$	$16^3 = 4,096$	$21^3 = 9,261$
$2^3 = 8$	$7^3 = 343$	$12^3 = 1,728$	$17^3 = 4,913$	$22^3 = 10,648$
$3^3 = 27$	$8^3 = 512$	$13^3 = 2,197$	$18^3 = 5,832$	$23^3 = 12,167$
$4^3 = 64$	$9^3 = 729$	$14^3 = 2,744$	$19^3 = 6,859$	$24^3 = 13,824$
$5^3 = 125$	$10^3 = 1,000$	$15^3 = 3,375$	$20^3 = 8,000$	$25^3 = 15,625$

(The foregoing tables may be memorized or used for reference.)

EVOLUTION.

303. A **root** of a number is one of its equal factors.

The second root, or square root, of a number is one of the two equal factors of the number.

The square root of 25 is 5; of 144 is 12.

The third root, or cube root, of a number is one of the three equal factors of the number.

The cube root of 8 is 2; of 27 is 3.

The fourth root of a number is one of its four equal factors; the fifth root, one of its five equal factors, and so on.

304. **Evolution** is the process of finding a required root of a number.

305. To indicate the square root of a number the **radical sign** ($\sqrt{\ }$) is used. To indicate any other root a number is written above the radical sign, and between its branches; thus, $\sqrt[3]{27}$ signifies the cube root of 27; $\sqrt[5]{243}$, the fifth root of 243.

306. The number written above the radical sign is called the **Index** of the root.

SQUARE ROOT.

307. Method of Factoring.

PROBLEMS.

1. Find the square root of 225.

PROCESS.

$225 = 3 \times 3 \times 5 \times 5 = 3 \times 5 \times 3 \times 5.$

$3 \times 5 \times 3 \times 5 = (3 \times 5) \times (3 \times 5) = 15 \times 15.$

The square root of 225 is 15.

2. Find the square root of 784.

3. Find the square root of 156,816.

PROCESS.

2	156816
2	78408
2	39204
2	19602
3	9801
3	3267
3	1089
3	363
11	121
	11

EXPLANATION.

The prime factors of 156,816 are 2^4, 3^4, 11^2.
The square root of 156,816 is the product of 2^2, 3^2, 11.

$$2^2 \times 3^2 \times 11 = 396.$$

4. Find the square root of 441; of 3,969; of 1,764.

5. Find the square root of 11,664; of 81,225; of 74,529.

To find the square root of a perfect square:

308. *Separate the number into its prime factors.*

Take one of every two equal factors; find the product of the factors thus taken.

SEPARATING THE POWER INTO PERIODS.

309. The square of a number is expressed by twice as many figures or by one less than twice as many figures as the number itself.

$1^2 = 1$	$10^2 = 100$	$100^2 = 10{,}000$
$5^2 = 25$	$25^2 = 625$	$125^2 = 15{,}625$
$9^2 = 81$	$99^2 = 9{,}801$	$999^2 = 998{,}001$

If a number is expressed by one or two figures, its square root is expressed by one figure; if a number is expressed by three or four figures, its square root is expressed by two figures; if a number is expressed by five or six figures, its square root is expressed by three figures; and so on. Hence,

To find how many figures express the square root of a number:

310. *Beginnina with the order of units, point off the number into periods of two figures. The number of periods shows how many figures express the root.*

311. Synthesis of the Power.

1. By adding partial products find the square of the sum of 2 tens and 5 units.

PROCESS.

	25	
	25	
Square of 5 units =	25	$25 = 5 \times 5 =$ the square of the units of the root.
5 units $\times$ 2 tens =	100	$200 = 5 \times 20, + 20 \times 5, =$ twice the product of
2 tens $\times$ 5 units =	100	the units of the root by the tens of the root.
Square of 2 tens =	400	$400 =$ the square of the tens of the root.
25^2 =	625	$625 =$ the square of the root.

From the foregoing process it is evident that

312. *The square of a number expressed by two figures is equal to the square of the units, plus twice the product of the units by the tens, plus the square of the tens.*

2. By adding partial products find the square of 405.

PROCESS.

405	
405	
25	= the square of the 5 units of the root.
4000	= twice the product of 5 units by 40 tens.
160000	= the square of 400, or 40 tens.
164025	= the square of 405.

From the foregoing process it is evident that

313. *The square of any number considered as tens and units is equal to the square of the units, plus twice the product of the units by the tens, plus the square of the tens.*

3. By adding partial products find the square of 9 tens and 8 units.

4. By adding partial products find the square of 45 tens and 4 units.

By adding partial products find the square of:

5. 361. 6. 432. 7. 19. 8. 175. 9. 215.

314. Extracting the Square Root.

Analytical Process.

PROBLEMS.

1. Find the square root of 625.

PROCESS.

$$\begin{array}{r|l} 625 & 20 \\ \underline{400} & \\ 40+5)225 & \underline{\ 5} \\ \underline{225} & 25 \end{array}$$

EXPLANATION.

Since the power consists of two periods, the root consists of tens and units.

The number 625 is the sum of the three partial products obtained from the root; (1) by taking the square of the tens; (2) taking twice the product of the tens by the units; (3) taking the square of the units; these partial products are to be subtracted from the power.

The greatest square in the first period is 4; the square root of 4 is 2; 20 (or 2 tens) is written as part of the root; 400, the square of 20, is subtracted from the power; the remainder is 225, which consists of twice the product of the tens by the units + the square of the units. Twice the tens of the root = 40; 40 times an unknown number less than 10, + the square of the same unknown number = 225. 40 times an unknown number less than 10 is much greater than the square of the same unknown number. Hence, the square of the units of the root forms but a small part of the number 225. What is required is the approximate number of times that 225 contains 40 + the unknown units; it is evident that 225 must not contain 40 an exact number of times; there must be a remainder that is the square of the units of the root; 225 contains 40 more than 5 times and less than 6 times; 5 is tried as the units' figure of the root; 40 times 5 = 200; the square of 5 = 25; 200 + 25 = 225, which is subtracted from the remaining part of the power. Nothing remains; the square root of 625 is 25.

2. Find the square root of 576. *Ans.* 24.
3. Find the square root of 1,024. *Ans.* 32.
4. Find the square root of 676. *Ans.* 26.
5. Find the square root of 1,225. *Ans.* 35.
6. Find the square root of 1,681. *Ans.* 41.
7. Find the square root of 2,025. *Ans.* 45.
8. Find the square root of 2,601. *Ans.* 51.
9. Find the square root of 14,400. *Ans.* 120.

315. Geometrical Illustration.

The square root of 4,624 is required.

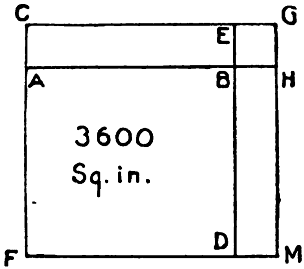
EXPLANATION.

PROCESS.

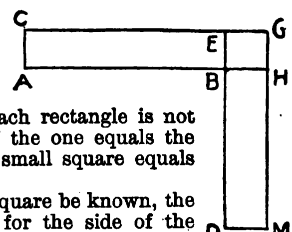
$$\begin{array}{r|r|l} & 4\dot{6}2\dot{4} & 60 \\ & \underline{3600} & \\ 2\times 60 = 120 & 1024 & \underline{\ \ 8} \\ \underline{8} & \underline{1024} & 68 \\ 128 & & \end{array}$$

Let it be supposed that the area of the square *CGMF* is 4,624 sq. in., and that the length of one side is required. The number of inches to the side is to be found by extracting the square root of 4,624.

Point off the number 4,624 into periods of two figures each; the left-hand period is 46. The highest perfect square in 46 is 36, the square root of which is 6. As there will be two figures in the root the 6 is 6 tens, or 60. Hence, the approximate length of the square is 60 inches.



Removing from the given square the square *ABDF*, each of whose sides is 60 inches long, and whose area is, therefore, 3,600 sq. in., the area remaining is 4,624 sq. in. – 3,600 sq. in. = 1,024 sq. in. This area, 1,024 sq. in., is included in the two rectangles *CEBA* and *BHMD*, and the small square *EGHB*.

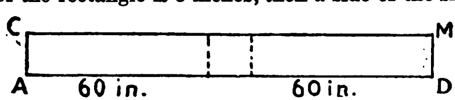


Each rectangle is known to be 60 inches long, and it is known that the two, if joined together, would make a rectangle 120 inches long. The width of each rectangle is not known, but it is known that the width of the one equals the width of the other, and that a side of the small square equals the width of either rectangle.

If the length of one side of the small square be known, the length of the given square can be found, for the side of the square *ABDF* + the side of the square *EGBH* = the side of the square *CGMF*.

Arrange the rectangles and small square as one rectangle.

CMDA is a rectangle somewhat more than 120 inches long, and it has an area of 1,024 sq. in. Supposing that the length of the rectangle is 120 inches, the width is found to be 8 inches approximately (1,024 ÷ 120 = 8+). If the width of the rectangle is 8 inches, then a side of the small square is



8 inches, and the entire length of the rectangle *CMDA* is 120 in. + 8 in. = 128 in. If the length of the rectangle is 128 inches, and its width is 8 inches, its area is 1,024 sq. in., which equals the area remaining after the square of 3,600 sq. in. was removed from the square of 4,624 sq. in.

Therefore, the length of the small square is 8 inches, and the length of the given square is 60 in. + 8 in. = 68 in., and the square root of 4,624 is 68.

316. To find the square root of a number:

(i) *Separate the number into periods of two figures by placing a dot over the units' figure, and every alternate figure to the left.*

(ii) *Determine the greatest perfect square contained in the period on the left, and take the square root of this perfect square for the first, or highest, figure of the square root.*

(iii) *Square the first figure of the root and subtract the square from the first period. To the remainder annex the second period; take the number thus formed for a dividend.*

(iv) *Consider as tens the part of the root already found; double it for a trial divisor; divide the dividend by the trial divisor, and take the quotient for the next figure of the root. Add the quotient to the trial divisor for a corrected divisor.*

(v) *Multiply the corrected divisor by the root-figure last found; subtract the product from the dividend.*

(vi) *If there be other periods proceed as before, regarding the part of the root already found as tens, and so continue until the entire root is found.*

PROOF. — *Square the square root.*

Find the square root of:

10. 1,369.	*Ans.* 37.	**16.** 361.	*Ans.* 19.
11. 19,600.	*Ans.* 140.	**17.** 289.	*Ans.* 17.
12. 4,096.	*Ans.* 64.	**18.** 5,329.	*Ans.* 73.
13. .0625.	*Ans.* .25.	**19.** 7,396.	*Ans.* 86.
14. .0961.	*Ans.* .31.	**20.** 84.64.	*Ans.* 9.2.
15. 1,444.	*Ans.* 38.	**21.** 9,409.	*Ans.* 97.

22. Extract the square root of 4,120,900.

PROCESS.

$$\begin{array}{r|l} & \dot{4}1\dot{2}0\dot{9}0\dot{0}\,(2030 \\ & 4 \\ \hline 40 & \;\;1209 \\ 400 & \;\;1209 \\ \hline 403 & \qquad 00 \end{array}$$

EXPLANATION.

The highest figure of the root is 2. The square of 2 is 4, which represents 4 tens; 12 does not contain 4 tens; write 0 in the root; bring down the next period; the trial divisor is now 40 tens; the next figure of the root is 3; subtract 3×403 from the remainder; bring down the last period which, being a period of noughts, gives a nought for the last figure of the root. $\sqrt{4120900} = 2030$.

23. Find the square root of $\frac{100}{169}$. *Ans.* $\frac{10}{13}$.

24. Find the approximate square root of 50.

PROCESS.

The process may be continued to any number of decimal figures that may be desired in the root.

Decimal periods are pointed off from the decimal point toward the right; if the last period is incomplete it is filled with a nought without altering the value of the number.

Step 1.	*Step* 2.	*Step* 3.
$\dot{5}0$(7.	$\dot{5}0.\dot{0}0$(7.0	$\dot{5}0.\dot{0}\dot{0}00$(7.07
49	49	49
14) 1	140) 1 00	1407) 1 0000
		9849
		151

25. Find the square root of 101. *Ans.* 10.049.

26. Find the square root of $\frac{7}{8}$. *Ans.* .935+.

A common fraction may first be reduced to a decimal, and the approximate root of the decimal may then be founa.

27. Find the square root of 200; of $\frac{4}{5}$; of $\frac{3}{7}$.

28. Find the square root of 63,504. *Ans.* 252.

29. Find the square root of 65,536. *Ans.* 256.

30. Find the value of $\sqrt{1,827,904}$. *Ans.* 1,352.

31. Find the value of $\sqrt{3,154,176}$. *Ans.* 1,776.

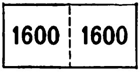
317. Applications of Square Root.

32. A rectangular field consists of 3,200 square rods; the field is twice as long as it is broad; the owner wishes to divide the field into two smaller fields, each of which will be square; how long will the cross-fence be?

PROCESS.

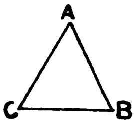
Each smaller field will consist of $\frac{1}{2}$ of 3,200 sq. rd. = 1,600 sq. rd. The cross-fence will be a side of each of the square fields. The length of the fence will be as many times one linear rod as there are units in the square root of 1,600.

$\sqrt{1600} = 40$; 40 times 1 rd. = 40 rd.

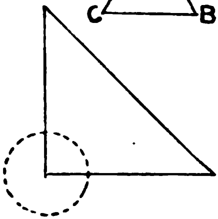


$\dot{1}6\dot{0}0$(40
16
40) 00

33. A rectangular field consists of 180 acres; it is twice as long as it is broad; find the length and the breadth of the field. *Ans.* 240 rd.; 120 rd.



318. A **Triangle** is a plane figure bounded by three straight lines. The figure *ABC* represents a triangle.



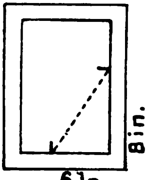
319. A **right-angled** triangle is a triangle, two of whose sides diverge from each other at an angle of 90°, or $\frac{1}{4}$ of the number of degrees in the circumference of a circle.

The side opposite the right-angle is the **Hypothenuse**; the two other sides are the **Base** and the **Perpendicular.**

320. Two of the sides of a right-angled triangle being given, the other side can be found.

321. To find one side of a right-angled triangle when the two other sides are given.

34. Mark 6 inches on one edge of a slate and 8 inches on an adjacent edge, as in the diagram.



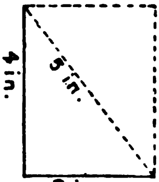
Draw a straight line connecting the two points; measure the line.

35. Find the square root of the sum of 8^2 and 6^2.

322. The square of the hypothenuse of a right-angled triangle is equal to the sum of the squares of the two other sides.

323. The square of either the base or the perpendicular of a right-angled triangle is equal to the difference of the squares of the two other sides.

36. The base of a right-angled triangle is 3 inches; the perpendicular is 4 inches; find the length of the hypothenuse.



PROCESS.

$$3^2 = 9$$
$$4^2 = 16$$
$$4^2 + 3^2 = 25$$
$$\sqrt{25} = 5$$

37. The base of a right-angled triangle is 30 feet; the hypothenuse is 50 feet; find the perpendicular.

PROCESS.

$50^2 = 2{,}500$; $30^2 = 900$; $2{,}500 - 900 = 1{,}600$; $\sqrt{1{,}600} = 40$.

38. Find the length of the longest straight furrow that could be ploughed in an unfenced square field containing 10 acres.

Ans. 56.56 + rods.

39. A, B, and C are three points: A is 60 yards north of C; B is 70 yards west of C; how far is A from B?

40. Of a rectangular field the length of one side is 40 rods; the distance between opposite corners of the field is 50 rods; find the number of acres in the field. *Ans.* $7\frac{1}{2}$ acres.

41. A ladder 39 feet long, when the foot is placed 15 feet from a wall, reaches the wall 1 foot below the top; how high is the wall? *Ans.* 37 feet.

CUBE ROOT.

324. Method by Factoring.

PROBLEMS.

1. Find one of the three equal factors of 125.

PROCESS.

$$125 = 5 \times 5 \times 5;\ \sqrt[3]{125} = 5.$$

2. Find the cube root of 1,728.

PROCESS.

$$1728 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3;$$
$$\text{or, } (2 \times 2 \times 3) \times (2 \times 2 \times 3) \times (2 \times 2 \times 3);$$
$$\text{or, } (2^2 \times 3) \times (2^2 \times 3) \times (2^2 \times 3).$$
$$\sqrt[3]{1728} = 2^2 \times 3 = 12.$$

3. Find the cube root of 46,656. *Ans.* 36.

4. Find the cube root of 91,125. *Ans.* 45.

5. Find the cube root of 15,625.

6. Find the cube root of 42,875. *Ans.* 35.

7. Find the cube root of 13,824.

8. Find the cube root of 35,937. *Ans.* 33.

325. To find the cube root of a perfect cube:

Separate the number into its prime factors; take one of every three equal factors; find the product of the factors thus taken.

Separating the Power into Periods.

326. If a number is expressed by one figure, its cube is expressed by no more than three figures.

$$1^3=1 \qquad 5^3=125 \qquad 9^3=729$$

Each additional figure of the root corresponds to an additional period of three figures in the cube.

$$\begin{array}{lll} 1^3 = 1 & 3^3 = 27 & 50^3 = 125{,}000 \\ 10^3 = 1{,}000 & 30^3 = 27{,}000 & 500^3 = 125{,}000{,}000 \end{array}$$

327. To find how many figures express the cube root of a number:

Beginning with the order of units, point off the number into periods of three figures each.

The number of periods shows how many figures express the root; the left-hand period may consist of one, two, or three figures.

328. Synthesis of the Power.

The third power of 25 is required.

PROCESS.

$$(20+5)\times(20+5)=25^2$$

$$25^2 = \left\{\begin{array}{l} 20\times 20 \quad = 20^2 \\ \left.\begin{array}{r} 20\times 5 \\ 5\times 20 \end{array}\right\} = \ 2\times 5\times 20 \\ 5\times 5 \quad = \ 5^2 \end{array}\right.$$

Let these partial results be brought down, and from them let the cube of 25 be obtained.

$$\begin{array}{llllll} & & 20^2 & = 20^3 & & \\ & 20\times\left\{\begin{array}{l}\\ \\ \\ \end{array}\right. & 2\times 5\times 20 = & & 2\times 5\times 20^2 & \\ & & 5^2 \qquad\quad = & & & 5^2\times 20 \\ 25^3 = \text{sum of} & \text{and} & & & & \\ & & 20^2 \qquad\quad = & & 5\times 20^2 & \\ & 5\times\left\{\begin{array}{l}\\ \\ \\ \end{array}\right. & 2\times 5\times 20 = & & & 2\times 5^2\times 20 \\ & & 5^2 \qquad\quad = & & & 5^3 \\ \hline & 25^3 & & \multicolumn{3}{l}{= 20^3 + 3\times 5\times 20^2 + 3\times 5^2\times 20 + 5^3} \end{array}$$

329. *The cube of a sum of tens and units is equal to the cube of the tens, plus three times the product of the square of the tens by the units, plus three times the product of the tens by the square of the units, plus the cube of the units.*

330. Extracting the Cube Root.

Analytical Process.

PROBLEMS.

Find the cube root of 15,625.

EXPLANATION.

Since the power consists of two periods, the root consists of tens and units. The number 15625 is the sum of four partial products obtained from the root by: (1) Taking the cube of the tens; (2) Taking three times the product of the square of the tens by the units; (3) Taking three times the product of the tens by the square of the units; (4) Taking the cube of the units. These partial products are to be subtracted from the power.

$\dot{1}562\dot{5}$

The Tens' Figure.

The greatest perfect cube in the highest period, 15, is 8. The cube root of 8 is written as the tens' figure of the root, and the cube of 2 tens is subtracted from the power. The remainder, 7625, consists of three partial products numbered above 2, 3, and 4.

$$\begin{array}{rr} & \dot{1}562\dot{5}(2 \text{ tens} \\ 20 \times 20 \times 20 = & \underline{8000} \\ & 7625 \end{array}$$

The Units' Figure.

It is now known that 7625 contains not only $3 \times 20 \times 20 = 1200$, but that it also contains $3 \times 20 = 60$. But 7625 contains *the product of 1200 by a number of units*, and it contains *the product of 60 by the square of the number of units*, and it contains the cube of the number of units. Now it is seen that 1260 is contained in 7625 as many as 6 times; therefore, 6 may be tried as the units' figure of the root. If the units' figure prove to be 6, there must be subtracted from 7625 the sum of 6×1200, $6 \times 6 \times 60$, and $6 \times 6 \times 6$.

But $6 \times 1200 = 7200$, and $6^2 \times 60 = 2160$; $7200 + 2160 = 9360$, and without going further, it is seen that 6 is too great for the units' figure of the root. Therefore 5 is tried.

To try five for the units' figure, is to subtract the sum of

$$\begin{array}{lrr} & & 7625 \\ 5 \times 1200 & = 6000 & \\ 5 \times 5 \times 60 & = 1500 & \\ 5 \times 5 \times 5 & = \underline{\ \ 125} & \underline{7625} \end{array}$$

Nothing remains; hence 5 is the units' figure of the root, and the cube root of 15625 is 25.

Geometrical Illustration.

The cube root of 42,875 is required.

$$
\begin{array}{rrr|l}
 & & \dot{4}2\dot{8}75 & 30 + 5 = 35 \\
 & & 27000 & \\
3 \times 30^2 = & 2700 & 15875 & \\
3 \times 30 \times 5 = & 450 & & \\
5 \times 5 = & 25 & & \\
\hline
 & 3175 & 15875 &
\end{array}
$$

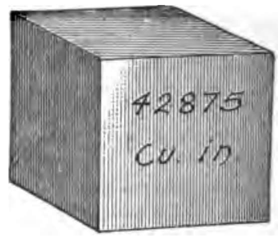


Fig. 1.

Let it be supposed that the volume of Fig. 1 is 42,875 cu. in. The number of inches in the length, breadth, or thickness of the cube is the cube root, or one of the three equal factors, of 42,875.

Point off the number 42,875 into two periods. The highest perfect cube in the left-hand period is 27, the cube root of which is 3. As there will be two figures in the root, the 3 is 3 tens, or 30. Hence the approximate length of either edge of the cube is 30 in.

Take out of the cube a smaller cube whose edge is 30 in. A volume of 27,000 cu. in. is thus subtracted, and a volume of 15,875 cu. in. remains. The volume remaining may be separated into three rectangular solids *C*, *D*, and *E* (Fig. 3), three smaller rectangular solids, *F*, *G*, and *H* (Fig. 4), and one small cube *B* (Fig. 5).

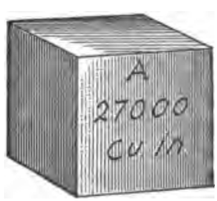


Fig. 2.

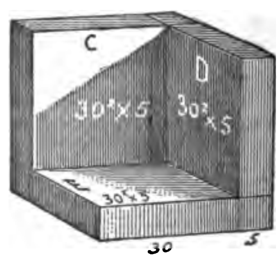


Fig. 3.

Fig. 4.

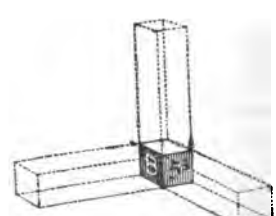


Fig. 5.

Comparison of the figures shows:

(1) That each of the larger solids, C, D, and E is 30 in. long and 30 in. wide, and that they are of equal thickness.

(2) That the length of each of the smaller solids F, G, and H is 30 in., and that the width as well as the thickness of each is equal to the thickness of C, D, or E.

(3) That each dimension of the small cube B is equal to the thickness of C, D, or E.

(4) That the edge of the original cube is equal to the edge of the cube A increased by the edge of the cube B.

Let it be supposed that the three solids C, D, and E contain all the remaining volume, 15,875 cu. in. If each of these solids were 1 in. thick, its volume would be 900 cu. in., and the volume of the three would be 2,700 cu. in. If the solids contain 2,700 cu. in. for 1 in. of thickness, to contain 15,875 cu. in. their thickness must be approximately 5 in.

$$15{,}875 \div 2{,}700 = 5+.$$

If the thickness of the solids is 5 in., the number of cubic inches contained in the solids C, D, and E is . . . . $5 \times 2{,}700 = 13500$

If the thickness is 5 in., the width also of the solids F, G, and H is 5 in., and as each is 30 in. long, their volume is . . . . $3 \times 30 \times 5 \times 5 = 2250$

The volume of the small cube is . . . . $5 \times 5 \times 5 = 125$

The sum of the volumes of C, D, E, F, G, H, and B, ... $= 15875$

This volume equals the volume remaining when the cube A was taken out of the original cube; subtracting, nothing remains. Therefore the edge of the original cube, being equal to the edge of the cube A (30 in.) increased by the edge of the cube B (5 in.), is 35 in. long, and the cube root of 15,875 is 35.

331. To find the cube root of a number:

(i) *Separate the number into periods by placing a dot over the units' figure, and every third figure to the left.*

(ii) *Determine the greatest perfect cube contained in the period on the left, and take the cube root of this perfect cube for the first, or highest, figure of the cube root.*

(iii) *Subtract the cube of the first root-figure from the first period; to the remainder annex the second period; take the number thus formed for a dividend.*

(iv) *Multiply the square of the first root-figure considered as tens by 3 for a trial divisor; take the quotient for the next figure*

of the root; for the true divisor add to the trial divisor 3 times the product of the last root-figure by the first root-figure considered as tens, and to this sum add the square of the last root-figure.

(v) *Multiply the true divisor by the last root-figure; subtract the product from the dividend, and to the remainder annex the next period for a new dividend.*

(vi) *Square the sum of the first root-figure considered as hundreds and the second root-figure considered as tens. Multiply the sum by 3. Use the product as a trial divisor, and continue as before.*

PROOF. — Cube the root.

PROBLEMS.

1. Find the cube root of 12,167; of 13,824; of 17,576.
2. Find the cube root of 19,683; of 29,791; of 35,937.
3. Find the cube root of 74,088; of 91,125; of 250,047.
4. Find the cube root of 1,860,867.

PROCESS.

			$\dot{1}86\dot{0}86\dot{7}$	1 (first root-figure).
			1	
1st trial divisor $= 3 \times 10^2$	=	300	860	2 (second root-figure).
To which add $3 \times 10 \times 2$	=	60		
To which add 2×2	=	4		
First correct divisor	=	364	728	
2d trial divisor 3×120^2	=	43200	132867	3 (third root-figure).
To which add $3 \times 120 \times 3$	=	1080		
To which add 3×3	=	9		
Second correct divisor	=	44289	132867	123 (root).

5. Find the cube root of 1,061,208,000. *Ans.* 1,020.
6. Find the cube root of $\frac{27}{64}$; of $\frac{125}{216}$; of $\frac{64}{343}$. *Ans.* $\frac{3}{4}$.
7. Find the approximate cube root of 11.

PROCESS.

Step 1.	*Step* 2.	*Step* 3.
11(2	1324)3.000(.2	146524).352000(.02
8	2 648	293048
3	352	58952

The process may be continued to any number of decimal figures in the root that may be desired; decimal periods must be full periods.

8. Find the cube root of 274.625. *Ans.* 6.5.

9. Find the cube root of $4\frac{1}{4}$, or 4.250. *Ans.* ——.

10. Find the cube root of 3. *Ans.* 1.44 +.

11. Find the cube root of 50. *Ans.* 3.68 +.

12. Find the cube root of $\frac{1}{64}$. *Ans.* $\frac{1}{4}$.

13. Find the cube root of $\frac{1}{1000}$. *Ans.* ——.

14. Find the cube root of .000001. *Ans.* .01.

15. Find the cube root of 34. *Ans.* 3.23+.

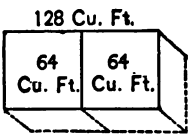
16. Find the number of square inches in the surface of one side of a cube containing 19,683 cubic inches.

332. Applications of Cube Root.

17. A pile of wood contains 128 cubic feet; its length is twice as great as its width, and its width is equal to its thickness; find the dimensions of the pile.

EXPLANATION.

If the pile were half as long as it is, the three dimensions would be equal, and the pile would contain 64 cu. ft.; the cube root of 64 is 4; therefore one half of the pile measures 4 ft. by 4 ft. by 4 ft.; and the whole pile measures 4 ft. by 4 ft. by 8 ft.



18. A block of stone contains 192 cubic feet; its length is three times as great as its width, and its width is equal to its thickness; find the dimensions of the block.

19. Find the approximate inside dimensions of a cubical box that will contain a bushel of corn.

Ans. Length, breadth, and thickness, each 12.907 inches.

20. Find the approximate inside dimensions of a cubical can that will contain a gallon of oil.

21. A cubical cistern is full of water; it contains 6,500 gallons; find the dimensions of the cistern.

22. A watering-trough is 8 times as long as it is wide; its width and depth are equal; it will hold 150 gallons of water; find the length (inside measure).

$\frac{1}{8}$ of 150 gal. = $18\frac{3}{4}$ gal.; $18\frac{3}{4} \times 231$ cu. in. = 4331.25 cu. in.

$8 \times \sqrt[3]{4331.25} = ?$

PROGRESSION.

333. A succession of numbers increasing or decreasing by a common difference, by a constant ratio, or by other fixed law, is called a **Series** of numbers.

In the series 1, 3, 5, 7 there is a common difference.

In the series 1, 3, 9, 27 there is a constant ratio.

334. The numbers that form a series are called the **Terms**.

The First Term and the Last Term are called the **Extremes**. All the intermediate terms are called the **Means**.

335. A series whose terms constantly increase from lower to higher numbers, is called an **Ascending Series**.

Thus, 1, 3, 5, 7, and 1, 3, 9, 27 are ascending series.

336. A series whose terms constantly decrease from higher to lower numbers is called a **Descending Series**.

ARITHMETICAL PROGRESSION.

337. A series whose terms increase or decrease by a common difference is called an **Arithmetical Progression**.

338. In Arithmetical Progression there are five numbers to be considered.

(1) The First Term. (2) The Last Term. (3) The Common Difference. (4) The Number of Terms. (5) The Sum of the Series.

Any three of these numbers being given, the other two numbers can be found.

339. To find the Last Term.

PROBLEMS.

1. A man has four sons; the common difference of their ages is 2 years; the youngest is 5 years old; find the age of the eldest son.

EXPLANATION.	PROCESS.
5 yr. + 2 yr. + 2 yr. + 2 yr. = 11 yr.	3 × 2 yr. = 6 yr.
The common difference has been added 3 times to the first term.	6 yr. + 5 yr. = 11 yr.

2. Find the amount of $100 at simple interest for 4 years at 6%.

EXPLANATION.

0 yr. 1st term.	Amt. 1 yr. 2d term.	Amt. 2 yr. 3d term.	Amt. 3 yr. 4th term.	Amt. 4 yr. 5th term.
$100.	$100 + $6.	$100 + $12.	$100 + $18.	$100 + $24 = $124.

PROCESS.

4 × $6 = $24; $100 + $24 = $124, Amount.

340. To find the last term of an ascending arithmetical series:

Multiply the common difference by the number of terms less 1. Add the product to the first term.

3. A man spends $20 a day; at the beginning of the 1st day he had $200; how much has he at the beginning of the 5th day?

EXPLANATION.

1st term.	2d term.	3d term.	4th term.	5th term.
$200.	$200 − $20.	$200 − $40.	$200 − $60.	$200 − $80 = $120.

PROCESS.

4 × $20 = $80; $200 − $80 = $120.

341. To find the last term of a descending arithmetical series.

Multiply the common difference by the number of terms less 1. Subtract the product from the first term.

4. The first term is 6; the common difference is 8; the number of terms is 9; find the last term.

5. The first term of a descending series is 38; the common difference is 3; the number of terms is 12; find the last term.

6. The first term is $\frac{1}{2}$; the common difference is $2\frac{1}{2}$; the number of terms is 9; find the last term.

7. The first term of a descending series is 300; the common difference is $7\frac{1}{2}$; the number of terms is 21; find the last term.

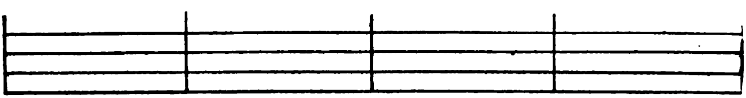
342. To find the Common Difference.

PROBLEMS.

1. There are 5 fence-posts in a direct line; the panels are of equal length; from the first post to the fifth post the distance is 32 feet; find the length of a panel.

EXPLANATION.

In 5 terms the common difference is used 4 times.
32 ft. $\div$ 4 = 8 ft. Each panel is 8 ft. long.



For 5 posts there are 4 panels.
32 ft. for 4 panels = 8 ft. for 1 panel.
(The first term in this case is 0 ft.)

2. The first term is 3; the last term is 129; the number of terms is 43; find the common difference.

EXPLANATION.

In 43 terms the common difference is used 42 times.
The whole difference is 129 − 3 = 126.
The common difference is 126 $\div$ 42 = 3.

To find the common difference:

343. *Divide the difference of the extremes by the number of terms less 1.*

3. The first term is $\frac{2}{3}$; the last term is $1\frac{3}{4}$; the number of terms is 14; find the common difference. *Ans.* $\frac{1}{12}$.

4. The first term is .001; the last term is 1,000; the number of terms is 1,000,000; find the common difference.

5. The first term is 1 rod; the last term is 1 mile; the number of terms is 639; find the common difference.

6. The first term is 2 pounds; the last term is 1 cwt. 2 lb.; the number of terms is 51; find the common difference.

7. A merchant begins business with $3,000; at the beginning of the 13th year he has $9,000; find his average annual gain, his yearly expenses being $1,000. *Ans.* $1,500.

8. The first term is $1\frac{7}{8}$; the last term is 15; the number of terms is 7; find the common difference.

344. To find the Number of Terms.

PROBLEMS.

1. The first term is 12; the last term is 72; the common difference is 5; find the number of terms.

Explanation.

The whole difference is $72 - 12 = 60$. The common difference is used $60 \div 5 = 12$ times. Hence the number of terms is $12 + 1 = 13$.

To find the number of terms:

345. *Divide the difference of the extremes by the common difference; add 1 to the quotient.*

2. The first term is $3\frac{1}{8}$; the last term is 100; the common difference is $3\frac{1}{8}$; find the number of terms.

3. The first term is $6\frac{1}{4}$; the last term is $106\frac{1}{4}$; the common difference is $12\frac{1}{2}$; find the number of terms.

4. The first term is $37\frac{1}{2}$; the last term is 475; the common difference is $6\frac{1}{4}$; find the number of terms.

5 The first term is $\frac{3}{16}$; the last term is $93\frac{1}{4}$; the common difference is $\frac{1}{16}$; find the number of terms.

6. The first term is .002; the last term is .02; the common difference is .009; find the number of terms.

346. To find the Sum of a Series.

PROBLEMS.

1. The first term is 4; the last term is 16; the number of terms is 5; find the sum of all the terms.

EXPLANATION.	PROCESS.
The number of terms less 1 is the number of times the common difference is used.	$5-1=4$
The whole difference is 12.	$12 \div 4 = 3$ (C. D.)
$12 \div 4 =$ the common difference.	4, 7, 10, 13, 16.
The common difference being 3, the terms are	$5 \times 10 = 50$.

There are 5 terms. If the terms were equal, the sum of the terms would be 5 times either term; the middle or average term is known. The sum of the terms is 5 times the average term.

The average term is 10; $5 \times 10 = 50$, the sum of the terms.

(Note that half the sum of the extremes is 10.)

PROOF.

$$\begin{array}{r} 4 \\ 7 \\ 10 \\ 13 \\ \underline{16} \\ 50 \end{array}$$

2. Find the sum of the series 7, 17, 27, 37, 47, 57, 67, 77.

EXPLANATION.

There is no average *term*, because the number of terms is even.

The average of the terms may be found by supposing a number in the middle.

7, 17, 27, 37,   47, 57, 67, 77.

A supposed number, or half-term, to occupy the blank space, and to be *half-way between* 7 and 77 would be $37 + 5$, or $47 - 5 = 42$; 42 is the average of the terms. In a long series of terms the average of the terms is found at once by taking the number *half*-way between the extremes; thus, $77 + 7 = 84$, half of which is 42.

The number of terms is 8; the average of the terms is 42; $8 \times 42 = 336$, the sum of all the terms.

347. To find the sum of the terms of an arithmetical series: *Multiply half the sum of the extremes by the number of terms.*

3. Find the sum of the numbers from 1 to 100, inclusive.

4. Solve problem 28, page 15.

5. The first term is $\frac{1}{2}$; the last term is $7\frac{1}{2}$; the number of terms is 15; find the common difference and the sum of the terms.

6. Find the sum of the numbers from 100 to 1000, inclusive.

GEOMETRICAL PROGRESSION.

348. A series whose terms increase or decrease by a constant ratio is called a **Geometrical Progression.**

In a descending series, the ratio is called a Divisor.

In an ascending series, the ratio is called a Multiplier.

349. In Geometrical Progression there are five numbers to be considered:

(1) The First Term.
(2) The Last Term.
(3) The Ratio.
(4) The Number of Terms.
(5) The Sum of the Series.

Any three of these numbers being given, the other two numbers can be found.

350. To find the Last Term.

PROBLEMS.

1. The first term is 2; the multiplier is 3; the number of terms is 6; find the last term.

EXPLANATION.

The terms found by multiplying each successive term by 3 are 2, 6, 18, 54, 162, 486.

$$
\begin{aligned}
\text{1st Term} &= 2 \\
\text{2d Term} &= 3 \times 2 \\
\text{3d Term} &= 3 \times 3 \times 2 &&= 3^2 \times 2 \\
\text{4th Term} &= 3 \times 3 \times 3 \times 2 &&= 3^3 \times 2 \\
\text{5th Term} &= 3 \times 3 \times 3 \times 3 \times 2 &&= 3^4 \times 2 \\
\text{6th Term} &= 3 \times 3 \times 3 \times 3 \times 3 \times 2 &&= 3^5 \times 2 = 486.
\end{aligned}
$$

2. The first term is 108, the divisor 3, and the number of terms 4; find the last term.

EXPLANATION.

The terms of the descending series may be found by dividing each successive term by 3.

108, 36, 12, 4.

The first term $= 108$.
The second term $= 108 \div 3$.
The third term $= 108 \div 3^2$.
The fourth term $= 108 \div 3^3 = 4$.

To find the last term of a Geometrical Progression.

351. For an ascending series: *Multiply the first term by the multiplier raised to a power whose index is 1 less than the number of terms.*

For a descending series: *Divide the first term by the divisor raised to a power whose index is 1 less than the number of terms.*

3. The first term is 24; the divisor is 2; the number of terms is 9; find the last term. *Ans.* $\frac{3}{32}$.

4. Find the amount of $100 at 6% compound interest for 5 years. *Ans.* ——.

5. The first term is 7, the multiplier is 3, and the number of terms is 10; find the last term. *Ans.* 137,781.

352. To find the Ratio.

PROBLEMS.

1. The first term is 5; the last term is 5,000; and the number of terms is 4; find the multiplier.

EXPLANATION.

Since there are 4 terms, there are 3 multipliers, or the same multiplier 3 times. The product of 5 times the cube of the multiplier is the last term, 5,000.

Then $5,000 \div 5 =$ the cube of the multiplier.

PROCESS.

$$5000 \div 5 = 1000$$
$$\sqrt[3]{1000} = 10$$

To find the ratio:

353. *Divide the greater by the less extreme; of the quotient find the root indicated by the number of terms less 1.*

2. Find the ratio of a descending series of 5 terms whose first term is 10,000 and whose last term is 1.

PROCESS.

$$10000 \div 1 = 10000.$$
$$\sqrt[4]{10000} = \sqrt{100} = 10.$$

3. Find the divisor in a descending series of 7 terms whose greatest term is 128 and whose least term is 2.

(The square root of the cube root = the 6th root, or the 6th root may be found by factoring.)

354. To find the Number of Terms.

PROBLEMS.

1. The first term is 6; the last term is 384; the multiplier is 4; find the number of terms.

EXPLANATION.

The last term, 384, is the product of 6 (the first term), and 4^3, the multiplier raised to a power whose index is 1 less than the number of terms. If, therefore, 384 be divided by the first term, 6, the quotient will contain that power of 4 whose index is one less than the number of terms. $384 \div 6 = 64$. $64 = 4^3$. The index of the power (3) is equal in number to 1 less than the number of terms. $3 + 1 = 4$.

$$\begin{aligned} & 6 \ (1) \\ 4 \times 6 = & \ 24 \ (2) \\ 4^2 \times 6 = & \ 96 \ (3) \\ 4^3 \times 6 = & \ 384 \ (4) \end{aligned}$$

$$384 \div 6 = 64$$
$$4\overline{)64}$$
$$4\overline{)16}$$
$$4$$

To find the number of terms in a geometrical series:

355. *Divide the greater extreme by the less; find how many times the result contains the ratio as a factor. This number (of factors) increased by 1 is equal to the number of terms.*

2. The first term is 6; the last term is 1,536; the multiplier is 4; find the number of terms. *Ans.* 5.

3. The extremes are 7 and 109,375; the ratio is 5; find the number of terms. *Ans.* 7.

356. To find the Sum of a Series.

PROBLEMS.

1. Find the sum of the series 3, 9, 27, 81, 243.

EXPLANATION.

The multiplier is seen to be 3. By the following process the intermediate terms are eliminated.

MULTIPLY THE SERIES BY 3.

The new series is	$9 + 27 + 81 + 243 + 729$	$= 3$ times the series.
The first series is	$3 + 9 + 27 + 81 + 243$	$= 1$ time the series.
Subtract	$729 - 3 = 726$	$= 2$ times the series.

$\frac{1}{2}$ of $726 = 363 =$ the sum of the series.

PROCESS.

$3 \times 243 = 729$; $729 - 3 = 726$; $\frac{1}{2}$ of $726 = 363$.

To find the sum of a geometrical series:

357. *Multiply the last term by the ratio. From the product subtract the first term. Divide the remainder by the ratio less 1.*

2. The first term is 5; the multiplier is 6; the last term is 6,480; find the sum of the series. *Ans.* 7,775.

3. The first term is 10,000; the last term 10; and the divisor, or direct ratio, 10; find the sum of the series.

(The sum is the same as it would be were it an ascending series whose first term is 10, last term 10,000, and multiplier 10.)

4. The first term is 5; the multiplier 6; and the number of terms 7; find the sum of the series. *Ans.* 279,935.

(The last term 233,280 is first found. See **350**.)

GENERAL REVIEW.

PROBLEMS.

1. What per cent of $\frac{14}{15}$ is the fractional unit of $\frac{14}{15}$?

2. How much seed-wheat is required to sow $46\frac{1}{2}$ acres, at 1 bu. 3 pk. 4 qt. to the acre?

3. A patchwork quilt $7\frac{1}{2}$ ft. by 78 in. is made of pieces each of which contains 4 square inches; find the number of pieces in the quilt.

4. At 8%, what principal will amount to \$898.04 in 5 yr. 4 mo. 15 da.?

5. The difference between $\frac{1}{3}$ and $\frac{1}{4}$ of the number of trees in an orchard is 45 trees; find the number of trees in the orchard.

6. If $2\frac{1}{4}$ quarts of salt can be made from $5\frac{1}{16}$ gallons of sea-water, how many bushels of salt can be made from 1,000 gallons of sea-water?

7. The first term of a series is $2\frac{1}{4}$; the last term is $49\frac{3}{4}$; the common difference is $4\frac{3}{4}$; find the number of terms.

8. The sum of two numbers is 550 and their difference is 250; find the two numbers.

(The sum + the difference = twice the greater number.)

9. Find the number of 4-ft. corn-rows in a square acre.

10. In a certain school there are 165 boys; the girls are 45% of the school; how many girls are in the school?

11. What per cent of $31\frac{1}{4}$ is $16\frac{2}{3}\%$ of $6\frac{1}{4}$?

12. The difference between $\frac{1}{4}$ per cent and $3\frac{1}{8}$ per cent of a number is 46; what is the number?

13. The first term of a series is 17; the last term is 1,377; the ratio is 3; find the sum of the series.

14. A company insured a house for $\frac{2}{3}$ of its value, and charged $1\frac{1}{2}\%$; the house burned, and the company's total loss was \$1,970; find the value of the house.

15. Find the least perfect square that is a greater number than 2,441.

16. If 5% of the tax is not collectible, and 2% is paid for collection, what sum must be levied in order to raise \$25,000?

17. A man who receives \$2 per day saves $\frac{1}{7}$ as much as he spends; in how many days will he save \$50?

18. Find the cost of a bill of exchange on New York for \$448, at $\frac{3}{8}\%$ premium.

19. A county contains 900 square miles; what part of a square inch represents the county on a map drawn to a scale of 60 miles to the inch?

20. A farmer bought a horse, giving his note for \$150 payable in 1 year, without interest; 6 months afterwards he paid the note, the holder allowing discount at 7% a year; how much did the farmer pay?

21. If $\frac{3}{4}$ of a dollar will pay for $\frac{15}{16}$ of a bushel of wheat, how much wheat can be bought for $\frac{9}{10}$ of a dollar?

22. A merchant buys on 60 days' credit a bill of goods invoiced at \$650; 5% discount being allowed for cash, he borrows money at 8%, and pays the bill; what does he save?

23. A tub could be filled with 3 bushels of oats; how many quarts of water would fill the tub?

24. Find the cost of 250 shares of stock at $117\frac{1}{2}$, brokerage being $\frac{1}{8}\%$.

25. If 4 men can build 50 panels of rail-fence in 3 hours, in what time can 5 men build 105 panels?

26. Last year a farmer sold 1,340 bu. corn at \$.80 per bushel; this year he sold 25% more corn at $22\frac{1}{2}\%$ less per bushel; find the difference of the proceeds from the two years' sales.

27. A standard gauge railway track is 4 feet $8\frac{1}{2}$ inches wide; what length of track occupies an acre?

28. What is the compound interest of \$250 for $2\frac{1}{2}$ years at 6%, payable semi-annually?

29. B, whose daily pay is $2.20, works 10% more hours than C; C, who receives 10% a day less than B, works 10 hours per day; find the difference in their wages per hour.

30. If the current rate of interest is 7%, what is the present worth of a note for $420, due without interest, in 1 yr. 6 mo.?

31. If $\frac{2}{3}$ of the width of the street exceeds $\frac{5}{8}$ of it by 11 yd. 2 ft., what is the width of the street?

32. Find the bank discount, at 9%, of a note of $150, due in 90 days.

33. A merchant sold a pair of shoes for $$3\frac{1}{4}$ and gained $$\frac{11}{20}$; what per cent did he gain?

34. If the carpet for a room 18 ft. by 20 ft. costs $28, what would be the cost of a similar carpet for a room 24 ft. by 30 ft.?

35. A railroad company paid to a man whose horse had been killed $\frac{4}{5}$ of the amount claimed. The difference between the claim and the payment was $24; find the amount paid.

36. A building was insured in the Ætna Insurance Co. for $4,000; in the Hartford Insurance Co. for $3,000; in the Globe Insurance Co. for $2,500; the building was damaged to the amount of $7,125; what is the loss of each company?

37. Three men cut cord-wood: A cuts $2\frac{3}{4}$ cords per day; B cuts $2\frac{7}{8}$ cords per day; in 6 days they all cut 51 cords; how much does C cut per day?

38. A note for $825 was made Jan. 2, 1893, payable on demand, with 8% interest: April 15, 1893, a payment of $375 was made; how much is due May 20, 1893.

39. Before crossing a bridge, a traveller tied his handkerchief to one of the spokes of his buggy-wheel; he counted 303 revolutions while on the bridge; he then found the circumference of the wheel to be $12\frac{1}{2}$ ft.; find the length of the bridge.

40. What per cent of the greatest prime number less than 1,000 is the greatest prime number less than 100?

41. A carpenter built a fence 199 ft. 10 in. long, with pickets 3 inches wide placed 2 inches apart; each picket required 2 nails; allowing 4% for waste, how many picket nails were required? *Ans.* 1,000 nails.

42. If 100 ears of corn make a bushel, and each stalk of corn produce an ear, how many bushels will be the yield of an acre on which there is a stalk to every 9 square feet?

43. A mechanic worked for $2.75 per day; if his daily wages had been $18\frac{2}{11}\%$ greater, he would have received $130; how many days did he work?

44. There are two numbers that are to each other as $\frac{5}{8}$ to $\frac{7}{9}$; the square root of the smaller number is $3\frac{1}{4}$; find the larger number. *Ans.* $13\frac{13}{90}$.

45. A farm consists of four fields, and a piece of woodland which contains 7 A. 3 rd.; the areas of the four fields are $28\frac{1}{8}$ A.; 16 A. $24\frac{1}{4}$ rd.; 20 A. $1\frac{1}{2}$ rd.; and $31\frac{3}{4}$ A.; find the number of acres, square rods, etc., in the farm.

46. There are two numbers whose square roots are to each other as 3 to $3\frac{1}{2}$; the larger number is 169; find the square root of the smaller number.

47. Two ships having met, one sails north at the rate of 8 miles per hour, while the other steams east at the rate of 16 miles per hour; how far apart are the two ships at the end of $1\frac{3}{4}$ hours? *Ans.* 31.30 + miles.

48. If the amount of $292 for 5 days is $292.25, what is the rate of interest per annum?

49. A can hoe $\frac{1}{20}$ of a row of corn in 1 minute; B can hoe $\frac{1}{25}$ of it in 1 minute; in how many minutes can A and B together hoe the row of corn? *Ans.* $11\frac{1}{9}$ minutes.

50. Find the difference between the true discount and the bank discount, each at 7%, of a note for $1,500 due in $4\frac{1}{2}$ months.

51. Find the length of single wire in a fence of 5 wires around a rectangular 80-acre field twice as long as it is broad.

52. A train is running at the rate of $\frac{5}{8}$ of a mile per minute; it is due at a station 20 miles distant in 30 minutes; to arrive on time, what per cent must its rate be increased? *Ans.* $6\frac{2}{3}\%$.

53. A can shell a peck of corn in 15 minutes; B can shell a peck of corn in 12 minutes; in what time can they together shell a peck of corn? *Ans.* $6\frac{2}{3}$ minutes.

54. A, B, and C are partners; of the capital, A owns $\frac{1}{3}$; B, $\frac{2}{5}$; and C, \$4,000; the profit is 20%; find each partner's gain.

55. Two bales of cotton weigh 946 pounds; $\frac{1}{10}$ of the weight of the lighter bale equals $\frac{1}{12}$ of the weight of the heavier bale; find the weight of each bale.

56. A block contains 64 cubic inches; its thickness is $\frac{1}{2}$ as great as its width, and its width is $\frac{1}{2}$ as great as its length; find the three dimensions.

57. B owns a fence $\frac{1}{8}$ of which separates his field from C's square field containing 40 acres; find the length of B's fence.

58. A teacher paid \$10 for a chair, a lamp, and a book; the lamp cost 40% less than the chair, and the book cost $33\frac{1}{3}\%$ less than the lamp; find the cost of each.

59. At \$1.25 per rod, find the cost of fencing a rectangular 60-acre field whose width is to its length as $\frac{2}{3}$ is to 1.

60. B has a field that is $\frac{1}{8}$ as great as the remainder of his farm; he buys $4\frac{1}{2}$ acres, which he adds to his field; the remainder of his farm is now six times as great as the field; what per cent did B add to the field? *Ans.* $33\frac{1}{3}\%$.

61. If the width of a certain floor were $12\frac{1}{2}\%$ greater, the floor would be square, and would contain 324 square feet; find the length and the width of the floor. *Ans.* 18 ft.; 16 ft.

62. Of two numbers the third power of the less is to the cube root of the greater as $\frac{1}{2}$ to 2; the less number is $1\frac{1}{2}$; find the greater number. *Ans.* $2,460\frac{3}{8}$.

63. A owes to B, C, and D equal sums of money; he pays $\frac{1}{2}$ of B's claim, $\frac{1}{3}$ of C's claim, and $\frac{1}{6}$ of D's claim; the total amount he now owes the three is \$60; find the amount now due to each creditor. *Ans.* \$15; \$20; \$25.

64. A man, walking from A to B, has walked 15% of the distance; when he shall have walked another mile, he will have walked $\frac{1}{5}$ of the distance; how far is it from A to B?

65. A rectangular field contains $13\frac{1}{3}$ acres; its width is $\frac{3}{4}$ as great as its length; find its width and length.

66. At what per cent profit must a dealer sell goods in order to counterbalance 10% loss by damage?

67. In a field there are 5 hands hoeing; they can complete the work by noon, but two hands leave at 11:30 A.M.; the others do not leave until they complete the work; when is the work completed? *Ans.* 12:20 P.M.

68. B's corn field is $\frac{1}{3}$ of his farm; he buys no more land, but by clearing he adds 10 acres to the field, which becomes $\frac{1}{2}$ of his farm; find the number of acres in the farm.

69. A fence around a rectangular orchard is 420 rods long; the width of the orchard is $\frac{2}{5}$ as great as the length; find the length of the orchard.

70. After making 65% of her voyage, a steamer broke her shaft; 10 days afterwards she completed her voyage, having proceeded at once under sail at the rate of 7 miles per hour; find the length of the voyage.

71. A train runs from A to C; when it arrives at B it has accomplished 16 miles more than $\frac{2}{5}$ of the distance from A to C; the distance from B to C is 74 miles; find the distance from A to C. *Ans.* 150 miles.

72. The course of a river through a certain county is 44 miles long, and the river averages 300 feet in width; the county contains 450 square miles; what per cent of the county does the river occupy? *Ans.* $\frac{5}{9}$ of 1%.

73. A, B, and C are employed to split rails; A splits $\frac{2}{3}$ as many as both B and C split; C splits $\frac{2}{3}$ as many as B; what part of the whole sum paid should B receive?

74. The sum of three fourths of a number and $7\frac{1}{2}$% of the other fourth is 615; find the number.

75. The inner dimensions of a watering-trough are $7\frac{1}{2}$ feet, $1\frac{3}{4}$ feet, and 11 inches; if the trough is full of water, how many horses could drink $2\frac{1}{2}$ gallons apiece?

76. The difference between 7% of $\frac{2}{3}$ of a number, and 15% of $\frac{1}{3}$ of the number is $1\frac{1}{2}$; find the number.

77. In a state which contains 49,000 square miles, there are 4,312 miles of railroads; counting the average width of the land (right-of-way) occupied by a railroad at 200 feet, what per cent of the state do the railroads occupy? *Ans.* $\frac{1}{3}$ of 1%.

78. A dealer asked 30% profit, but sold for 10% less than he asked; what per cent did he gain?

79. A can pick a hundredweight of cotton in 7 hours; B can pick as much in 6 hours; C can pick as much in 5 hours; in what time can the three, working together, pick a hundredweight of cotton?

80. Find the length in yards of one side of a square field that contains $51\frac{79}{121}$ acres.

81. A blacksmith, sharpening ploughs at 15 cents apiece, made $3 in a day, doing $\frac{2}{3}$ as much work in the afternoon as in the forenoon; how many did he sharpen in the forenoon?

82. Find the length of the longest straight furrow that could be ploughed in a rectangular field 600 yd. by 800 yd.

83. A train 384 feet long is running 36 miles per hour; while the train is crossing a bridge the rate of speed is reduced $83\frac{1}{3}$%, and the crossing is made in 3 minutes from the instant the engine reaches the bridge till the last car passes over; find the length of the bridge. *Ans.* 1,200 feet.

84. Express in inches the length of one edge of a cubical block containing 91 cu. ft. 216 cu. in.

85. A boat left A at 9 A.M. and is due at B, 108 miles away, at 6 P.M.; 40% of the time has passed and she has made but 30% of the distance; at what rate per hour must she now run in order to reach B on time?

86. Find the number of cotton-plants to the acre, the rows being $3\frac{1}{2}$ feet apart and the plants 10 inches apart in the row.

87. Post-holes are dug 8 feet apart from centre to centre around a square field of 40 acres, and around a rectangular field of $38\frac{2}{5}$ acres whose width is to its length as 2 is to 3; find the number of posts needed for the two fields.

Ans. 1,320 posts.

88. A floor is 18 feet by 16 feet; three rugs lie on the floor; each of two rugs is $2\frac{1}{2}$ feet by 6 feet; the other rug is 3 feet by 6 feet; what per cent of the floor do the rugs cover?

89. A lot 44 feet wide contains $\frac{1}{8}$ of an acre; find the length of the lot.

90. Find the distance from the north-east corner to the south-west corner of a section of land.

91. A dealer sold $\frac{5}{8}$ pound rice for as much as $\frac{3}{4}$ pound cost him; what per cent did he gain?

92. An orchard contains 434 trees of three kinds; $\frac{2}{3}$ of the peach-trees equal in number $\frac{3}{4}$ of the apple-trees, and $\frac{2}{3}$ of the apple-trees are as many as $\frac{3}{4}$ of the pear-trees; find the number of trees of each kind. *Ans.* peach, 162; apple, 144; pear, 128.

93. If a ton of hay per week is bought for 15 horses, how much should be bought for 21 horses for 7 weeks?

94. A's field is $\frac{2}{3}$ as large as B's field; B's field produces $\frac{3}{4}$ as much corn per acre as A's field produces; the difference in the entire yield of the two fields is 40 bushels; find the number of bushels produced by each field. *Ans.* 320 bu.; 360 bu.

95. The difference between two numbers is 56; $\frac{1}{3}$ of the greater number equals $\frac{2}{3}$ of the less number; find the two numbers.

96. The employés of a factory demanded that wages be increased 25%; the matter was compromised by raising daily wages $12\frac{1}{2}$%, and reducing work-hours $6\frac{1}{4}$%; before the compromise the pay for a day of 12 hours was $16\frac{2}{3}$¢ per hour; find the pay per hour agreed upon. *Ans.* 20¢.

97. A box, without a lid, is to be made of inch-plank; its outside dimensions are to be 2 feet by $1\frac{1}{2}$ feet by 1 foot; find the number of cubic inches of plank required to make the box.

98. In 4-ft. cotton-rows, 220 yards long, a man ploughs 4 acres in a day, putting 2 furrows to the row; allowing nothing for turning at the ends of the rows, how far does he walk?

For ordinary purposes a bushel is considered 2,150.4 cubic inches.

99. How many bushels of corn could be contained in a box which measures on the inside 28 in. by 24 in. by 16 in.?

100. A square garden contains, within the fence, 3,600 square yards; a walk 3 feet wide is between the fence and the cultivated portion of the garden; what per cent of the whole garden is the walk? *Ans.* $6\frac{5}{9}$%.

MENSURATION.

SURFACES.

358. A **Surface** has length and breadth, without thickness.

359. A **Plane** is a level, or flat, surface; a plane figure is a plane bounded by lines.

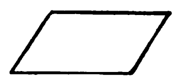
360. A **Polygon** is a plane bounded by straight lines.

The total length of the lines which bound a polygon is called its **Perimeter.**

A regular **Polygon** is one whose sides are equal, and whose angles are equal.

361. A **Quadrilateral** is a polygon of four sides.

362. A **Parallelogram** is a quadrilateral whose opposite sides are parallel.

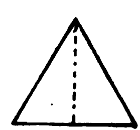


A **Rectangle** is a parallelogram whose angles are right angles.

A **Square** is a rectangle whose four sides are equal.

An angle greater than a right angle is called an obtuse angle.
An angle less than a right angle is called an acute angle.

363. The **Altitude** of a triangle is the shortest distance to the base from the angle opposite the base or side upon which the triangle is supposed to stand.



(Any side of the triangle may be considered the base.)

PROBLEMS.

1. How many square feet are in a floor 18 feet long and 16 feet wide? *Ans.* 288 square feet.

2. How many yards of carpet $\frac{3}{4}$ yd. wide would be required for a floor 18 feet by 24 feet? *Ans.* 64 yards.

364. To find the area of a parallelogram:

Multiply the base by the altitude.

All rules of mensuration are given as though only abstract numbers were implied; the result of an operation by any of the rules must be written as that concrete number which answers the conditions of the given problem.

3. A foot of lumber is 1 foot wide, 1 foot long, and 1 inch thick; how many feet of lumber are in a plank 12 feet long, 9 inches wide, and 1 inch thick? *Ans.* 9 feet.

4. How many feet of lumber in a plank 20 feet long, 18 inches wide, and 2 inches thick? *Ans.* 60 feet.

5. A sheet of paper 8 inches by 10 inches is divided into two equal triangles by cutting the sheet diagonally from corner to corner; find the number of square inches in each triangle.

365. To find the area of a triangle whose base and altitude are given:

Find half the area of a rectangle which has the same base and altitude as the triangle.

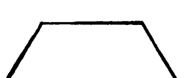
366. To find the area of a triangle whose three sides are given:

From half the sum of the three sides subtract each side separately; find the square root of the product of the half-sum and three remainders.

6. The base of a triangular lot is 240 feet; the altitude is 450 feet; find the area. *Ans.* 54,000 square feet.

7. How many square feet are in the surface of a triangle whose base is 36 feet, and whose altitude is 40 feet?

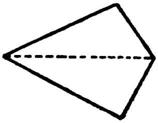
367. A quadrilateral with only two sides parallel is called a **Trapezoid**.



368. To find the area of a trapezoid:

Multiply half the sum of the parallel sides by the altitude.

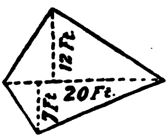
8. Find the surface of a plank 12 in. wide at one end, 6 in. wide at the other end, and 10 ft. long. *Ans.* $7\frac{1}{2}$ sq. ft.



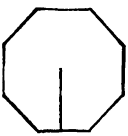
369. A quadrilateral with no two of its sides parallel is called a **Trapezium.**

370. To find the area of a trapezium:

Divide the trapezium into two triangles; find the sum of the areas of the triangles.



9. The base of either of two triangles into which a trapezium may be divided is 20 feet, the altitudes of the triangles are 7 feet and 12 feet respectively; find the area of the trapezium. *Ans.* 190 square feet.

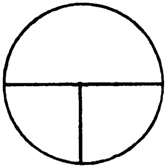


371. To find the area of a regular polygon:

Multiply the perimeter by half the length of a perpendicular line from the centre to one of the sides.

10. The perimeter of a regular polygon is 25 feet; the perpendicular distance from its centre to any side is $4\frac{1}{2}$ feet; find the area. *Ans.* $56\frac{1}{4}$ square feet.

372. A **Circle** is a plane bounded by a curved line, every point of which is equally distant from a point within called the centre. The line which bounds a circle is called the **Circumference.**



A straight line passing through the centre and joining two points in the circumference of a circle is called the **Diameter.**

Half the diameter is called the **Radius.**

373. To find the circumference of a circle:

Multiply the diameter by 3.14159 (or by $3\frac{1}{7}$).

374. To find the diameter of a circle:

Divide the circumference by 3.14159 (or by $3\frac{1}{7}$).

375. To find the area of a circle:

Multiply the square of the diameter by .7854; or, *multiply the circumference by half the radius.*

PROBLEMS.

1. Find the circumference of a wheel whose diameter is 3 ft. 10 in. *Ans.* 12 ft. .513+ in.

2. Find the diameter of a circle whose circumference is 25,000 miles. *Ans.* 7,957+ miles.

3. Find the area of a circle whose diameter is 18 feet. *Ans.* 254+ square feet.

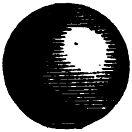
4. A horse is tied to a stake; the rope is 16 feet long; find the area of the circle upon which the horse can graze.

5. Find the diameter of a log whose girth is 13 ft. 6 in.

6. Find the number of square yards of carpet on the floor of a circular hall 36 feet in diameter.

SOLIDS.

376. A **Sphere** is a solid bounded by a curved surface, every part of which is equally distant from a point within called the centre.



The surface of a sphere is 4 times as great as the surface of a circle whose diameter is equal to the diameter of the sphere.

377. To find the surface of a sphere:

Multiply the circumference by the diameter.

PROBLEMS.

1. Find the surface of a sphere whose circumference is 25,000 miles.

25,000 mi. ÷ 3.14159 = Diam.

2. Find the surface of a sphere whose diameter is 16 inches.

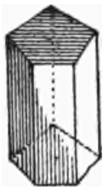
3. Find the surface of a sphere $\frac{1}{4}$ of whose circumference is 9 feet.

378. To find the solid contents of a sphere:

Multiply the cube of the diameter by .5236.

4. Find the contents of a sphere whose diameter is 20 inches.

5. How many cubic inches are in a ball whose diameter is 3 inches?



379. A **Prism** is a solid whose ends, or bases, are equal and parallel polygons, and whose sides are parallelograms.

The sum of the surfaces of the sides is called the convex surface; the sum of the convex surface and the areas of the bases is called the entire surface.

380. To find the convex surface of a prism:

Multiply the perimeter by the altitude.

6. Find the convex surface of a prism whose perimeter is 3 feet and whose altitude is 4 feet. *Ans.* 12 square feet.

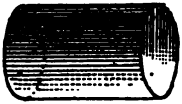
7. Find the convex surface of a prism whose perimeter is 9 feet and whose altitude is 3 feet.

381. To find the entire surface of a prism:

Add the areas of the bases to the convex surface.

382. To find the contents of a prism:

Multiply the area of the base by the altitude.



383. A **Cylinder** is a round solid of uniform diameter, with equal and parallel circles for its bases, or ends.

384. To find the convex surface of a cylinder:

Multiply the circumference by the altitude.

8. Find the convex surface of a cylinder whose circumference is 16 feet and whose altitude is 10 feet.

Ans. 160 square feet.

9. Find the convex surface of a cylinder whose diameter at the base is 3 feet and whose altitude is 7 feet.

Ans. 65.97 + square feet.

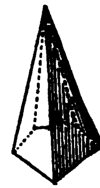
385. To find the contents of a cylinder:

Multiply the area of the base by the altitude.

10. Find the contents of a cylinder the area of whose base is $4\frac{1}{2}$ square feet and whose altitude is 8 feet.

Ans. 36 cubic feet.

11. Find the contents of a round log of wood 9 feet long and 3 feet in diameter. *Ans.* 63.617+ cubic feet.



386. A **Pyramid** is a solid whose base is a polygon, and whose sides are triangles meeting in a point.

The point in which the triangles meet is called the **Vertex.**

The **Altitude** is the shortest distance from the vertex to the base; the **Slant Height** is the shortest distance from the vertex to one of the sides of the base.

387. To find the convex surface of a pyramid:

Multiply the perimeter of the base by half the slant height.

12. Find the convex surface of a pyramid whose base is 3 inches square, and whose slant height is 24 inches.

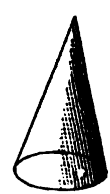
13. Find the convex surface of a pyramid with four sides, the base being 764 feet square, and the slant height 610 feet.

388. To find the contents of a pyramid:

Multiply the area of the base by one third the altitude.

14. Find the contents of a pyramid whose base is 40 feet square, and whose altitude is 66 feet. *Ans.* 35,200 cubic feet.

15. Find the contents of a pyramid whose base is 9 feet square, and whose altitude is 12 feet. *Ans.* 324 cubic feet.



389. A **Cone** is a solid whose base is a circle, and whose surface tapers uniformly to a point called the vertex.

The altitude is the shortest distance from the vertex to the base.

The slant height is the shortest distance from the vertex to the circumference of the base.

390. To find the convex surface of a cone:

Multiply the circumference of the base by half the slant height.

16. Find the convex surface of a cone whose base is 60 inches in circumference, and whose slant height is 40 inches.

Ans. 1,200 square inches.

17. Find the convex surface of a cone whose diameter at the base is 6 inches, and whose slant height is 10 inches.

Ans. 94.2477 square inches.

391. To find the contents of a cone:

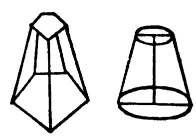
Multiply the area of the base by one third the altitude.

18. Find the contents of a cone the diameter of whose base is 8 feet, and whose altitude is 12 feet.

Ans. 201.062+ cubic feet.

19. Find the contents of a cone the circumference of whose base is 36 inches, and whose altitude is 10 inches.

392. If a pyramid or a cone be cut into two parts by a plane parallel to the base, the upper part is still a pyramid or a cone; the lower part, or part remaining, is called the **Frustum** of the pyramid or cone.



393. To find the contents of a frustum:

Add the sum of the areas of the two bases to the square root of the product of the areas of the two bases; multiply the result by one third the altitude.

20. Find the contents of a frustum of a pyramid whose bases are 18 feet square and 10 feet square, and whose altitude is 12 feet. *Ans.* 2,416 cubic feet.

21. Find the contents of a frustum of a cone, the diameter of the two bases being 5 feet and 8 feet respectively, and the altitude 6 feet. *Ans.* 202.6332 cubic feet.

MISCELLANEOUS PROBLEMS.

1. A room is 18 feet by 18 feet; how many yards of carpeting 27 inches wide are required for the floor, allowing a waste of 8 inches to each strip for matching figures?

Ans. $49\frac{7}{9}$ yards.

2. The walls of a room 16 feet by 14 feet by 10 feet are to be papered; the paper is bought in whole rolls at 20 cents per yard; a roll is 8 yards long, 18 inches wide; what will be the cost, allowing $\frac{1}{6}$ for doors and windows? *Ans.* $22.40.

No rule can be given for allowances to be made for doors and windows in plastering the walls of rooms, etc. Local custom usually determines the rate of deduction; very frequently the allowance is stipulated in the contract.

3. Find the cost, at 30¢ per square yard, for plastering the walls and ceiling of a hall 25 feet by 40 feet by 14 feet, no allowance being made for doors and windows. *Ans.* $94.

4. Find the cost, at 40¢ per square yard, for plastering the walls and ceiling of the same hall, making full allowance for 10 windows 5 feet by 10 feet, and 5 doors 5 feet by 8 feet.
Ans. $94.22.

5. Find the cost, at 36¢ per square yard, for plastering the walls and ceiling of the same hall, allowing one half for doors and windows, and one foot all round for the base-board.
Ans. $93.60.

6. A roof 30 feet in length has rafters 15 feet long; the roof is covered with shingles 4 inches wide which show 4 inches to the weather; the lower course of shingles, on either side, is double; find the number of shingles in the roof.
Ans. 8,280 shingles.

In roofing, flooring, etc., a square is a surface 10 feet by 10 feet.

7. From eaves to ridge a roof is 25 feet; the length of the roof is 40 feet; find the cost of the slate at $9 a square.

8. A fence is built around a rectangular lot 40 rods long and 20 rods wide; the fence is 5 planks high; the base-board, or bottom plank, is 1 inch thick and 12 inches wide, the 4 other planks are $\frac{3}{4}$ inch thick and 6 inches wide; allowing nothing for waste, find the cost of the planks at $14 per thousand feet.
Ans. $83.16.

Lumber less than 1 inch thick is sold as inch-lumber.

9. Find the cost of a load of lumber consisting of 50 planks 16 feet long, 8 inches wide, and $1\frac{1}{2}$ inches thick at $15 per M.
Ans. $12.

10. A wagon-body is $9\frac{1}{2}$ feet long, 4 feet wide, and 32 inches deep; how many bushels of shelled corn will it hold?
Ans. 81.4 + bushels.

394. To measure grain in a crib or a bin:

Measure the length, breadth, and depth in inches; divide the product of the three numbers by 2150.42; the quotient will be the number of bushels.

The rule allows nothing for the increased density of the grain at the bottom. Weigh a half-bushel of grain taken as nearly as possible from the centre of the crib, and it can be determined what allowance should be made for increased density.

Let it be supposed that the grain to be measured is shelled corn, and that a half-bushel from the centre of the crib weighs 30 pounds; the average standard weight of a bushel of corn is 56 pounds; then 4 pounds must be added to every bushel, and the estimate by the rule given above will be $\frac{14}{15}$ of the contents of the crib.

A box 16 inches square and $8\frac{1}{2}$ inches deep holds very little more than a bushel; a box 16 inches square and $8\frac{3}{8}$ inches deep holds very little less than a bushel. A bushel of ear-corn is estimated as $\frac{1}{2}$ bushel shelled corn.

395. To measure the contents of a cask:

Multiply the square of the mean diameter in inches by the length in inches; multiply the product of these numbers by .0034; the result will be the number of gallons.

The rule gives an approximate result. The mean diameter may be estimated from the cask's diameter half-way between the bung and the head, but to get the inside diameter allowance must be made for the thickness of the staves; the measurement of the length must also allow for the thickness of the heads. Estimated by the mean diameter the cask is considered a cylinder, whose contents are found by **385**, but instead of multiplying by .7854 and afterward dividing by 231 (the number of cubic inches in a gallon) the product of the square of the diameter and the length is multiplied by .0034 ($= .7854 \div 231$).

PROBLEMS.

1. Find the contents in bushels of a bin of corn, 20 feet square and 10 feet deep, a bushel measure from the centre weighing 60 pounds. *Ans.* 3,443.8 + bushels.

2. Find the contents of a cask whose mean diameter is 2 ft. 1 in., and whose length is 2 feet (inside measurements).

3. Find the contents of a crib of ear-corn 20 feet by 24 feet and 12 feet deep, a bushel measure from the centre shelling out 29 pounds. *Ans.* 4,793.8 + bushels ear-corn.

THE METRIC SYSTEM.

396. Near the close of the eighteenth century a commission, created by the French Government, selected a standard unit upon which to base a new system of weights and measures. This new system is called the **Metric System**.

397. The unit chosen is called the Metre; it was supposed to be one ten-millionth part of the distance from the earth's equator to either pole; it has since been shown, however, that the distance from the equator to the pole is somewhat greater than 10,000,000 metres, yet the length of the metre, 39.37 + inches, remains as it was fixed by the commission.

398. The Metric System is used in France, and other European countries, and in many South American states. Its use in the United States is authorized by an act of Congress.

399. The metre is the standard unit of length; the units of other measures are derived from the metre.

The standard unit of surface, the square metre, is a square, each of whose dimensions is 1 metre.

The standard unit of volume, the cubic metre, is a cube, each of whose dimensions is 1 metre.

The standard unit of capacity for dry measures and for liquid measures is the litre, whose volume is equal to that of a cube, each of whose dimensions is one-tenth of a metre.

The standard unit of weight is the gram, whose weight equals that of a cubic centimetre ($\frac{1}{100}$ metre) of distilled water at the temperature of melting ice.

400. By using Latin and Greek prefixes, denominations higher and lower than the standard units are formed, the Greek numerals indicating multiples, and the Latin ordinals indicating decimal divisions.

These prefixes are:

Deka-, meaning ten;	Deci-, meaning tenth;
Hekto-, meaning hundred;	Centi-, meaning hundredth;
Kilo-, meaning thousand;	Milli-, meaning thousandth;

Myria-, meaning ten thousand.

TABLES OF METRIC MEASURES.

401. LINEAR MEASURES.

10 millimetres (mm)	= 1 centimetre, (cm).
10 centimetres	= 1 decimetre, (dm).
10 decimetres	= 1 metre, (M).
10 metres	= 1 decametre, (DM).
10 dekametres	= 1 hektometre, (HM).
10 hektometres	= 1 kilometre, (KM).
10 kilometres	= 1 myriametre (MM).

Equivalents.

1 centimetre	= .3937 inches.
1 decimetre	= 3.937 inches.
1 metre	= 39.37 inches.
1 kilometre	= .6214 mile.

1 DECIMETRE = 10 CENTIMETRES = 100 MILLIMETRES.

402. MEASURES OF SURFACE.

100 square millimetres (sq. mm)	= 1 square centimetre, (sq. cm).
100 square centimetres	= 1 square decimetre, (sq. dm).
100 square decimetres	= 1 square metre, (sq. M).
100 square metres	= 1 square dekametre, (sq. DM).
100 square dekametres	= 1 square hektometre, (sq. HM).
100 square hektometres	= 1 square kilometre, (sq. KM).

In measuring land the unit is a square dekametre, and is called an **Are**. A square metre is called a **Centiare**.

403. LAND MEASURES.

10 centiares (ca)	= 1 deciare (da).
10 deciares	= 1 are (A).
10 ares	= 1 dekare (Da).
10 dekares	= 1 hektare (Ha).

Equivalents.

1 square centimetre	= .155 square inch.
1 square decimetre	= .1076 square foot.
1 square metre	= 1.196 square yards.
1 are	= 119.6 square yards.
1 hektare	= 2.471 acres.

404. MEASURES OF VOLUMES OR SOLIDS.

1000 cubic millimetres (cu. mm.)	= 1 cubic centimetre (cu. cm).
1000 cubic centimetres	= 1 cubic decimetre (cu. dm).
1000 cubic decimetres	= 1 cubic metre (cu. M.).

Equivalents.

1 cubic centimetre	= .061 cubic inch.
1 cubic decimetre	= 61.022 cubic inches.
1 cubic metre	= 1.308 cubic yards.

In measuring wood the standard unit is called the **Stere**, and is equal to .2759 cord; the stere is a cubic metre.

405. WOOD MEASURES.

10 decisteres (ds.)	= 1 stere (S).
10 steres	= 1 dekastere (Ds).

$3\frac{7}{11}$ steres are about 1 cord.

406. MEASURES OF CAPACITY.

		Equivalents.
10 millilitres (ml.)	= 1 centilitres (cl.)	
10 centilitres	= 1 decilitre (dl.).	
10 decilitres	= 1 litre (L.) =	61.022 cu. in. 1.057 liquid qt. .908 dry qt.
10 litres	= 1 dekalitre (Dl.).	
10 dekalitres	= 1 hektolitre =	3.531 cu. ft. 26.417 gal. 2.837 bu.
10 hektolitres	= 1 kilolitre (Kl.).	

407. MEASURES OF WEIGHT.

		Equivalents.
10 milligrams (mg.)	= 1 centigram (cg.).	
10 centigrams	= 1 decigram (dg.).	
10 decigrams	= 1 gram (G.) =	1 cu. cm., or 1 ml. water. 15.432 gr. Troy. .0022046 lb. Av.
10 grams	= 1 dekagram (Dg.).	
10 dekagrams	= 1 hektogram (Hg.).	
10 hektograms	= 1 kilogram (Kg.). =	1. cu. dm., or 1 L. water. 2.2046 lb. Av.
10 kilograms	= 1 myriagram (Mg.).	
10 myriagrams	= 1 quintal (Q.).	
10 quintals	= 1 tonneau (T.) =	1 cu. metre, or 1 kl. water. 1.1023 tons.

REDUCTION.

PROBLEMS.

1. Reduce 13 kilolitres to millilitres.

13 kilolitres = 130 hektolitres.
130 hektolitres = 1,300 dekalitres.
1,300 dekalitres = 13,000 litres.
13,000 litres = 130,000 decilitres.
130,000 decilitres = 1,300,000 centilitres,
1,300,000 centilitres = 13,000,000 millilitres.

2. Reduce 32,975 centimetres to higher denominations.

32975 centimetres	= 3297.5 decimetres.
3297.5 decimetres	= 329.75 metres.
329.75 metres	= 32.975 dekametres.
32.975 dekametres	= 3.2975 hektometres.
3.2975 hektometres	= .32975 kilometres.

3. How many metres are in 75 Hm. 32 Dm.? *Ans.* 7,820 M.

4. Reduce 2,348 metres to hektometres; to millimetres.

5. At $5 per are, how many dekares of land can be bought for $2,500? *Ans.* 50 Da.

6. How much matting 1 metre wide is required to cover a floor 6.3 metres long and 5.4 metres wide?

7. How many miles are in 4,800 kilometres?

8. A dealer bought 1,400 bushels of wheat at $1 per bushel, and sold it at $3 per hektolitre; find the profit.

9. A pile of wood is 14.5 M. long, 3.8 M. wide, and 1.4 M. high; find its volume. *Ans.* —— steres.

10. Find the weight of 27 Hl. 8 Dl. of water at the temperature of melting ice.

11. Find the cost of excavating 6.25 cu. M. of earth, at $.035 per cu. cm.

12. A merchant bought 100 metres of silk at $1 per M. and sold it at $1.15 per yard; what per cent did he gain?

13. A field is 5 Km. long and $3\frac{1}{2}$ Km. wide; find its value at $18.75 per dekare.

14. Find the capacity in hektolitres of a bin 5 M. long, 4 M. wide, and 3 M. deep.

15. Reduce 4 lb. Troy to grams; 2 lb. Avoirdupois to grams.

16. What per cent of 185 DG. is 740 dg.?

17. How many hektolitres are in 20 bu. 3 pk.? in 63 gallons?

18. Find the least common multiple of 75 cm., 15 M., and 2 Km.

19. Reduce $\frac{1}{875}$ Kl. to cl.; $\frac{1}{450}$ mg. to cg.

20. Find the cost, at $6.85 per cu. M., of building a wall 24.8 M. long, 5 M. wide, and 1.7 M. high.

APPENDIX.

SPANISH MEASURES.

In making the first surveys of lands in Texas, and other parts of the Southwestern States originally held by Spain, the old Spanish Land Measures were employed. The following tables were compiled by Judge William Bramlette, Chief Clerk, and Gen. X. B. Debray, Spanish Clerk, of the General Land Office of Texas.

THE LINEAR MEASURES.

Unit. The **Vara**, corresponding nearly to the English or American yard, and equal to .9259+ American yards, or about $33\frac{1}{3}$ inches. The Vara was divided into three equal parts called Piés, corresponding nearly to American feet.

The pié was divided into twelve equal parts called Pulgadas, corresponding nearly to American inches.

It is customary to speak of piés as Mexican feet, and of pulgadas as Mexican inches.

TABLE.

12 Mexican inches	= 1 Mexican foot.
3 Mexican feet	= 1 Vara (Vr.) = $33\frac{1}{3}$ American inches.
5000 Varas	= 1 Mexican league = 2.63+ miles.

THE SQUARE MEASURES.

Unit. The **square vara** (sq. vr.), corresponding nearly to the American square yard.

TABLE.

25,000,000	sq. vrs.	= 1 square league, also called a Sitio (site) for large cattle = 4,428.4 American acres.
11,111,111+	sq. vrs.	= 1 Sitio (site) for small cattle, = 1,968.1777 American acres.
1,000,000	sq. vrs.	= 1 labor (field) = 177.136 American acres.
26,000,000	sq. vrs.	= 1 league and labor = 4,605.536 American acres.
30,000,000	sq. vrs.	= 1 Porcion (share) = 5,314.08 American acres.

AMERICAN MEASURES WITH MEXICAN EQUIVALENTS.

1 yard = 1.08 varas or 38.8+ Mexican inches.

1 mile = 1,900.8 varas.

1 acre = 5,645.376 square varas.

EXERCISES.

1. How many Mexican inches in a vara? How many American inches in a vara? What decimal part of an American inch is a Mexican inch?

2. How many American inches in a Mexican foot?

3. How many varas in 5 yards? in 7 yards? in 20 yards?

4. What part of a vara is $16\frac{3}{8}$ American inches?

5. How many varas in a rod? in 80 rods?

6. Find the width in American measures of a lot 48 varas in width. *Ans.* 8 rd. 1 ft. 4 in.

7. How many varas are 15 miles?

8. How many miles are 20,908.8 varas? *Ans.* 11 miles.

9. Find the length in varas of one side of a quarter-section of land.

10. How many varas are 26 rd. 2 yd. 1 ft. 6 in.? *Ans.* 157.14 varas.

11. How many square varas are in a section of land? In a half section? In a quarter section?

12. How many acres of land in 1,500,000 square varas?

13. Find the length in varas of one side of a square league.

14. Find the length, in American measures, of one side of a square league of land.

15. How many acres in a square field 1,000 vr. to the side?

16. How many square varas in a field containing 87 acres?

17. How many sq. vr. in a ranch containing 25,000 acres?

18. How many acres in a rectangular field 579 varas in length and 381 varas in width? *Ans.* 39.07 + acres.

CIRCULATING DECIMALS.

A circulate is a decimal whose expression repeats indefinitely. The figures repeating are called **repetends.**

(*a*) $\frac{1}{9} = .111+$, the figure 1 repeating indefinitely.

(*b*) $\frac{6}{9}$ or $\frac{2}{3} = .666+$, the figure 6 repeating indefinitely.

(*c*) $\frac{27}{99}$ or $\frac{3}{11} = .2727+$, the figures 27 repeating indefinitely.

(*d*) $\frac{216}{999}$ or $\frac{8}{37} = .216216+$, the figures 216 repeating indefinitely.

Circulates are expressed by placing a dot over the figure, or over the first figure and the last figure of the repetend; as $.\dot{1}$; $.\dot{2}\dot{7}$; $.\dot{2}1\dot{6}$.

REDUCTION OF CIRCULATES.

EXAMPLES.

1. Reduce $.\dot{6}$ to a common fraction.

(See (*a*) above.) $.\dot{1} = \frac{1}{9}$; $.\dot{6} = 6 \times \frac{1}{9} = \frac{6}{9}$.

2. Reduce $.\dot{2}\dot{7}$ to a common fraction.

$$\begin{cases} 100 \times .\dot{2}\dot{7} = 27.\dot{2}\dot{7}, \\ \quad 1 \times .\dot{2}\dot{7} = \quad .\dot{2}\dot{7}, \\ 99 \times .\dot{2}\dot{7} = 27. \quad , \\ \quad 1 \times .\dot{2}\dot{7} = \frac{1}{99} \times 27, \\ .\dot{2}\dot{7} = \frac{27}{99}. \end{cases}$$

3. Reduce the mixed circulate $.3\dot{1}\dot{8}$ to a common fraction.

$$.3 = \frac{3}{10};\ .0\dot{1}\dot{8} = \frac{18}{990};\ \frac{3}{10} + \frac{18}{990} = ?$$

4. Reduce to common fractions:

$$.\dot{2}1\dot{6};\ .\dot{3}2\dot{4};\ .5\dot{3}\dot{6};\ .4\dot{2}\dot{7};\ .\dot{0}\dot{1};\ .53\dot{4}\dot{7};\ .\dot{1}4285\dot{7}.$$

RULE. — *For the numerator, write the repetend; for the denominator, write as many nines as there are figures in the repetend.*

Questions from Examinations for Illinois Teachers' Certificates.

1. What must one's salary be to enable him to set three-fourths of it aside for family expenses and to deposit the other fourth in a savings bank at the rate of 50 cents the first week, \$1 the second, \$1.50 the third, \$2 the fourth, and so on for the year?

2. Find the sum of these four expressions:
$(\frac{3}{4} \text{ of } \frac{4}{5}) - (\frac{1}{2} \text{ of } \frac{2}{3})$; $\left(\frac{3\frac{1}{4}}{6\frac{1}{2}} \times 72\frac{1}{2}\right) \div (\frac{2}{3} \text{ of } \frac{3}{5} \text{ of } 9\frac{3}{8})$; $\sqrt{4.426816}$; $(\frac{7}{5} \div \frac{4}{25})$.

3. A, B, and C hire a pasture for \$180. A puts in 8 cows for 10 weeks; B, 20 cows for 5 weeks; C, 30 cows for 9 weeks; how much should each pay?

4. What rate of income will U.S. $3\frac{1}{2}\%$ bonds yield if they are bought at 102 and are payable at par in 25 years?

5. What is the product of .875 and its reciprocal?

State the principle upon which cancellation is based.

What is the difference between a ratio and a common fraction?

Questions from Examinations for Tennessee Teachers' Certificates.

1. $1.0075 \times .006875 = ?$ Explain fully the principle involved.

2. Divide \$5376 among three heirs proportionally to $1\frac{2}{3}$, $1\frac{1}{15}$, $2\frac{1}{5}$.

3. If the divisor is 19, the quotient 37, and the remainder 11, what is the dividend?

4. A boy bought eggs at the rate of 3 for 5 cents and sold them at the rate of 4 for 7 cents, clearing 9 cents; how many did he buy?

5. (*a*) If $4\frac{1}{3}$ yd. of velvet cost \$26.78, what would be the cost of 145.25 yd. at the same rate?

(*b*) A man sold a horse for \$175, thereby making $16\frac{2}{3}\%$; what did the horse cost him?

Questions from Examinations for Georgia Teachers' Certificates.

1. If $5\frac{8}{9}$ yards of silk brocade cost \$ 53, how many yards will \$ $654\frac{3}{4}$ pay for?

2. Give three processes of dividing $\frac{6}{19}$ by $\frac{2}{3}$, and write what you consider the most satisfactory analysis.

3. Which is worth more, and how much, a Georgia State bond of \$ 1,000 at 5%, or \$ 850 worth of stock yielding 8%?

4. Find exact interest of \$ 1,000 for 73 days at 5%.

5. Find the cost of plastering a room 16 feet long, 14 feet wide, 12 feet high, making no deduction for doors or windows, at 22 cents a square yard. Find the expense of carpeting the same room with carpet $\frac{3}{4}$ of a yard wide, at \$ 1.25 a yard.

Questions from Examinations for Michigan Teachers' Certificates.

1. One-fourth the value of a farm is four times the value of a horse; both taken together are worth \$ 1,700. Find the value of each. Write out a complete analysis.

2. A field of five acres in form of a square is to be surrounded by a fence $4\frac{1}{2}$ feet high, to be built of boards 6 inches wide, placed horizontally; the lower board is to be 4 inches above the ground, and there is to be a space of 5 inches between the boards; what will be the cost of the boards required at \$ 18 per M.?

3. A geographic mile is $\frac{1}{60}$ of $\frac{1}{360}$ of the earth's circumference; the equatorial circumference is 131,483,200 feet; how many common or statute miles are equal to 50 geographic miles on the equator?

4. A can mow $\frac{2}{3}$ of a field of grass in 7 days; B can mow $\frac{2}{3}$ of it in 8 days; how long will it take both working together to mow the field? Explain as to a class.

5. If 81 yards of cloth $1\frac{1}{2}$ yards wide make 27 coats, how many yards $1\frac{1}{4}$ yards wide will be required to make 50 jackets, each jacket containing $\frac{4}{5}$ as much cloth as a coat? Solve by proportion.

QUESTIONS FROM EXAMINATIONS FOR INDIANA TEACHERS' CERTIFICATES.

1. When wheat is worth 75 cents a bushel, a baker's loaf weighs 9 ounces; what should it weigh when wheat is 60 cents a bushel?

2. A certain government map is drawn to a scale of 3 miles to the inch; how long a line will represent one side of a square farm containing 160 acres?

3. A and B worked for 17 days and received \$72.25; $\frac{2}{3}$ of A's daily wages equals $\frac{3}{4}$ of B's daily wages; how much should each receive?

4. Will a keg of nails selling at 5 cents a pound yield a greater or less revenue when the duty is 25% ad valorem than when the specific duty is 1 cent per pound?

5. A sold a carriage to B and gained $7\frac{1}{2}\%$; B sold it to C and lost $12\frac{1}{2}\%$; B received \$141.90 for the carriage; what did it cost A?

QUESTIONS FROM EXAMINATIONS FOR WISCONSIN TEACHERS' CERTIFICATES.

1. If 12 cows are worth 29 hogs, 15 hogs worth 8 sheep, 29 sheep worth 9 oxen, how many oxen are worth 30 cows?

2. How many rods of fence will be necessary to enclose a field in the shape of a right-angled triangle, its base being 52 rods, and its area $16\frac{3}{4}$ acres?

3. A certain sum will pay A's wages for $8\frac{1}{2}$ days or B's wages for $10\frac{1}{4}$ days; for how many days will it pay their combined wages?

4. The distance between Chicago and Ashland is 435 miles. If a train leaves Chicago for Ashland at 11:30 P.M., and another leaves Ashland for Chicago at 5:45 the next morning over the same road, at what time will these two trains meet, and how far from Ashland, the former train running 32 miles an hour, and the latter 28 miles an hour?

5. Divide $\frac{3}{7}$ by $\frac{4}{9}$, and write out your explanation of the operation as you would present it to a class just beginning division of fractions.

Questions from Examinations for South Carolina Teachers' Certificates.

1. Find the cost of carpet 30 inches wide at \$ 1.25 per yard for a room 18 feet by 14 feet, if the strip runs lengthwise.

2. If 8 horses consume $4\frac{1}{2}$ tons of hay in 32 days, how many days will $6\frac{3}{4}$ tons last 9 horses?

3. What number diminished by the difference between $\frac{3}{4}$ and $\frac{3}{5}$ of itself, leaves a remainder of 34?

4. How many cubic yards of earth must be removed in digging a cellar 36 feet 10 inches long, 22 feet 3 inches wide, and 5 feet 2 inches deep?

5. A sells his horse for \$ 198, which is 10% less than his asking price, and his asking price was 10% more than the cost; what did the horse cost him?

Questions from Examinations for Nebraska Teachers' Certificates.

1. Find the product of the G. C. D. and the L. C. M. of 16 and 24.

2. How many fractional units are in the reciprocal of $\frac{4}{5}$? Reduce the reciprocal of $\frac{7}{16}$ to a mixed number, giving analysis.

3. If $1\frac{7}{8}$ pounds of tea cost \$ 1.05, what will 16 pounds of tea cost? Analysis.

4. The divisor is six times the quotient; three-fourths of the quotient is $\frac{3}{8}$; find the dividend.

5. The product was 5 T. 6 cwt. 50 lb.; the multiplicand was 4 cwt. 52 lb.; what was the multiplier?

6. In a proportion the ratio of the third term to the fourth is $6\frac{1}{4}$; the second term is $212\frac{1}{2}$; find the first term.

7. A man sold his farm, which he had been renting for 9% of the price received, and loaned the money at 6%; his income was decreased \$ 150; for what sum did he sell the farm?

8. A rectangular field contains 180 acres; it is twice as long as it is broad; find the length and breadth of the field.

9. At what price should 4% bonds be bought (brokerage $\frac{1}{4}$ of 1%), to yield an income of 5%?

Questions from Examinations for New York Teachers' Certificates.

1. Reduce (*a*) .37652 yr. to integers of lower denominations; (*b*) $3\frac{3}{4}$ quarts to the fraction of a bushel.

2. A printer's bill for a lot of handbills was $6.70. If the contract was that the price of the first hundred should be $1.50, and for each succeeding hundred $.40, how many handbills were charged for?

3. If a certain bell metal is composed in weight of 73 parts copper, 18 parts tin, 6 parts zinc, 12 parts lead, and 3 parts iron, how many pounds of each metal are there in a bell made from this composition, and weighing 616 pounds?

4. Find the proceeds for a 90-day note for $275, made and discounted to-day, at 6%, by the exact interest method at bank, in Albany, N.Y. (Contracts made in New York after Jan. 1, 1895, have no grace.)

5. If a picture dealer receives a discount of 40% from the list price on etchings, and sells them at list price, what is his per cent of gain on his investment?

Questions from Examinations for Mississippi Teachers' Certificates.

1. A horse costs $1\frac{1}{4}$ times as much as a wagon; both cost $270; find the cost of the wagon.

2. At 55 cents per cubic yard of earth, find the cost of raising $\frac{3}{8}$ of an acre 9 inches.

3. A walk 12 feet wide surrounds a square garden; the area of the walk is half an acre; find the area of the square it encloses.

4. A's capital is to his partner B's as 5 to 8. At the end of 4 months A withdraws half of his capital, and B withdraws two-thirds of his capital. At the end of the year their whole gain is $4,000; find the share of each.

5. Illustrate by examples two methods of finding the L. C. M. of three or more numbers.

TEACHERS' EXAMINATION QUESTIONS.—NEW JERSEY.

1. A square field contains 40 acres; what will it cost to fence it at \$1.12½ per yard?

2. A, B, and C engage in business. At the start A puts in \$5,000; B, \$7,000; and C, \$12,000. Three months later A puts in \$3,000 additional, and B withdraws \$2,000. At the end of 9 months C withdraws \$10,000; what is each partner's share of the net loss for the year, which amounts to \$13,500?

3. How many yards of carpet $\frac{3}{4}$ of a yard wide will be required to cover a floor 36 feet 9 inches long and 27 feet wide, the strips to run lengthwise and an allowance of 3 inches per strip to be made for matching figures?

4. A gentleman has an income of \$3,290, derived from bonds paying a semi-annual dividend of 3½%; what is the face value of the bonds? If the bonds were bought at 90, what per cent do they pay on the investment?

5. A schoolroom contains two square blackboards whose sides are 4 feet and 8 feet, respectively; compare their areas.

TEACHERS' EXAMINATION QUESTIONS.—COLORADO.

1. Write a number that is in all the following classes: odd, composite, simple, integral, denominate.

2. For how much must I draw a note at 90 days in order that when discounted in a bank at 7% its proceeds will pay for 137$\frac{3}{4}$ yards of cloth at \$2$\frac{5}{8}$ per yard?

3. How much will it cost to paint the walls and ceiling of a room 57½ feet long, 38 feet wide, and 10 feet 6 inches high, at 29½ cents per square yard?

4. Subtract $\frac{3}{4}$ of $\frac{4}{9}$ from $\frac{3}{8}$ of $\frac{3\frac{1}{2}}{4\frac{1}{5}}$; add to the remainder $\frac{5}{16}$; divide the result by 6$\frac{1}{4}$, and change the quotient to a decimal.

5. If the height of a room 16 feet square is 10 feet, what is the distance from one corner of the floor to the diagonally opposite upper corner of the ceiling?

Questions from Examinations for Arkansas Teachers' Certificates.

1. New York being 3° E. from Washington, and San Francisco 45° 25′ W., what time will it be at New York when it is noon at San Francisco?

2. A bookseller wishes to mark up the price of a book which he now sells for $ 2, so that he can deduct 15%, and yet receive the present price; what must be the marked price?

3. The distance around each of two gardens is 25 rods; one is in the form of a circle, the other a square; which contains the more land, and how much?

4. A broken tree rested on the stump 20 feet from the ground, and its top touched the ground 50 feet from the stump; how high was the tree?

5. The shares of the joint stock of a firm consisting of three partners are as $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$; they divide a profit of $ 3,900; what is each partner's share?

Questions from Examinations for Iowa Teachers' Certificates.

1. How many planks laid crosswise, 1 foot wide, will it take for a board walk 1 mile 16 rods long?

2. A man sold $\frac{5}{12}$ of his farm at one time, $\frac{2}{7}$ at another, and the remainder for $ 180 at $ 45 per acre; how many acres were there in the farm?

3. A stock range in Montana containing 1,000 acres in the form of a square was fenced in at a cost of $ 2,000. If the range had been in the form of a rectangle 800 rods long, what would it have cost to enclose it with the same kind of fence?

4. How many yards of carpet, $\frac{7}{8}$ of a yard wide, will it require to cover a room 35 feet long, 18 feet wide? What will the carpet cost at $ 1.75 per yard?

5. Define common multiple, least common multiple, common divisor, greatest common divisor, prime factor, two numbers prime to each other, and illustrate each answer by an example.

Teachers' Examination Questions. — Kentucky.

1. One pound avoirdupois equals 1 lb. 2 oz. 11 pwt. 16 gr. Troy. Find the weight Troy, equivalent to 7 lb. 11 oz. avoirdupois.

2. A tailor buys 38 yd. of cloth at \$2.45 per yard; sponging causes a shrinkage of 4%. What must be his selling price per yard to gain 10%?

3. What will it cost, at \$1.15 per linear yard, to carpet a flight of stairs 11 ft. 4 in. high, the tread of each stair being 10 inches and the rise 8 inches?

4. Divide 90 into three parts so that $\frac{3}{4}$ of the first, $\frac{2}{3}$ of the second, and $\frac{6}{7}$ of the third shall all be equal.

5. A druggist bought 10 lb. of quinine, by avoirdupois weight, at $47\frac{1}{2}$ cents per ounce, and sold it by apothecaries' weight, in 3 gr. capsules at 10 cents a dozen. The capsules cost him 25 cents a gross. What was his profit? What was his gain per cent?

Teachers' Examination Questions. — Minnesota.

1. The floor of a public hall 80 ft. by 45 ft. is increased 12 per cent in width and 8 per cent in length. Find the per cent of increase in area.

2. How many square feet of inch boards are there exactly in a covered box 4 ft. 8 in. by 3 ft. 6 in. by 2 ft. 4 in.?

3. Three successive trade discounts of 20 per cent, 15 per cent, and 8 per cent equal what single discount?

4. Divide 459.25 into three parts that shall be to one another as $\frac{3}{4}$, $\frac{5}{6}$, and 3, respectively.

5. A piece of land is $8\frac{1}{4}$ feet wide; how long must it be to contain 3 acres?

6. Write a problem to involve this question: $\frac{5}{7} \div \frac{1}{4} = ?$. Solve the problem and explain it.

7. Reduce .096 to a common fraction (*a*) whose numerator is 144; (*b*) whose denominator is 375.

Ontario Examination Questions.—Public School Leaving.

1. A clerk pays $7.50 taxes on his salary. What is his total salary if $400 of it is exempt from taxation and a $2\frac{1}{2}\%$ rate is levied on the remainder?

2. A miller bought 20,000 bushels of wheat and had it insured for $\frac{4}{5}$ of its cost, at $1\frac{1}{16}\%$, paying a premium of $136; at what price per bushel must he sell it to gain 20%?

3. For what sum must a note be drawn on June 1st, payable in 90 days, so that when discounted on June 14th, at 8%, the proceeds will be $717.20?

4. A farmer bought 80 acres on the 1st December, 1893, for $3,600, payable one-third cash, one-fourth on the first of February, 1894, and the balance on the 1st of June, 1894; find the equated time for the payment if made in one sum.

5. Find the number of cubic feet in a hewn log 12 inches square at one end and $9\frac{3}{4}$ inches square at the other, its length being 27 feet.

Ontario Examination Questions.—High School Primary.

1. Divide $916 among A, B, and C, so that 5% of A's share may equal $7\frac{1}{2}\%$ of B's, and $12\frac{1}{2}\%$ of B's may equal 20% of C's.

2. A buys 600 yd. of silk at 95 cents a yard and sells it at once, receiving in payment a 90-day note for $700, which he at once discounts at a bank at 6% per annum; find the gain.

3. A merchant reduces the marked price of an article by a certain per cent. He gives the same per cent off this reduced price for cash. The cash price is now $\frac{25}{36}$ of the original marked price; find the rate per cent.

4. How many cords are there in a cylindrical log 20 feet long and 3 feet 6 inches in diameter?

5. Find the diameter of a circle whose area is equal to the sum of the areas of two circles whose diameters are 12 inches and 16 inches, respectively.

ONTARIO EXAMINATION QUESTIONS.—UNIVERSITY MATRICULATION.

1. From 1870 to 1880, the population of a town increased 30%; from 1880 to 1890 it decreased 30%. The population in 1870 exceeded that in 1890 by 2,781; find the population in 1880.

2. (*a*) A man borrows $12,000 for a year at 8% and loans it at 2% per quarter-year, compounding interest at the end of each quarter; how much money will he have made at the end of the year?

(*b*) A borrows from B a sum of money and agrees to pay him by three annual payments of $200 each. If money is worth 5% per annum, compound interest, find the sum borrowed.

3. A commission merchant received 500 bbl. of flour, which he sold at $5 a bbl., charging 2% commission; he was instructed to invest the net proceeds, deducting a purchase commission of 2%, in tea; find the value of the tea bought, and the total commission.

4. A man holds $15,600 stock worth 60; to transfer to 4% stock at 78 will increase his annual income $12; he effects the transfer, but not until each stock has increased 2 in price; find the increase of his income.

5. A merchant marks his goods at an advance of 25% on cost. After selling $\frac{1}{3}$ of the goods, he finds that some of the goods in hand are damaged so as to be worthless; he marks the salable goods at an advance of 10% on the marked price and finds in the end that he has made 20% on cost; what part of the goods was damaged?

6. A grocer, by selling 12 pounds of sugar for a certain sum, gained 20%. If sugar advances 10% in the wholesale market, what per cent will the grocer now gain by selling 10 pounds for the same sum?

7. A note made June 1st, at three months, was discounted immediately at 8% per annum, and produced $357.40; what was the face of the note?

8. What rate per cent per annum, compounded half-yearly, is equivalent to 6% per annum, compounded yearly?

9. Two candles are of equal length. The one is consumed uniformly in 4 hours, and the other in 5 hours. If the candles are lighted at the same time, when will one be three times as long as the other?

10. Calculate the number of acres in the surface of the earth, considering the earth a sphere of 8,000 miles diameter.

Teachers' Examination Questions. — Texas.

1. Write the analysis of each of the following:

(*a*) A boy has 75 cents, with which he can buy 5 melons. Find the average price of a melon.

(*b*) A boy has 75 cents, with which he buys melons at the average price of 5 cents each. How many melons does he buy?

2. A trader bought a plantation at $ 14 per acre, and sold it for $ 15,824, gaining $ 2 per acre; find the cost.

3. Find the product of the smallest prime number greater than 153, and the greatest composite odd number less than 230.

4. From the sum of $29\frac{3}{7}$ and $42\frac{5}{8}$, take the difference of $20\frac{1}{6}$ and $10\frac{3}{16}$.

5. The product of two factors is $\frac{3}{16}$; one of the factors is $\frac{3}{8}$; find the other.

6. What per cent is gained by buying wheat at $62\frac{1}{2}$ cents per bushel and selling at $67\frac{1}{2}$ cents?

7. In a proportion the inverse ratio of the first term to the second term is $3\frac{1}{5}$; the fourth term is 160; find the third term.

8. Give solution and analysis: Find the present worth and true discount of a note for $ 135.75, due 1 yr., 8 mo., 15 da. hence, money being worth 8 per cent.

9. What may X offer for a house which pays $ 895 rent per year that he may receive 8% interest on the investment?

10. Reduce to lowest terms: $.66\frac{2}{3}$; .125; $.37\frac{1}{2}$.

QUESTIONS FOR EXAMINATION OF TEACHERS: SELECTED FROM PAPERS OF VARIOUS STATES.

1. The sum of minuend, subtrahend, and remainder is 1,296; the minuend is three times the subtrahend; find the remainder.

2. The product of dividend, divisor, and quotient is 576; the divisor is six times the quotient; find the three numbers.

3. Multiply (without reducing to improper fractions) $21\frac{3}{4}$ by $7\frac{2}{3}$.

4. Express in Roman the product of $(3\frac{1}{2})^2$ and 2^3.

5. Show that 227 is a prime number.

6. Without dividing, reduce the following to decimals: $\frac{7}{20}$; $\frac{9}{40}$; $\frac{17}{125}$.

7. Solve by proportion: The interest of a sum of money for 1 yr. 4 mo. 18 da. is \$115.50; find the interest for 3 yr. 5 mo. 15 da.

(Avoid the absurdity of multiplying one concrete number by another concrete number.)

8. Find the ratio of $\frac{5}{18}$ of one bushel to $13\frac{1}{3}$ quarts.

9. Divide 600 into parts proportional to the fractions $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$.

10. What is the distance, in rods, from the centre to each corner of a section of land?

11. What decimal of an acre is 180 square feet? Carry to the sixth decimal.

12. Bought 75 yards of cloth at 10% less than first cost and sold it at 10% more than first cost and gained \$25; what was the first price paid per yard?

13. The difference between the simple and the annual interest on a certain principal for 4 years and 2 months at 6% is \$12; find the principal.

14. A certain principal at a certain rate amounts to \$750 in three years, and the interest is one-fourth of the principal; find the principal and the rate.

15. Multiply the G. C. D. of 84, 112, and 168 by their L. C. M.

16. Half of $\frac{4}{5}$ of A's money equals $\frac{1}{3}$ of $\frac{6}{7}$ of B's; they bot[h] have the sum of $2,400; what has each?

17. If a merchant loses 10% by selling goods at 27 cent[s] a yard, for what should they be sold to gain 20%?

18. A farmer exchanged 60 bushels of corn for oats, giving $3\frac{3}{4}$ bushels of corn for $5\frac{1}{3}$ bushels of oats; how many bushels of oats did he get?

19. The greatest common factor of two numbers is 12, and their least common multiple is 504; find the product of the numbers. (Entire solution with nine figures.)

20. Find the face of a draft on St. Louis costing $781.95, exchange being $\frac{1}{4}$% premium.

21. What is the present worth of a debt of $640 due in 1 yr. 5 mo. 15 da., money being worth 8%?

22. What principal at $8\frac{1}{2}$% simple interest will produce $150.10 interest in 3 months and 2 days?

23. What must be the price of 5% stock in order that the purchaser may receive 7% from his investment?

24. Find by factoring: The square root of 1,764; the cube root of 3,375.

25. A man is 144 miles due south from home; he travels due west 165 miles; how far is he then from home?

26. Find by two methods the cube root of 74,088.

27. Solve by proportion: At 75 cents per bushel, corn for 30 horses, 8 days, costs $54; find cost of corn for 45 horses, 16 days, at 60 cents per bushel.

28. What will be the cost of painting 10 circular pillars, each 40 inches in circumference and 18 feet high, at $12\frac{1}{2}$ cents per square yard?

29. A room whose length, breadth, and height are as the numbers 3, 2, and 1, contains 4,374 cu. ft.; what are its dimensions? Find the distance between its remotest corners.

30. A merchant sends $24,600 to his agent at St. Louis to purchase flour at $5 per barrel; how many barrels can he purchase if he charges $2\frac{1}{2}$% commission on the sum expended for flour?

31. How much less will it cost to fence a field 72 rods square than a rectangular field 3 times as long and $\frac{1}{3}$ as wide, if fencing cost \$2.50 a rod?

32. How many bunches of laths will be required for the walls and ceiling of a room 18 ft. long, 14 ft. wide, 10 ft. high, each bunch being estimated to cover 5 sq. yd.?

33. What will it cost to plaster the sides and bottom of a cylindrical cistern 10 ft. in diameter and 9 ft. deep, at \$1.25 a sq. yd.?

34. What will be the net proceeds of a sale of 160 barrels of potatoes at \$1.75 a barrel, allowing $2\frac{1}{2}\%$ commission and \$6.75 for other charges?

35. What is $\frac{3}{4}$ of that number of which 16 is $\frac{4}{7}$?

36. A package case which is twice as long as it is broad or deep will hold 93,312 cu. in.; what are its dimensions?

37. A number diminished by $\frac{1}{4}\%$ of itself equals 25% of 7,400. What is the number?

38. A field containing 20 acres is twice as long as it is broad. At \$10 a thousand for lumber, how much will the lumber cost to make a five board fence around the field, if the boards are 6 in. wide?

39. If a loaf of bread weighing 8 ounces cost 5 cents when wheat is worth \$1.00 a bushel, how many of the same sized loaves should be sold for 44 cents when wheat is worth 45 cents per bushel?

40. A wood-rack is 8 ft. long and 3 ft. 4 in. wide. How high must it be to hold one cord?

41. Divide \$870 among A, B, and C, so that B shall have \$110 more than C, and A shall have \$56 more than B.

42. The difference between the interest of \$500, and of \$650, for the same time, at 5%, is \$18.75; find the time.

43. A man sold two houses for \$1200 each; on one he gained $\frac{1}{3}$ of the cost, and on the other he lost $\frac{1}{5}$ of the cost; how much did he gain or lose on the two houses?

44. How many sq. ft. are in a board 16 ft. long and 11 in. wide at one end and 14 in. wide at the other?

SCALES OF NOTATION.

Number, being pure thought, is independent of the inventions for expressing it.

One hundred (written in the words of any language), 100, C, and any other symbol that might be accepted, are mere signs of a reality ; and the spoken word is just as much and just as little a sign as is the chalk-mark on the blackboard.

In the decimal system of notation, 1 unit of any order has to 1 unit of the next lower order the ratio 10. This constant ratio is called the radix, or root, of the scale. Constancy of ratio (above 1) is what gives to a symbol the value of position. (See the comparison of the Roman and the Arabic methods, pp. 1 and 2.)

In the decimal scale, to express numbers ten symbols are used ; the nine digits and 0 ; in the binary scale one digit and 0 ; in the ternary scale two digits and 0, — any scale requiring as many characters as there are units in its radix ; thus, since 10 decimal means 1 *ten* and 0 unit, 10 binary means 1 *two* and 0 unit. As 100 decimal means 1 × 10 × 10, or 1 hundred, 0 ten, and 0 unit, so 100 binary means 1 × 2 × 2, or 1 four, 0 two, and 0 unit.

Scales.	Characters Used.	Radix, or Ratio.
Decimal,	1, 2, 3, 4, 5, 6, 7, 8, 9, 0	Ten, expressed 10.
Nonary,	1, 2, 3, 4, 5, 6, 7, 8, 0	Nine, expressed 10.
Octary,	1, 2, 3, 4, 5, 6, 7, 0	Eight, expressed 10.
Septenary,	1, 2, 3, 4, 5, 6, 0	Seven, expressed 10.
Senary,	1, 2, 3, 4, 5, 0	Six, expressed 10.
Quinary,	1, 2, 3, 4, 0	Five, expressed 10.
Quaternary,	1, 2, 3, 0	Four, expressed 10.
Ternary,	1, 2, 0	Three, expressed 10.
Binary,	1, 0	Two, expressed 10.

The value of the zero, or nought, is seen to consist in serving to occupy a place which else would be filled by the digit at its left ; thus, in 10, if the zero were erased, the 1 would become of the first order — it would be one rather than ten. Hence, as the 0 merely serves to restrict the digits to their assigned positions, a *unary scale*, which would have no zero, and in which every place would be filled by the digit 1 without regard to position, would be useless except as a means of mere counting.

Undenary Scale,	**1, 2, 3, 4, 5, 6, 7, 8, 9, (10 ?), 0.**
Duodecimal Scale,	**1, 2, 3, 4, 5, 6, 7, 8, 9, (10, 11 ?), 0.**

It is clearly necessary that in every scale every order should be filled with a single character; if in the undenary scale ten should be expressed 10, one place, or order, would be more than filled; hence, when it is necessary to express a number greater than 9 with a character that will occupy but one place, a letter of the alphabet may be used, *a*, *b*, *c*, etc., meaning 10, 11, 12, etc.

The first order of the decimal scale is units; the higher orders are the first power, second power, third power, etc., of 10; that is to say, tens, hundreds, thousands, and so on.

The first order of the binary scale is units; the higher orders are the first power, second power, third power, etc., of 2; that is to say, twos, fours, eights, and so on.

The first order of the nonary scale is units; the higher orders are the first power, second power, third power, etc., of 9; that is to say, nines, eighty-ones, seven hundred twenty-nines, and so on.

In any scale of notation 1 unit of the second order is the first power of the radix; 1 unit of the third order is the second power of the radix, and so on.

In the Decimal Scale	111 means 1 hundred,	1 ten,	and 1 unit.
In the Binary Scale	111 means 1 four,	1 two,	and 1 unit.
In the Quaternary Scale	111 means 1 sixteen,	1 four,	and 1 unit.

EXAMPLES.

1. Express the decimal number 2 by the binary scale.

There is but one digit in the binary scale, the digit 1; as in the decimal scale the digit 1, removed by means of the zero to the next higher place, is thus multiplied by 10, so in the binary scale the digit 1, removed by means of the zero to the next higher place, is thus multiplied by 2.

PROCESS.

$$2\underline{)2}$$
1 unit of 2d order + 0 unit of 1st order = 10.

2. Express the decimal number 3 by the binary scale.

Since 2 decimal = 10 binary,
2 + 1 decimal = 10 + 1 binary = 11.

PROCESS.

$$2\underline{)3}$$
1 unit of 2d order + 1 unit of 1st order = 11.

3. Express the decimal number 11 by the binary scale.

$$\begin{array}{r|l} 2 & 11 \\ \hline 2 & \;\;5 \text{ and 1 unit of the 1st order;} \\ \hline 2 & \;\;2 \text{ and 1 unit of the 2d order;} \\ \hline & \;\;1 \text{ and 0 unit of the 3d order.} \end{array}$$

The expression will be 1011; the omission of the zero would cause the 1 in the fourth order to take the place of the third order.

4. Express 381 decimal in terms of the ternary scale.

Ans. 112010.

5. Express 876 decimal in terms of the scale of nine.

Ans. 1173.

6. Express 1128 decimal in terms of the duodecimal scale.

$$\begin{array}{r|l} 12 & 1128 \\ \hline 12 & \;\;94 + 0 \\ \hline & \;\;\;9 + 10 \end{array}$$

Since the 10 of the second order cannot be expressed with one ordinary digit, a symbol must be used which will occupy the place of but one order. In such cases the letters of the alphabet may be used; a for 10, b for 11, and so on.

1128 decimal = 7 a 0, duodecimal.

7. Change from the decimal scale: 250 to the septenary; 410 to the nonary; 850 to the scale with radix thirteen.

Ans. to each, 505, respectively.

8. Change 384 from the nonary scale to the decimal.

Radix = 9.

3 units of 3d order = 27 units of 2d order; 27 + 8 = 35.

35 units of 2d order = 35 × 9 units of 1st order = 315; 315 + 4 = 319.

384 nonary = 319 decimal.

PROCESS.

$$\begin{array}{r} 384 \\ \underline{9} \\ 27 \\ \underline{8} \\ 35 \\ \underline{9} \\ 315 \\ \underline{4} \\ 319 \end{array}$$

9. With c the symbol for 12, transpose $6\,c\,7$ from the scale with radix fifteen to the decimal scale.

Radix = 15.

6	units of 3d order =	$6 \times 15 \times 15$	= 1350 decimal
c = 12	units of 2d order =	12×15	= 180 decimal
7	units of 1st order =	7×1	= 7 decimal
$6\,c\,7$	units scale of 15		= 1537 decimal

10. Express the sum by the decimal scale:

1066 septenary, 1000 nonary, and 981 of a scale with radix sixteen. *Ans.* 3553.

11. With a, b, c, and d, meaning 10, 11, 12, 13, express decimally $1\,d\,70\,ca$, of a scale with radix fourteen.

Ans. 1056618.

12. Subtract 434 quinary from 1000 decimal.

Ans. 881 decimal.

13. Subtract 1110 binary from 220 ternary. (Change both to the decimal scale.) *Ans.* 10.

14. Change 3245 from the senary to the undenary scale.

Each figure denotes 6 times its value one place to the right; therefore the 3 in the fourth order denotes 18 in the third order; 18 third order + 2 third order = 20 third order, *decimal*; $20 \div 11 = 1$, with 9 remainder, etc.

11	3245	senary
11	152	1
11	10	2
11	0	6
Undenary		621

First Division.

$3 \times 6 + 2 = 20$; $20 \div 11 = 1$, with 9 remainder.
$9 \times 6 + 4 = 58$; $58 \div 11 = 5$, with 3 remainder.
$3 \times 6 + 5 = 23$; $23 \div 11 = 2$, with 1 remainder.

Second Division.

$1 \times 6 + 5 = 11$; $11 \div 11 = 1$, with 0 remainder.
$0 \times 6 + 2 = 2$; $2 \div 11 = 0$, with 2 remainder.

Third Division.

$1 \times 6 + 0 = 6$; $6 \div 11 = 0$, with 6 remainder.

15. Multiply 322, radix 5, by 43, radix 5.

$$\begin{array}{r} 322 \\ \underline{43} \\ 2021 \\ \underline{2343} \\ 31001 \end{array}$$

$3 \times 2 = 6 = 1$ unit of first order + 1 unit of second order, etc. In writing the partial products, and in the adding, 5 units must make 1 unit of the next higher order. The result also is on the scale of radix 5.

16. Divide 31001, radix 5, by 43, radix 5.

```
43)31001(322, radix 5
   234
    210
    141
     141
     141
```

PROOF BY CASTING OUT NINES.

Any number of tens is equal to the same number of nines plus the same number of ones, or units.

1 ten = 1 nine + 1 unit; 93 tens = 93 nines + 93 units.

If the greatest possible multiple of 9 found in the sum of the digits of any number be subtracted from that sum, the remainder, or *excess* of *nines*, will equal the number itself less its greatest multiple of 9.

$$93 = 10 \text{ nines} + 3 \text{ units.}$$
$$9 + 3 = 1 \text{ nine} + 3 \text{ units.}$$

EXAMPLES.

1. Add 703, 812, 625, and test by excess of nines.

The greatest multiple of 9 being subtracted

from	703	$(7 + 3 = 10 = 1 \text{ nine} + 1 \text{ unit})$	excess, 1
from	812	$(8 + 1 + 2 = 11 = 1 \text{ nine} + 2 \text{ units})$	excess, 2
from	625	$(6 + 2 + 5 = 13 = 1 \text{ nine} + 4 \text{ units})$	excess, 4
from	2140	$(2 + 1 + 4 = 7 = 0 \text{ nine} + 7 \text{ units})$	excess, 7

The foregoing shows that the excess of nines in the sum equals the sum of the excesses in the three numbers; the addition, therefore, is assumed to be correct.

2. Add and test: 7963 + 8762 + 5086.

7963	excess, 7	
8762	excess, 5	$7 + 5 + 1 = 13$; excess, *4*.
5086	excess, 1	
21811	excess, *4*	

3. Subtract and test: 434 − 293.

434	excess, 2	The remainder of the excesses should equal the remainder of the numbers. In this case another 9 is added to the excess in the minuend. $9 + 2 - 5 = 6$.
293	excess, 5	
141	excess, *6*	

4. Multiply and test: 51×39.

$$\begin{array}{rl} 39 & \text{excess, } 3 \\ \underline{51} & \underline{\text{excess, } 6} \\ 1989 & \text{excess, } 0 \end{array} \left.\vphantom{\begin{array}{c}1\\1\end{array}}\right\} 6 \times 3 = 18;\ \text{excess, } 0.$$

5. Divide and test: $8273 \div 41$.

DIVISOR $\times$	QUOTIENT $+$	REMAINDER $=$	DIVIDEND.
(Excess in)	(Excess in)	(Excess in)	(Excess in)

$$\left.\begin{array}{lrl} \text{Divisor,} & 41; & \text{excess of nines, } 5 \\ \text{Quotient,} & 201; & \text{excess of nines, } 3 \\ \text{Remainder,} & 32; & \text{excess of nines, } 5 \end{array}\right\} 5 \times 3 + 5 = 20;\ \text{excess, } 2.$$

Dividend, 8273; excess, 2.

ANSWERS.

Page 6. — 1. $3.45. 2. 996 bbl. 3. $8.65. 4. $6,959.25.

Page 7. — 5. 8,989 a. 6. 699. 7. 6,319. 8. 1,743 yr. 9. 2,335. 10. 26,433. 11. 257,122. 12. 272,355.

Page 8. — 13. 678,901. 14. 7,335,677. 15. $27,644.94. 16. $627,121.40.

Page 9. — 17. 12,823. 18. 15,858. 19. 19,944. 20. 23,411. 21. 7,018,086. 22. 7,089,290. 23. 7,921,471. 24. 6,963,773. 25. 856,097,369. 26. 1,147,781,419. 27. 968,666,591. 28. 1,102,188,913.

Page 10. —

29.	BUSHELS.		BUSHELS.		BUSHELS.
Ala.	30,048,976.	Miss.	23,524,444.	Va.	42,586,777.
Ark.	27,668,356.	Mo.	248,658,734.	Corn.	543,293,022.
Fla.	3,645,733.	N. C.	35,540,400.	Wheat.	64,587,612.
Ga.	32,013,148.	S. C.	15,472,011.	Oats.	60,219,775.
Ky.	89,457,164.	Tenn.	74,974,391.	Rye.	2,183,491.
La.	10,142,076.	Tex.	36,551,690.	Total.	670,283,900.

30.	BUSHELS.		BUSHELS.		BUSHELS.
Ala.	33,527,330.	Kan.	337,520,859.	Utah.	2,232,100.
Ark.	39,134,044.	Ky.	98,341,970.	Wash.	9,018,892.
Cal.	44,957,546.	Mich.	92,619,656.	Wis.	110,712,772.
Del.	4,986,739.	Minn.	128,208,147.	Corn.	1,570,871,207.
Fla.	4,106,264.	Neb.	271,395,778.	Wheat.	320,773,160.
Ga.	35,212,211.	N. H.	4,699,571.	Oats.	627,723,349.
Ill.	467,231,539.	Ohio.	190,595,414.	Rye.	18,717,568.
Ind.	178,531,085.	Ore.	15,548,227.	Total.	2,538,085,284.
Iowa.	469,505,140.				

Page 12. — 1. 1,242 bu. 2. 3,664. 3. 2,582. 4. 4,133. 5. 1,146.

Page 13. — 6. 2,776,040 sq. mi. 7. 166,367,648 bu. 8. $30,528,393. 9. 112,802.

Page 14.—**10.** 7,406 votes. **11.** 3,679 persons. **12.** 634,960 bu. **13.** 270,254 bu. **14.** 59,054,095 lb. **15.** $1,147,104. **16.** $1,801,187. **17.** $250,125. **18.** 12,582 lb. **19.** 1,000,136. **20.** $200,119. **21.** 2,324,911. **22.** 25.23 mi. **23.** 1,850,895. **24.** 13,861.

Page 15.—**25.** 1,499,000. **26.** 15,924,591. **27.** 3,861. **28.** 16,244. **29.** DCXLIII. **30.** 197,350. **31.** $88,910.91. **32.** 10,011 too small. **33.** 3,001,012; 1,401,497. **34.** 13,579,974. **35.** 53,699,777. **36.** 19,751,298. **37.** $602,002.30. Review Work. 943,147,858 bu.

Page 17.—**1.** 37,773.

Page 18.—**2.** $1,400. **3.** 10,857,468,265 bu. **4.** $3,027,957,543. **5.** 11,175,435,180 lb. **6.** 32,206,250,123 lb. **7.** 7,632,383,922 bu. **8.** 27,468,129,556 bu. **9.** $1,897,465,316.47. **10.** $2,825,572,018.92. **11.** $57.50. **12.** $290.

Page 19.—**13.** $112.50. **14.** $8,500. **15.** $14,000. **16.** $8,500. **17.** $500. **18.** $6,000. **19.** $2,000. **20.** $10,000. **21.** $360. **22.** $560. **23.** $15,000. **24.** $8,450. **25.** 1,726. **26.** 1,726.

Page 20.—**27.** $10,356. **28.** $28.50. **29.** $332.50. **30.** $28,000. **31.** 9,100,000. **32.** 691,310. **33.** 1,345,710. **34.** 5,620,965. **35.** 6,731,748. **36.** 2,563,368. **37.** $721.44.

Page 21.—**38.** 826,281. **39.** 9,036,036. **40.** 3,097,600 sq. yd. **41.** 43,560 sq. ft. **42.** 27,878,400 sq. ft. **43.** 256 trees. **44.** 625. **45.** 729. **46.** 1,296. **47.** 3,125. **48.** 116,281. **49.** 6,859. **50.** 4,096. **51.** 7,776. **52.** 900 sq. mi. **53.** 61,875 sq. mi. **54.** $109,353. **55.** $36,173.80. **56.** $477,470.84. **57.** $628,434,951.20. **58.** $448,043,773.09. **59.** $287,984,041.38.

Page 22.—**60.** $190,315,993.75. **61.** $51,069,610.80. **62.** $6,825,507.15. **63.** 274,071. **64.** 185,661; 182,714; 241,654. **65.** 406,980; 687,990; 784,890. **66.** 2,516,085; 2,407,548; 1,509,651. **67.** 384,813. **68.** 110,292; 152,712; 118,776. **69.** 69,264; 623,376; 617,604. **70.** 1,302,114; 1,704,296; 2,580,336. **71.** 4,292,364; 5,055,001; 5,102,244.

Page 23.—**72.** 9,689,031. **73.** 977,778,153. **74.** 487,144,565,802. **75.** 7,713,921,357; 3,853,099,857; 5,397,428,457. **76.** 320. **77.** 490,906. **78.** 0. **79.** 0. **80.** Nine hundred ten thousand, nine hundred eleven. **81.** 1,909,999. **82.** 2,325,655,800. **83.** 320,000. **84.** $55.23. **85.** $12.88.

Page 24.—**86.** $21.92. **87.** $33.89. **88.** $184.50. **89.** $52.50. **90.** $384.74. **91.** $197.46.

Page 27.—**1.** 604.

Page 28. — **2.** \$300. **3.** 99,395,948$\frac{4}{9}$ bu. **4.** 1,830,727,642$\frac{7}{9}$ lb. **5.** \$112,635,809. **6.** \$202,089,435$\frac{5}{9}$. **7.** 221 a. **8.** 428 lb. **9.** 276,105,497$\frac{1}{7}$ lb. **10.** 1,487,858; 1,623,117$\frac{9}{11}$.
11. 27,328,572$\frac{3}{7}$; 23,912,500$\frac{7}{8}$; 21,255,556$\frac{3}{9}$; 17,390,909$\frac{8}{11}$.

Page 29. — **12.** 2,103$\frac{1}{21}$.

Page 30. — **13.** 196 lb. **14.** 56 lb. **15.** 70 lb. **16.** 70. **17.** 70$\frac{70}{99}$. **18.** 69$\frac{30}{100}$. **19.** 32 lbs. **20.** 56 lbs. **21.** \$1.25.

Page 31. — **22.** 10,000 thousands. **23.** \$31. **24.** 15,000,000 hundreds. **25.** \$34. **26.** 1. **27.** 10. **28.** \$2. **29.** 2. **30.** 6. **31.** 17 hr. **32.** 15. **33.** \$3$\frac{1}{3}$. **34.** 1$\frac{12}{18}$. **35.** 66$\frac{2}{3}$. **36.** 77. **37.** 3. **38.** 11. **39.** 1. **40.** 2. **41.** 1. **42.** 2.

Page 32. — **43.** 9. **44.** 9. **45.** 25. **46.** 16. **47.** 3. **48.** 3$\frac{3}{4}$.

Page 33. — **49.** 1,280. **50.** 800. **51.** 720. **52.** 74$\frac{38}{53}$. **53.** 180. **54.** $\frac{1}{15}$. **1.** 378,389,505 units. **2.** 12 sq. mi. **3.** 910. **4.** 1,140 lb. **5.** 428. **6.** 260$\frac{3}{4}$ bu. oats. **7.** 252. **8.** 3,200 gal. **9.** \$33,264. **10.** \$.08.

Page 34. — **11.** 20. **12.** \$3. **13.** \$356. **14.** 288 bu. **15.** 44$\frac{140}{231}$. **16.** 24 a. **17.** \$14,970. **18.** 51,033$\frac{14}{39}$. **19.** 218,019. **20.** 63,360 in. **21.** 4. **22.** 24 sheets. **23.** 832. **24.** 3 mi.; 1,408 rev.

Page 35. — **25.** 18 bu. wheat. **26.** 81. **27.** 901,321,663. **28.** 15,552. **29.** 1. **30.** 1,020. **31.** 4. **32.** 5$\frac{1}{2}$. **33.** 7,722. **34.** 64 trillions. **35.** 364 da. **36.** 90. **37.** 428,574. **38.** 840. **39.** MMMMIX.

Page 38. — **1.** 7, 11. **2.** 2, 2, 2. **3.** 2, 2, 2, 2. **4.** 2, 3, 5. **5.** 2, 2, 2, 2, 3, 3, 5.

6.

$21 = 3 \times 7.$
$22 = 2 \times 11.$
$24 = 2 \times 2 \times 2 \times 3.$
$25 = 5 \times 5.$
$26 = 2 \times 13.$
$27 = 3 \times 3 \times 3.$
$28 = 2 \times 2 \times 7.$
$30 = 2 \times 3 \times 5.$
$32 = 2 \times 2 \times 2 \times 2 \times 2.$
$33 = 3 \times 11.$
$34 = 2 \times 17.$
$35 = 5 \times 7.$
$36 = 2 \times 2 \times 3 \times 3.$
$38 = 2 \times 19.$
$39 = 3 \times 13.$
$40 = 2 \times 2 \times 2 \times 5.$
$42 = 2 \times 3 \times 7.$
$44 = 2 \times 2 \times 11.$
$45 = 3 \times 3 \times 5.$
$46 = 2 \times 23.$
$48 = 2 \times 2 \times 2 \times 2 \times 3.$
$49 = 7 \times 7.$
$50 = 2 \times 5 \times 5.$
$51 = 3 \times 17.$
$52 = 2 \times 2 \times 13.$
$54 = 2 \times 3 \times 3 \times 3.$
$55 = 5 \times 11.$
$56 = 2 \times 2 \times 2 \times 7.$
$57 = 3 \times 19.$
$58 = 2 \times 29.$
$60 = 2 \times 2 \times 3 \times 5.$

7.

$62 = 2 \times 31.$
$63 = 3 \times 3 \times 7.$
$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2.$
$65 = 5 \times 13.$
$66 = 2 \times 3 \times 11.$
$68 = 2 \times 2 \times 17.$
$69 = 3 \times 23.$
$70 = 2 \times 5 \times 7.$
$72 = 2 \times 2 \times 2 \times 3 \times 3.$
$74 = 2 \times 37.$
$75 = 3 \times 5 \times 5.$
$76 = 2 \times 2 \times 19.$
$77 = 7 \times 11.$
$78 = 2 \times 3 \times 13.$
$80 = 2 \times 2 \times 2 \times 2 \times 5.$
$81 = 3 \times 3 \times 3 \times 3.$
$82 = 2 \times 41.$
$84 = 2 \times 2 \times 3 \times 7.$
$85 = 5 \times 17.$
$86 = 2 \times 43.$
$87 = 3 \times 29.$
$88 = 2 \times 2 \times 2 \times 11.$
$90 = 3 \times 3 \times 2 \times 5.$
$91 = 7 \times 13.$
$92 = 2 \times 2 \times 23.$
$93 = 3 \times 31.$
$94 = 2 \times 47.$
$95 = 5 \times 19.$
$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3.$
$98 = 2 \times 7 \times 7.$
$99 = 3 \times 3 \times 11.$
$100 = 2 \times 2 \times 5 \times 5.$

8.

$102 = 2 \times 3 \times 17.$
$104 = 2 \times 2 \times 2 \times 13.$
$105 = 3 \times 5 \times 7.$
$106 = 2 \times 53.$
$108 = 2 \times 2 \times 3 \times 3 \times 3.$
$110 = 2 \times 5 \times 11.$
$111 = 3 \times 37.$
$112 = 2 \times 2 \times 2 \times 2 \times 7.$
$114 = 2 \times 3 \times 19.$
$115 = 5 \times 23.$
$116 = 2 \times 2 \times 29.$
$117 = 3 \times 3 \times 13.$
$118 = 2 \times 59.$
$119 = 7 \times 17.$
$120 = 2 \times 2 \times 2 \times 3 \times 5.$
$121 = 11 \times 11.$
$122 = 2 \times 61.$
$123 = 3 \times 41.$
$124 = 2 \times 2 \times 31.$
$125 = 5 \times 5 \times 5.$
$126 = 2 \times 3 \times 3 \times 7.$
$128 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2.$
$129 = 3 \times 43.$
$130 = 2 \times 5 \times 13.$
$132 = 2 \times 2 \times 3 \times 11.$
$133 = 7 \times 19.$
$134 = 2 \times 67.$
$135 = 3 \times 3 \times 3 \times 5.$
$136 = 2 \times 2 \times 2 \times 17.$
$138 = 2 \times 3 \times 23.$
$140 = 2 \times 2 \times 5 \times 7.$
$141 = 3 \times 47.$
$142 = 2 \times 71.$
$143 = 11 \times 13.$
$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3.$
$145 = 5 \times 29.$
$146 = 2 \times 73.$
$147 = 3 \times 7 \times 7.$
$148 = 2 \times 2 \times 37.$
$150 = 2 \times 3 \times 5 \times 5.$
$152 = 2 \times 2 \times 2 \times 19.$
$153 = 3 \times 3 \times 17.$
$154 = 2 \times 7 \times 11.$
$155 = 5 \times 31.$
$156 = 2 \times 2 \times 3 \times 13.$
$158 = 2 \times 79.$

$159 = 3 \times 53.$
$160 = 2 \times 2 \times 2 \times 2 \times 2 \times 5.$
$161 = 7 \times 23.$
$162 = 2 \times 3 \times 3 \times 3 \times 3.$
$164 = 2 \times 2 \times 41.$
$165 = 3 \times 5 \times 11.$
$166 = 2 \times 83.$
$168 = 2 \times 2 \times 2 \times 3 \times 7.$
$169 = 13 \times 13.$
$170 = 2 \times 5 \times 17.$
$171 = 3 \times 3 \times 19.$
$172 = 2 \times 2 \times 43.$
$174 = 2 \times 3 \times 29.$
$175 = 5 \times 5 \times 7.$
$176 = 2 \times 2 \times 2 \times 2 \times 11.$
$177 = 3 \times 59.$
$178 = 2 \times 89.$
$180 = 2 \times 2 \times 3 \times 3 \times 5.$
$182 = 2 \times 7 \times 13.$
$183 = 3 \times 61.$
$184 = 2 \times 2 \times 2 \times 23.$
$185 = 5 \times 37.$
$186 = 2 \times 3 \times 31.$
$187 = 11 \times 17.$
$188 = 2 \times 2 \times 47.$
$189 = 3 \times 3 \times 3 \times 7.$
$190 = 2 \times 5 \times 19.$
$192 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3.$
$194 = 2 \times 97.$
$195 = 3 \times 5 \times 13.$
$196 = 2 \times 2 \times 7 \times 7.$
$198 = 2 \times 3 \times 3 \times 11.$
$200 = 2 \times 2 \times 2 \times 5 \times 5.$

9. $380 = 2 \times 2 \times 5 \times 19$; $864 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3.$
10. $900 = 2 \times 2 \times 3 \times 3 \times 5 \times 5$; $625 = 5 \times 5 \times 5 \times 5.$
11. $1{,}000 = 2 \times 2 \times 2 \times 5 \times 5 \times 5$; $1{,}500 = 2 \times 2 \times 3 \times 5 \times 5 \times 5.$
12. $2{,}000 = 2^4 \times 5^3$; $2{,}500 = 2^2 \times 5^4.$

Page 39.—**13.** $1{,}440 = 2^5 \times 3^2 \times 5$; $3{,}003 = 3 \times 7 \times 11 \times 13$; $4{,}090 = 2 \times 5 \times 409$. **14.** 2, 3, 5, 5, 7. **15.** 2^7, 3^2, 5. **16.** 2, 3^4, 5. **17.** 2, 2, 11, 37. **18.** 3, 5, 29. **19.** 3, 7, 47. **20.** 2^7, 5^2. **21.** 2^7, 3, 5^2. **22.** 2, 401. **23.** 2, 2, 3, 5, 17. **24.** 3, 3, 109. **25.** 3^3, 37. **26.** 2, 3^2, 5^2, 11. **27.** 11, 401. **28.** 2^7, 7. **29.** 2, 3, 3, 61. **30.** 3, 5^2, 11. **31.** 2, 3, 3, 5, 89. **32.** 2, 3, 7, 7, 13. **33.** 5, 1,721. **34.** 2, 3, 7, 13.

Page 40.—**36.** 2, 2, 5, 359. **37.** Prime. **38.** Prime. **39.** 2, 1,409. **40.** 7,563. **41.** 47,131. **42.** 2, 1,499. **43.** 3, 3, 3, 263. **44.** 2^9, 11. **45.** 19,479. **46.** Prime. **47.** 2, 2, 31, 53.

Page 41.—**1.** 3. **2.** 4. **3.** 3. **4.** 2 and 3. **5.** 15. **7.** 3. **8.** 31. **9.** 24. **10.** 14. **11.** 5. **12.** 25. **13.** 39.

Page 42.—**14.** 36. **15.** 3. **16.** 1. **17.** 7. **18.** 4. **19.** 2 and 1. **20.** 1 and 7.

Page 43.—**21.** 14.

Page 44.—**22.** 8. **23.** 32. **24.** 45. **25.** 64. **26.** $50.

Page 45.—**1.** 72. **2.** 72. **3.** 108. **4.** 64. **5.** 432. **6.** 2,400. **7.** 3,600. **8.** 4,480. **9.** 10,692. **10.** 19,824. **11.** 462. **12.** 148,302. **13.** 2,940.

Page 47. — **14.** 1,200. **15.** 3,600. **16.** 7,260. **17.** 270,300. **18.** 900. **19.** 58,750. **20.** 35,625. **21.** 10,010. **22.** 108,000. **23.** 327,600. **24.** 1,026. **25.** 1,640. **26.** 576. **27.** 1,716. **28.** 57,531. **29.** 7,770.

Page 48. — **30.** 77,216. **31.** 186,656. **32.** 26,307,572. **33.** 349,419,200. **34.** 9,442,368. **35.** 1,038,007. **36.** 252,121. **37.** 33,410,276. **38.** 876,096. **39.** 6,319,614. **40.** 1,101,100. **41.** 7,776. **1.** 2,961. **2.** 6,480. **3.** 91.

Page 49. — **4.** 64 a. **5.** One hundred forty-two and nine twenty-firsts. **6.** 3 days. **7.** 151. **8.** (1) 4,452 bu.; (2) 1,364$\frac{16}{138}$ lb. **9.** 1. **10.** $1,885.44. **11.** $3,780. **12.** 800 panels. **13.** 151. For Subtractional Division, the factor found is the quotient. For Partitive Division the factor found is the divisor. **14.** 555. **15.** 5,982. **16.** VI.

Page 50. — **17.** 17 and 59. **18.** $5,000. **19.** $10,000. **20.** $10,500. **21.** 107,307. **22.** **24.** 299 head. **25.** $5. **26.** 9. **27.** (1) 64 bales; (2) 57,600 lb. **28.** 310. **29.** 729. **30.** 3125.

Page 51. — **31.** 144. **32.** 32. **33.** 3,003. **34.** 1,000,001. **35.** 2,002. **36.** 1. **37.** 188 posts. **38.** $5. **39.** 12,725. **40.** $\frac{1}{12}$.

Page 58. — **1.** $\frac{15}{5}$. **2.** $\frac{56}{8}$, $\frac{70}{10}$. **3.** $\frac{189}{21}$, $\frac{198}{22}$. **4.** $\frac{198}{33}$, $\frac{96}{16}$. **5.** 40 half-dollars; 80 quarter-dollars. **6.** $\frac{272}{9}$, $\frac{1728}{16}$. **7.** $\frac{88}{4}$. **8.** 11,179 twelfths. **9.** $\frac{905}{3}$. **10.** $\frac{921}{16}$.

Page 59. — **11.** $\frac{51}{15}$. **12.** 812 lb. **13.** 411. **14.** 7. **15.** 8. **16.** 9. **17.** 9. **18.** 12. **19.** 17. **20.** $7\frac{1}{9}$.

Page 60. — **21.** 11. **22.** $66\frac{6}{9}$. **23.** 6. **24.** 2. **25.** $32\frac{2}{5}$. **26.** 342,621 twenty-fifths. **27.** $\frac{2125}{7}$. **28.** $\frac{180018}{17}$. **29.** $\frac{1}{2}$. **30.** $\frac{2}{3}$.

Page 61. — **31.** $\frac{3}{5}$. **32.** $\frac{2}{3}$; $\frac{2}{3}$; $\frac{7}{9}$; $\frac{14}{27}$; $\frac{3}{5}$; $\frac{2}{3}$; $\frac{2}{3}$.
33. $\frac{1}{20}$; $\frac{1}{10}$; $\frac{1}{5}$; $\frac{1}{4}$; $\frac{24}{25}$; $\frac{41}{50}$; $\frac{11}{20}$; $\frac{17}{20}$. **34.** $\frac{7}{11}$; $\frac{11}{13}$; $\frac{13}{23}$; $\frac{31}{37}$; $\frac{1}{32}$; $\frac{1}{13}$; $\frac{1}{13}$.
35. $3\frac{1}{3}$; $4\frac{1}{3}$; $8\frac{1}{2}$; $11\frac{4}{11}$; $21\frac{1}{7}$; $9\frac{2}{3}$; $25\frac{7}{12}$. **36.** $\frac{7}{11}$. **37.** 5,000.
38. 14 fr. units each $\frac{1}{25}$. **39.** $\frac{1}{5}$. **40.** $\frac{150}{180}$. **41.** $\frac{155}{160}$. **42.** $\frac{217}{224}$.
43. $\frac{90}{320}$.

Page 62. — **44.** $\frac{48}{64}$. **45.** $\frac{79}{1264}$. **46.** $\frac{14}{21}$. **47.** $\frac{36}{24}$. **48.** 75 fr. units. **49.** $\frac{48}{64}$. **50.** $\frac{15}{30}$; $\frac{20}{30}$; $\frac{4}{30}$. **51.** 1 dime; 4 dimes; 5 dimes. **52.** 19 pk.; 34 pk.; 25 pk. **53.** $\frac{8}{12}$; $\frac{9}{12}$; $\frac{10}{12}$. **54.** $\frac{63}{84}$; $\frac{70}{84}$; $\frac{36}{84}$. **55.** $\frac{32}{40}$; $\frac{30}{40}$; $\frac{28}{40}$.

Page 63. — **56.** $\frac{30}{60}$; $\frac{20}{60}$; $\frac{15}{60}$; $\frac{12}{60}$. **57.** $\frac{27}{42}$; $\frac{20}{42}$; $\frac{35}{42}$; $\frac{21}{42}$.
58. $\frac{36}{75}$; $\frac{55}{75}$; $\frac{60}{75}$; $\frac{50}{75}$. **59.** $\frac{8}{108}$; $\frac{42}{108}$; $\frac{45}{108}$; $\frac{96}{108}$. **60.** $\frac{70}{336}$; $\frac{63}{336}$; $\frac{48}{336}$; $\frac{168}{336}$.
61. $\frac{48}{84}$; $\frac{70}{84}$; $\frac{63}{84}$; $\frac{56}{84}$. **62.** $\frac{588}{924}$; $\frac{792}{924}$; $\frac{770}{924}$; $\frac{693}{924}$. **63.** $\frac{27}{6}$; $\frac{31}{6}$; $\frac{4}{6}$.
64. $\frac{200}{60}$; $\frac{255}{60}$; $\frac{312}{60}$. **65.** $\frac{154}{63}$; $\frac{225}{63}$; $\frac{67}{63}$. **66.** $\frac{1638}{504}$; $\frac{1080}{504}$; $\frac{1575}{504}$; $\frac{2072}{504}$.
67. $\frac{692}{56}$; $\frac{170}{56}$; $\frac{441}{56}$. **68.** $\frac{14}{32}$; $\frac{8}{32}$. **69.** $\frac{261}{63}$; $\frac{14}{63}$. **70.** 600.

Page 64. — 1. $\frac{1}{2}$. 2. $2\frac{23}{60}$. 3. $\frac{13}{24}$. 4. 3. 5. $2\frac{177}{280}$. 6. $1\frac{1}{16}$. 7. $19\frac{1}{12}$. 8. $35\frac{1}{8}$. 9. $30\frac{9}{32}$. 10. $3\frac{1}{4}$ mi. 11. $1\frac{7}{16}$. 12. $3\frac{1}{16}$.

Page 65. — 13. $1\frac{1}{12}$. 14. $6{,}320\frac{1}{21}$. 15. $\frac{25}{42}$. 16. $803\frac{3}{4}$. 17. $\frac{91}{132}$. 18. MCCCXIX. 19. 87 sixtieths. 20. $2.02. 21. $1{,}442\frac{491}{1848}$. 22. $2{,}280\frac{611}{672}$. 23. $\frac{4951}{10296}$. 24. $2\frac{293}{504}$. 25. $3\frac{251}{252}$. 26. $752\frac{1}{4}$. 27. $1{,}128\frac{257}{1386}$. 28. $3{,}059\frac{2}{819}$. 29. $992\frac{11}{24}$. 30. $419\frac{7}{18}$. 31. $\$150\frac{19}{20}$. 32. $1\frac{473}{504}$. 33. $2\frac{23}{24}$. 34. $1\frac{13954}{21879}$. 35. $30\frac{53}{198}$. 36. $\$621\frac{3}{20}$. 37. $\$92\frac{3}{20}$. 38. $5\frac{29}{32}$. 39. $4\frac{40}{81}$. 40. $5\frac{19}{50}$. 41. $6\frac{31}{96}$. 42. $77\frac{25}{84}$. 43. $120\frac{7}{8}$. 44. $54\frac{7}{9}$. 45. $40\frac{19}{25}$. 46. $60\frac{3}{5}$. 47. $65\frac{3}{32}$. 48. $\$768\frac{3}{5}$.

Page 66. — 1. $\frac{23}{144}$. 2. $\frac{9}{32}$. 3. $\frac{11}{63}$. 4. $\frac{1}{12}$. 5. $\frac{3}{32}$. 6. $\$74\frac{3}{4}$. 7. $\$\frac{13}{20}$. 8. $\frac{1}{12}$. 9. $69\frac{3}{10}$. 10. $300\frac{35}{117}$.

Page 67. — 11. 1. 12. $1\frac{2003}{2100}$. 13. $\frac{1}{18}$. 14. $\frac{3}{8}$. 15. $\$2\frac{3}{4}$. 16. $\frac{17}{136}$. 17. $\$.08\frac{7}{8}$. 18. $1\frac{19}{24}$. 19. $\$6{,}646\frac{1}{20}$. 20. Ten thousand one and ten twenty-firsts. 21. $1\frac{43}{63}$. 22. $91\frac{29}{604}$. 23. $10\frac{58}{63}$.

Page 69. — 1. $9. 2. $28. 3. 12. 4. $12. 5. $8.75. 6. $14. 7. $\$20\frac{1}{4}$. 8. $\$13\frac{1}{2}$. 9. $\$26\frac{1}{4}$. 10. $42\frac{3}{4}$. 11. $94\frac{1}{2}$.

Page 70. — 12. 111. 13. $138\frac{1}{4}$. 14. $248\frac{5}{8}$. 15. $418\frac{2}{7}$. 16. $665\frac{7}{9}$. 17. $1{,}041\frac{3}{4}$. 18. $1{,}641\frac{1}{8}$. 19. $2{,}081\frac{7}{32}$. 20. $3{,}321\frac{1}{2}$. 21. $1{,}158\frac{3}{16}$. 22. 1,790. 23. $2{,}723\frac{5}{32}$. 24. $8{,}779\frac{3}{4}$. 25. $15{,}881\frac{5}{8}$. 26. $34{,}837\frac{15}{32}$. 27. $40{,}123\frac{3}{7}$. 28. $15{,}216\frac{3}{4}$. 29. $6{,}234\frac{19}{32}$. 30. 15 mi. 31. 4,224 ft. 32. 84 lb. 33. 250 lb. 34. 154 lb. 35. $352\frac{1}{2}$ lb. 36. 202 lb. 37. 315 lb. 38. 664 lb. 39. 707 lb. 40. $31.56.

Page 71. — 41. $128\frac{1}{4}$. 42. $154\frac{7}{8}$. 43. 147. 44. $1{,}179\frac{3}{7}$. 45. $826\frac{3}{8}$. 46. $1{,}347\frac{7}{8}$. 47. 729. 48. 1,093. 49. 1,743. 50. $2{,}767\frac{1}{2}$. 51. 300. 52. $528\frac{1}{8}$. 53. $177\frac{3}{4}$. 54. $207\frac{1}{2}$. 55. $2{,}008\frac{3}{4}$. 56. $\frac{4}{9}$. 57. $\frac{1}{4}$. 58. $\frac{21}{32}$ bu. 59. $\frac{13}{40}$. 60. $\frac{8}{15}$ mi. 61. $\frac{64}{147}$. 62. $12\frac{1}{2}$. 63. $14\frac{5}{8}$; 7. 64. $11\frac{3}{32}$; $12\frac{2}{7}$. 65. $21\frac{3}{32}$; $28\frac{1}{8}$. 66. $\frac{2}{7}$. 67. $\frac{3}{1568}$. 68. $\frac{49049}{810000}$. 69. $\frac{128}{1575}$. 70. $\frac{1232}{6045}$. 71. $\frac{1}{8640}$.

Page 72. — 72. $321\frac{5}{8}$. 73. $325\frac{7}{8}$. 74. $189\frac{7}{8}$. 75. $1{,}400\frac{7}{16}$. 76. $954\frac{14}{25}$. 77. $204\frac{3}{16}$. 78. $212\frac{16}{49}$. 79. $42\frac{3}{8}$. 80. $181{,}051\frac{17}{32}$.

Page 73. — 1. $\frac{3}{16}$. 2. $\frac{3}{20}$. 3. $\frac{8}{27}$ A. 4. $\frac{7}{48}$. 5. $\frac{13}{80}$. 6. $\frac{17}{192}$. 7. $\frac{1}{56}$. 8. $\frac{3}{32}$. 9. $\frac{3}{112}$. 10. $\frac{1}{28}$. 11. $\frac{4}{63}$. 12. $\frac{2}{21}$. 13. $\frac{5}{112}$. 14. $\frac{5}{64}$. 15. $\frac{5}{49}$. 16. $\frac{1}{16}$. 17. $\frac{3}{100}$. 18. $\frac{8}{121}$. 19. $\frac{9}{256}$. 20. $\frac{5}{81}$. 21. $\frac{9}{16}$.

Page 74. — 22. $1{,}750\frac{11}{32}$. 23. $8\frac{1}{8}$ bu. corn. 24. $12\frac{5}{8}$. 25. $16\frac{7}{9}$. 26. $20\frac{5}{9}$. 27. $31\frac{1}{3}$ ft. 28. $14\frac{3}{40}$. 29. $11\frac{10}{63}$. 30. $69\frac{1}{8}$. 31. $300\frac{27}{40}$. 32. $112\frac{23}{135}$. 33. $333\frac{17}{36}$. 34. $111{,}151\frac{35}{36}$. 35. $\frac{7}{40}$. 36. $\$\frac{8}{15}$. 37. $83\frac{109}{324}$. 38. $778\frac{59}{98}$. 39. $340\frac{2}{225}$. 40. $410\frac{1}{120}$. 41. $16\frac{7}{80}$. 42. $111\frac{37}{63}$. 43. $891\frac{25}{32}$. 44. $17\frac{3}{70}$. 45. $322\frac{3}{35}$. 46. $612\frac{68}{117}$. 47. $9\frac{43}{80}$. 48. $115\frac{55}{128}$.

49. $800\frac{27}{100}$. **50.** $10\frac{3}{80}$. **51.** $166\frac{121}{162}$. **52.** $71\frac{709}{1250}$. **53.** $300\frac{19}{160}$. **54.** $200\frac{311}{1250}$. **55.** $45\frac{311}{1600}$.

Page 75.—**56.** $57\frac{3}{5}$ min. **57.** $96\frac{3}{7}$ bu. **58.** $187\frac{1}{2}$ bu. **59.** 175 bu. **60.** $17\frac{1}{2}$ times. **61.** 256 cartridges.

Page 76.—**62.** 50 bu. **63.** $43\frac{3}{4}$ bu. **64.** $46\frac{2}{3}$. **65.** 96. **66.** $281\frac{1}{4}$. **67.** $286\frac{2}{3}$. **68.** 144. **69.** $35\frac{3}{7}$. **70.** $93\frac{5}{7}$. **71.** $295\frac{3}{4}$. **72.** 328. **73.** 152. **74.** 114.

Page 77.—**75.** 240. **76.** $217\frac{1}{7}$. **77.** 1,120. **78.** $3{,}626\frac{2}{3}$. **79.** $151\frac{1}{2}$. **80.** 459. **81.** $25\frac{3}{5}$. **82.** $2{,}154\frac{2}{3}$. **83.** $508\frac{1}{2}$. **84.** $158\frac{5}{8}$. **85.** $10\frac{2}{3}$. **86.** $21\frac{1}{3}$. **87.** $106\frac{2}{3}$. **88.** 120. **89.** 240. **90.** 4,800. **91.** 960. **92.** 150. **93.** 8 times. **94.** $2\frac{2}{3}$. **95.** $12\frac{1}{4}$. **96.** $1\frac{1}{63}$. **97.** $\frac{5}{9}$. **98.** $\frac{24}{35}$. **99.** $\frac{14}{15}$. **100.** Twice. **101.** Twice. **102.** Twice. **103.** $\frac{1}{2}$. **104.** Twice. **105.** 2. **106.** $\frac{1}{2}$. **107.** 7 times. **108.** 5 half dollars.

Page 78.—**109.** 14. **110.** $6\frac{3}{4}$. **111.** 2. **112.** $1\frac{7}{8}$. **113.** $1\frac{7}{48}$. **114.** $\frac{70}{81}$. **115.** 2. **116.** $\frac{2}{3}$. **117.** $3\frac{5}{9}$. **118.** $4\frac{1}{8}$. **119.** $4\frac{1}{3}$. **120.** $13\frac{2}{3}$. **121.** $8\frac{3}{4}$. **122.** $22\frac{1}{2}$. **123.** $1\frac{9}{25}$. **124.** $\frac{124}{171}$. **125.** $1\frac{47}{124}$. **126.** 5. **127.** $\frac{1}{5}$. **128.** $1\frac{17}{39}$. **129.** $1\frac{7}{8}$. **130.** $1\frac{1}{20}$. **131.** $1\frac{1}{48}$. **132.** $\frac{9}{16}$. **133.** $\frac{1}{2}$. **134.** $1\frac{53}{54}$. **135.** $\frac{1}{5}$. **136.** 6 times. **137.** 3 times. **138.** $\frac{1}{2}$. **139.** $\frac{9}{16}$. **140.** $\frac{9}{50}$. **141.** $1\frac{7}{8}$. **142.** $\frac{289}{324}$. **143.** $\frac{2}{3}$. **144.** $1\frac{97}{128}$. **145.** $\frac{128}{225}$. **146.** 302.

Page 79.—**147.** $\frac{3}{14}$. **148.** 20. **149.** 5. **150.** $\frac{6}{7}$. **151.** $1\frac{47}{72}$. **152.** 8. **153.** $\frac{1}{3}$. **154.** $2\frac{2}{3}$. **155.** $1\frac{1}{7}$. **156.** $1\frac{1}{2}$. **157.** 6. **158.** $\frac{1}{3}$. **159.** 2. **160.** $5\frac{1}{4}$. **161.** 15. **162.** $\frac{1}{15}$. **163.** $\frac{3}{7}$. **164.** $\frac{1}{4}$. **165.** $8\frac{1}{3}$. **166.** $\frac{1}{10}$. **167.** $3\frac{1}{3}$. **168.** $\frac{5}{9}$. **169.** $13\frac{1}{3}$. **170.** $3\frac{1}{3}$. **171.** $\frac{1}{20}$. **172.** $\frac{16}{27}$. **173.** $\frac{1}{15}$. **1.** $\frac{3}{640}$. **2.** 0. **3.** (1) $3\frac{49}{120}$; (2) $2\frac{85}{504}$. **4.** $\frac{1}{3}$. **5.** $1\frac{17}{72}$. **6.** 3.

Page 80.—**7.** $\frac{1}{5421}$. **8.** $\frac{4}{5}$; $\frac{15}{16}$. **9.** $1\frac{1}{6}$. **10.** $\frac{7}{80}$. **11.** $\frac{6}{7}$; $\frac{5}{7}$. **12.** $1\frac{23}{40}$. **13.** 2. **14.** $\frac{189}{320}$. **15.** $\frac{3}{4}$. **16.** $\frac{2}{5}$. **17.** $\frac{1}{3}$. **18.** $\frac{3}{10}$. **19.** $1\frac{13}{36}$. **20.** $1\frac{4}{5}$. **21.** $\frac{1}{15}$. **22.** $1\frac{17}{63}$. **23.** $6\frac{3}{7}$. **24.** $1\frac{79}{369}$. **25.** $1\frac{1}{6}$. **26.** $\frac{3}{112}$. **27.** $\frac{1}{3}$. **28.** $\frac{2}{15}$. **29.** $\frac{1}{30}$. **30.** $1\frac{17}{46}$. **31.** $1\frac{178}{7360}$. **32.** $5\frac{25}{27}$. **33.** $\frac{5}{147}$. **34.** $\frac{1}{3}$. **35.** $\frac{1}{6}$. **36.** $\frac{1}{48}$. **37.** 5. **38.** $\$10\frac{1}{2}$. **39.** $\$17\frac{1}{5}$. **40.** $\frac{1}{2}$.

Page 81.—**41.** $\frac{231}{248}$. **42.** $9\frac{1}{2}$. **43.** 2,715 fifths. **44.** $\$7\frac{13}{16}$. **45.** $10\frac{2}{3}$. **46.** $213\frac{1}{3}$. **47.** 1,000 eighths. **48.** $\frac{1}{200}$. **49.** $\frac{1}{10}$. **50.** 625. **51.** $\frac{1}{140}$. **52.** $\frac{315}{512}$. **53.** $\frac{4}{7}$. **54.** 17 da. **55.** $1\frac{3}{4}$. **56.** $\$425\frac{17}{40}$. **57.** $1\frac{3}{4}$. **58.** \$646.67+. **59.** $2\frac{417}{512}$. **60.** 55¢. **61.** $2\frac{19}{128}$. **62.** $158\frac{5}{8}$ a. **63.** $\frac{81}{128}$. **64.** $\frac{50}{81}$. **65.** $\$4\frac{1}{20}$. **66.** $\$17\frac{23}{32}$ or \$17.71+.

Page 82.—**67.** $27\frac{218}{225}$. **68.** \$31.52+. **69.** $6\frac{32}{243}$. **70.** \$9.11+. **71.** $\frac{9}{16}$. **72.** $\frac{9}{16}$. **73.** $9\frac{9}{14}$. **74.** \$24.33+. **75.** $\frac{3}{50}$. **76.** $\frac{3}{8}$ mile per min. **77.** 1,320 yd. **78.** $\frac{2}{5}$. **79.** $\frac{375}{2048}$. **80.** $42{,}461\frac{1}{4}$ lb. **81.** 98,000 lb. **82.** $\frac{5}{6}$. **83.** 440 ft. **84.** 6 da. **85.** $2\frac{1}{3}$ da.

Page 83.—86. $8\frac{21}{32}$ a. 87. $\frac{3}{8}$. 88. $133\frac{1}{3}$ a. 89. $\frac{5}{42}$. 90. $724\frac{10}{71}$ lb. 91. 17,138 lb. 92. $\frac{8}{15}$ bu. wheat. 93. $\frac{1}{8}$. 94. $5\frac{1}{7}$ da. 95. $20\frac{3}{4}$ bu. 96. $\$8\frac{8}{15}$. 97. $\frac{55}{256}$. 98. $1\frac{1}{3}$ lb. 99. $28\frac{4}{7}$ mi. 100. $\frac{373}{42336}$.

Page 87.—3. $\frac{1}{8}$. 4. 10^2; $\frac{1}{25}$. 5. $\frac{2}{25}$; $\frac{4}{25}$; $\frac{37}{200}$; $\frac{27}{50}$; $\frac{31}{200}$. 6. $\frac{41}{400}$; $\frac{403}{2000}$; $\frac{1}{80}$; $\frac{1203}{2000}$; $\frac{2}{125}$. 7. $\frac{17}{5000}$; $\frac{67}{5000}$; $\frac{101}{2000}$; $\frac{3203}{20000}$; $\frac{323}{20000}$. 8. $\frac{31}{32}$; $\frac{3}{800}$; $\frac{3}{4000}$; $\frac{5}{32}$; $\frac{5}{8}$. 9. $\frac{5}{16}$; $\frac{7}{16}$; $\frac{1}{16}$; $\frac{15}{16}$; $\frac{3}{80}$. 10. $\frac{5}{6}$. 11. $\frac{1}{3}$ 12. $\frac{2}{3}$. 13. $\frac{1}{6}$. 14. $\frac{5}{9}$. 15. $\frac{1}{15}$. 16. $\frac{15}{16}$; $\frac{1}{15}$; $\frac{1}{12}$; $\frac{7}{16}$; $\frac{1}{16}$; $\frac{1}{8}$. 17. .5.

Page 88.—18. .18. 19. .75; .12. 20. .6; .075; .1875. 21. .575; .4875; .875. 22. .192; .59375; .5625. 23. .175. 24. .45. 25. 5.15. 26. 4.35. 27. 48.5625.

Page 89.—28. 50.3333+. 29. .6666+. 30. .4285+; .8888+; .8333+; .08333+. 31. .9166+; .1176+; .2727+; .4444+; .1538+. 1. $8.20. 2. 26.1111. 3. 34.31865. 4. 286.00751. 5. 11.170155. 6. 142.99625. 7. 172.1565. 8. 4016.8125.

Page 90.—1. $4.95. 2. .396. 3. .1544. 4. 2.0075. 5. .005. 6. 1 thousandth. 7. 1 thousandth. 8. .025. 9. 1 thousandth. 10. 4. 11. 2.104685. 1. $\frac{1}{100}$. 2. $\frac{1}{1000}$. 3. $\frac{1}{10000}$.

Page 91.—4. $4\frac{829}{1000}$. 5. 30.68105. 6. 12.5678; 125.678; 1256.78. 7. $\$51.21\frac{1}{4}$. 8. $\$24.76\frac{1}{4}$. 9. 76.4465625. 10. 101.0101. 11. 924.7474655. 12. 916.50714155. 13. 25.76721875. 14. .000729. 15. 43.8729. 16. 1.59375. 1. $22.50.

Page 92.—2. $98.25. 3. 10; 1; 10; 100. 6. 3.21749. 7. 3.007. 8. 54.312.

Page 93.—9. 321.2. 10. 2.274. 11. .000032. 12. 3,200. 13. .48 14. 18.56. 15. .43. 16. 34,500. 17. 12.0428+. 18. 2.067+. 19. .024. 20. .04. 21. 4. 22. 800. 23. 13.7142+. 24. 55. 25. 100. 26. 10,000. 27. .0005. 28. 100. 29. 330,000. 1. $.555\frac{5}{9}$. 2. $\frac{367}{400}$. 3. $1.31\frac{1}{4}$. 5. $11\frac{1023}{2000}$. 6. $7.731\frac{2}{3}$. 7. 1.375. 8. 8.75. 9. 14.4. 10. 31.25. 11. .75. 12. .08. 13. 6.875. 14. 2.16. 15. 46.8. 16. 1.6875. 17. 2.578125. 18. 12.625. 19. 7.6875. 20. .054. 21. .25. 22. 2.165.

Page 94.—1. 21. 2. 49. 3. $302.25. 4. .046875. 5. $9.24+. 6. $1,000. 7. $245.55. 8. $.60. 9. 594. 10. 594. 11. $26.35. 12. $3.75. 13. $218.50. 14. 490 lb. 15. $20\frac{4}{7}$ bu. 16. 272 A. 17. $28\frac{4}{7}$ bu. 18. 0.

Page 95.—19. $46\frac{2}{3}$ bu. 20. $4.57+. 21. 256 lots. 22. $3,693.75; $41,616.25. 23. 56 B/C. 24. 800 Mex. D. 25. $9\frac{2}{3}$ hr. 26. 10,000 lb. 27. $572.78+. 28. $126. 29. $19,800. 30. $1.96.

Page 96.—**31.** 105 lb. **32.** 497 lb. **33.** 50 lb. **34.** 15,545 nails. **35.** \$614.46. **36.** $8\frac{1}{3}$ tons. **37.** 13,068 stalks. **38.** \$12.15. **39.** 3,136 lb. **40.** 50 bu. corn. **41.** $16\frac{4}{11}$ bu. **42.** \$2.40. **43.** \$13.17+. **44.** \$31.50.

Page 97.—**45.** \$81,000. **46.** \$16.57+. **47.** \$3. **48.** (B) \$.74. **49.** A \$.03. **50.** $31\frac{1}{3}$ bu. **51.** 32 A. **52.** $6\frac{2}{3}$ more days. **53.** \$1,120. **54.** 24¢. **55.** \$3.60.

Page 108.—**1.** 102 in. **2.** 437 pt. **3.** 5,026 d. **4.** 253 gi. **5.** 124,884 oz. **6.** 9,492 gr. **7.** 22,532 gr. **8.** 261,392 in. **9.** 8,079, 1. **10.** 416,070 sq. rd. **11.** 138,240 A. **12.** 1,399,680 cu. in. **13.** 31,622,400 sec. **14.** 650,440″. **15.** 41,760 min. **16.** 28,800 sheets. **17.** 1,617 cu. in.

Page 109.—**18.** 2 bu., 2 pk., 3 qt., 1 pt. **19.** 8*s.* $4\frac{1}{2}$*d.* **20.** 9 gal., 3 qt., 1 pt., 1 gi. **21.** 21 T., 10 cwt., 90 lb. **22.** 2 lb., 5 oz., 10 pwt., 11 gr. **23.** 3 mi., 8 rd. **24.** 2 bbl., 9 gal., 1 qt., 1 pt. **25.** 2 mi., 40 rd. **26.** 4 rd. **27.** 1,440 rd. **28.** 7,051 yd. **29.** 1 T., 3 cwt., 27 lb. **30.** 220 yd. **31.** 1 mi., 41 rd., 1 yd., 6 in. **32.** 5,960 ft.

Page 110.—**33.** 7 rd., 1 ft., 6 in. **34.** 4 sq. rd. **35.** 242 sq. yd. **36.** 14 rd., 2 yd. **37.** 14 rd., 3 yd. **38.** 45 sq. rd., 27 sq. yd., 5 sq. ft., 108 sq. in. **39.** 79,200 sq. in. **40.** 185,945 cu. in. **41.** 9,292,800 sq. yd. **42.** 1,089 sq. ft. **43.** 14 rd., 4 yd. **44.** 15 rd., 1 ft., 6 in. **45.** 2,988 in. **46.** 62 sq. rd., 13 sq. yd., 3 sq. ft., 72 sq. in. **47.** 17,000 sq. ft. **48.** ———. **49.** $134,536\frac{1}{4}$ sq. ft.

Page 111.—**50.** 5 A., 141 sq. rd., $3\frac{1}{4}$ sq. yd. **51.** 1,500 gal. **52.** 64 oz. **53.** 4 lb. **54.** \$15. **55.** 2,880 Av. oz. **56.** 525 Tr. lb. **57.** $53\frac{1}{3}$ sq. ch. **58.** 14,964.78 cu. in. **59.** 870 sq. ft. **60.** 210 cu. in. **61.** 94 rd., 3 yd. **62.** 29 sq. rd., 22 sq. yd., 6 sq. ft., 108 sq. in. **63.** 27,648 qts. **64.** 80 A. **65.** \$44. **66.** \$137.70. **67.** \$530. **68.** \$148.50. **69.** $2,430\frac{2}{3}$ Tr. lb. **70.** 1 mi., 24 rd., 1 yd. **71.** 1,805 yd. **72.** 64,776 in. **73.** 1,320 yd.

Page 112.—**74.** 108,900 sq. ft. **75.** $\frac{2}{3}$°. **76.** $\frac{3}{4}$ da. **1.** $\frac{4}{5}$ qt. **2.** $\frac{1}{32}$ gal. **3.** $\frac{1}{1280}$ sq. mi. **4.** $\frac{1}{84000}$ T. **5.** $\frac{1}{48}$ da. **6.** $\frac{3}{8}$ hr. **7.** $\frac{7}{288}$ gal. **8.** $\frac{1}{90}$°. **9.** $\frac{5}{6}$ da. **10.** 2 hr., 40 min. **11.** 10 hr., 40 min. **12.** 101 sq. rd., 24 sq. yd., 6 sq. ft., 108 sq. in. **13.** 7*s.* 6*d.* **14.** 6 oz., 13 pwt., 8 gr. **15.** 125 lb. **16.** 782 yd., 8 in. **17.** 4 da., 16 hr. **18.** 40 da., 16 hr. **19.** 162 da., 5 hr., 20 min. **20.** 3 pk., $2\frac{2}{3}$ qt. **21.** 133 lb., $5\frac{1}{3}$ oz.

Page 113.—**22.** $\frac{59}{64}$ bu. **23.** $\frac{1}{8}$ mi. **24.** $\frac{13}{18}$ yd. **25.** $\frac{1}{3}$ da. **26.** $\frac{13}{72}$ da. **27.** $\frac{1}{8}$ T. **28.** $\frac{11}{75}$ yr. **29.** $\frac{43}{55}$ rd. **30.** $\frac{1}{8}$ wk. **31.** $\frac{7}{8}$ wk.

32. $\frac{31}{36}$ yd. 33. $\frac{5}{63}$ of 3 wk. 34. $\frac{7}{10}$ of 5 mi. 35. $\frac{1}{20}$ yr. 36. $\frac{37}{732}$ yr.
37. $\frac{1}{12}$ of $15. 38. .15,625 of 2 bu. 39. 1,320 yd. 40. $\frac{7}{8}$ mi.
41. $\frac{1}{32}$ da. 42. $\frac{19}{80}$ mi. 43. $\frac{3}{242}$ A. 44. .9,375 gal. 45. .875 gal.
46. .5,625 bu. 47. .68 hr. 48. .1 da. 49. .375 T. 50. .375 sq. ft.
51. $\frac{7}{8}$ sq. mi. 52. .125 cu. ft.

Page 114. — 1. 20 gal., 1 gi. 2. £43 9s. 5*d*.
3. 39 mi., 291 rd., 4 yd., 11 in. 4. 17 bu., 1 pk. 5. 19 T., 9 cwt.
6. 113 A., 44 sq. rd. 7. 8 yr., 41 da., 21 hr. 8. 55 rd., 4 yd., 2 ft., 6 in.
9. 369 A., 71 sq. rd., 2 sq. yd., 2 sq. ft., 108 sq. in.

Page 115. — 10. 3,080 ft. 11. 15,939 in. 12. 175$\frac{1}{2}$ in.
13. 3 mi., 292 rd., 2 yd., 2 ft., 6 in. 14. 18 lb., 6$\frac{1}{2}$ oz. 15. 194 lb., 1 oz.
16. 1 bu., 3 pk., 4 qt. 17. 2 A., 123 sq. rd., 27 sq. yd., 1 sq. ft., 99 sq. in.
18. 2 bu., 1 pk., 5 qts. 19. 8 hr., 20 min. 20. 83 T., 14 cwt., 50 lb.
21. 7$\frac{529}{640}$ mi. 22. 122°, 26′, 15″. 23. 8.265,625 bu. 24. 1.273 + mi.
25. 67.883$\frac{1}{3}$ da.

Page 116. — 1. 4 yd., 6 in. 2. 4 yd., 6 in.
3. 7 mi., 309 rd., 4 yd., 1 ft., 6 in. 4. 2 pk., 2 qt. 5. 18 A., 120 sq. rd.
6. 20 da., 18 hr. 7. 1 rd., 2 yd., 2 ft. 8. 69 A., 20 sq. rd. 9. 896 sq. mi.
10. 1 T., 12 cwt., 16 lb.

Page 117. — 11. 23 gal., 2 qt., 1 pt. 12. 990 ft. 13. 1 ft., 9 in.
14. 9 min., 50 sec. 15. 582$\frac{5}{16}$ sq. yd. 16. 5 mo., 20 da.
17. 116 yr., 7 mo., 18 da. 18. 30 yr., 5 mo., 17 da.
19. 1 cwt., 3 lb., 8$\frac{9}{10}$ oz. 20. 2 bu. 21. 1.6,875 gal. 22. 3.124 T.
23. 53 A., 129$\frac{1}{2}$ sq. rd. 24. 1,135$\frac{3}{4}$ yd.

Page 118. — 1. 47 bu., 2 pk. 2. 37 mi., 170 rd., 3 yd.
3. 91 A., 91 sq. rd., 24 sq. yd., 5 sq. ft., 36 sq. in. 4. 320 bu.
5. £11 7*s*. 6*d*. 6. $163.75. 7. $61.08. 8. 826$\frac{7}{8}$ bu. 9. 1,285$\frac{1}{8}$ bu.
10. 2516.71875 bu.; $2,139.21. 11. 609 A., 32 sq. rd.

Page 119. — 12. $3,009,375. 13. $689.85.
14. 47 mi., 40 rd., 2 yd., 2 ft., 8 in. 15. 63 bu., 1 pk., 4 qt.
16. 2 T., 9 cwt., 90 lb., 8 oz. 17. 2 T., 11 cwt., 11 lb., 5 oz. 18. $90.25.
19. 13 mi., 160 rd. 20. 718 bu., 2 pk., 3 qt. 21. 1 mi. 22. 11,296.25 lb.
23. 3,413.25 pk.; $494.92 +. 24. 91 rd., 3$\frac{1}{2}$ yd.

Page 120. — 1. 1 da., 12 hr., 41 min. 2. 5 da., 8 hr., 56 min., 6 sec.
3. 80 rd., 2 ft., 5$\frac{1}{7}$ in. 4. 2 cwt., 93 lb., 10$\frac{3}{4}$ oz. 5. 22 bu., 2 pk.
6. 1 bu., 2 pk., 1 qt. 7. 5. 8. $$\frac{22}{45}$. 9. $\frac{221}{1210}$ of garden.
10. 704 sheets. 11. 8 such strips.

Page 121. — 12. 1 mi., 2 rd., 2 yd., $\frac{4}{7}$ ft. 13. 3$\frac{1}{2}$ bu. 14. $\frac{3}{16}$ of 4 bu.
15. 6 hr., 40 min. 16. .23 of 2 T. 17. 88 A. 18. $\frac{101}{600}$ of 3 cwt.

19. $23\frac{127}{226}$. **20.** $\frac{1}{128}$ bu. **21.** 1 wk., 3 da., 16 hr., 27 min., 12 sec.
22. $\frac{13}{36}$ da. **23.** 38 rd., 2 yd., 2 ft. **24.** $\frac{13}{32}$ A. **25.** 1 T., $33\frac{3}{8}$ lb.
26. 5 qt., $1\frac{1}{3}$ pt. **27.** $56\frac{16}{19}$ A. **28.** 296 rd. **29.** $41\frac{5}{7}$ bu.
30. 1 mi., 104 rd., 2 yd., $11\frac{7}{19}$ in. **31.** 8,688 rails.

Page 122.—**1.** 1 hr., 15 min., 46 sec.

Page 123.—**2.** 56 min., 40 sec. **3.** 1 hr., 17 min., 24 sec.
4. 2 hr., 49 min., 56 sec. **5.** 1 hr., 20 min., 20 sec.
6. 7 hr., 3 min., 40 sec. **7.** 6 hr., 23 min., 4 sec. **8.** 11 hr., 39 min.
9. 1 hr., 5 min., 52 sec. **10.** 34°. **11.** 7°, 30′.

Page 124.—**1.** \$94.05. **2.** $.0297\frac{4}{101}$. **3.** 20 times. **4.** \$0.60 per C.
5. .5609. **6.** 264 panels. **7.** 3, 7, 11, 13. **8.** \$11. **9.** 90¢. per yd.
10. 400. **11.** 65¢. per cwt. **12.** .02536875. **13.** \$12.07+. **14.** 1,000.
15. $1\frac{1}{4}$ corn to 1 wheat. **16.** 3,600. **17.** \$6.25. **18.** .7333.

Page 125.—**19.** 33 rd., 1 yd., 1 ft., 2 in. **20.** 30 t. p. **21.** 396.
22. 60¢. per bu. **23.** \$88.75. **24.** $5\frac{3}{20}$. **25.** \$126. **26.** 5,280 ties.
27. 51,700,347. **28.** $39\frac{2}{7}$ bu. per A. **29.** .931. **30.** 50. **31.** 132 A.
32. $34\frac{4}{5}$ mi. per hr. **33.** $7\frac{65}{126}$. **34.** \$.08 per lb. **35.** 19,194.2 cu. in.

Page 126.—**36.** 18 hr., 40 min. **37.** $4{,}693\frac{1}{3}$. **38.** \$17.50.
39. $\$.02\frac{1}{2}$ per mi. **40.** 54 lb., $15\frac{1}{3}$ oz. **41.** \$4,455. **42.** \$5.50 per week.
43. 400. **44.** $14\frac{2}{7}$ mi. **45.** $1{,}115\frac{3}{7}$ A. **46.** 10,560. **47.** $\frac{83}{288}$ mi. per sec.
48. 30 chickens. **49.** 153. **50.** 1,000. **51.** 112.

Page 127.—**52.** \$22.50. **53.** 2 mi., 200 rd.
54. $\frac{2}{3}$ mi. per min; 40 mi. per hr.; $1\frac{1}{2}$ min. to the mile. **55.** 201.
56. 41 tickets. **57.** 15 wk., 3 da., 4 hr. **58.** \$127.63. **59.** $1\frac{602}{729}$.
60. \$.25 per cwt. **61.** 5,760 rails. **62.** 960 rails. **63.** Lost \$86.96.

Page 129.—**1.** 2. **2.** $\frac{6}{7}$. **3.** 56. **4.** $\frac{5}{9}$. **5.** 5. **6.** 10. **7.** 10.
8. $\frac{2}{3}$. **9.** $\frac{1}{4}$. **10.** 7. **11.** 4. **12.** 2. **13.** $\frac{2}{9}$. **14.** $3\frac{5}{9}$. **15.** $1\frac{7}{9}$. **16.** $\frac{8}{9}$.
17. 3. **18.** $\frac{7}{3}$; $2\frac{2}{9}$. **19.** 2. **20.** 10; 2. **21.** $1\frac{1}{3}$; $1\frac{2}{4}$.

Page 130.—**22.** $\frac{3}{4}$; $\frac{3}{7}$. **23.** $\frac{3}{5}$. **24.** $\frac{1}{2}$. **25.** $\frac{1}{3}$. **26.** $\frac{1}{4}$. **27.** $\frac{1}{5}$.
28. $\frac{1}{6}$. **29.** $\frac{1}{8}$. **30.** $\frac{1}{10}$. **31.** $\frac{1}{12}$; $\frac{1}{30}$. **32.** $\frac{1}{16}$; $\frac{1}{75}$. **33.** $\frac{1}{20}$; $\frac{1}{50}$.
1. \$24. **2.** \$19. **3.** \$42. **4.** \$123. **5.** \$81. **6.** \$204. **7.** \$24.
8. \$43. **9.** \$40. **10.** \$100. **11.** \$111. **12.** \$111. **13.** \$30.
14. \$30. **15.** \$20. **16.** \$20. **17.** \$40.

Page 131.—**18.** \$241. **19.** \$146. **20.** \$80. **21.** \$210. **22.** \$220.
23. \$14.25. **24.** \$28. **25.** \$21. **26.** \$3. **27.** \$8. **28.** \$15.
29. \$9.25. **30.** \$4. **31.** \$41. **32.** \$13. **33.** \$9. **34.** \$112.50.
35. \$93.75. **36.** \$217. **37.** \$30. **38.** \$225. **39.** \$225. **40.** \$45.
41. \$285. **42.** \$22.50. **43.** \$58.50. **44.** \$112.50. **45.** \$262.50.
46. \$20. **47.** \$84.

Page 132.—**48.** 2 hr., 9 min., 22 sec. **49.** 1 hr., 13 min., 4 sec. **50.** 1 hr., 25 min. **51.** 34°. **52.** 2 hr., 1 min., 4 sec. **53.** 67°, 30′. **54.** $\$14.06\frac{1}{4}$. **55.** 130°. **56.** $12.50. **57.** $30.

Page 133.—**1.** $\frac{1}{4}$. **2.** $\frac{1}{2}$. **3.** $2\frac{1}{2}$. **4.** $1\frac{1}{7}$. **5.** 3. **6.** $3\frac{1}{4}$; $4\frac{1}{5}$; $\frac{3}{19}$. **7.** $\frac{63}{64}$. **8.** 63. **9.** $\frac{1}{2}$; $\frac{1}{3}$. **10.** 3. **11.** $1\frac{1}{2}$. **12.** $6. **13.** 6. **14.** 18 mi. **15.** 3. **16.** 90 lb.

Page 134.—**17.** 6. **18.** 15 doz. **19.** $2\frac{1}{2}$; $6\frac{3}{4}$. **20.** $2\frac{1}{2}$. **21.** 60 ¢. **22.** 4. **23.** 4. **24.** 12. **25.** 10.

Page 135.—**1.** $\frac{12}{16}$. **2.** 12. **3.** 24. **4.** 21. **5.** 8. **6.** $\frac{40}{72}$. **7.** $\frac{21}{168}$. **8.** $8\frac{2}{11}$. **9.** $\frac{36}{54}$. **10.** $\frac{18}{81}$. **11.** 13. **12.** 24. **13.** 24. **14.** $\frac{162}{427}$ bu. **15.** 3. **16.** 200. **17.** 8. **18.** $12\frac{1}{2}$. **19.** 25. **20.** $10. **21.** $266\frac{2}{3}$. **22.** $22\frac{2}{9}$. **23.** $29\frac{1}{3}$. **24.** 8 bu. **25.** 12 bu. **26.** 320 mi. **27.** $4\frac{5}{7}$ da. **28.** 60 hr. **29.** 24 hr. **30.** 1 hr. **31.** $1\frac{1}{2}$ mi. **32.** $44\frac{1}{3}$.

Page 136.—**33.** (2d) $1\frac{7}{8}$ bu.; (4th) $2\frac{5}{8}$. **34.** $2\frac{1}{4}$; 48; 12. **35.** 20; 3; $1\frac{1}{5}$. **1.** 12 A. **2.** 5 times; 800 ft. **3.** $11\frac{3}{7}$ da.

Page 137.—**4.** 194 leap yr. **5.** $\$11\frac{1}{4}$. **6.** 450 bu. **7.** $\$3\frac{1}{5}$. **8.** $12\frac{1}{2}$ yd. **9.** $608\frac{1}{3}$ bu. **10.** $9\frac{3}{5}$ hr. **11.** $80\frac{5}{14}$ bu. **12.** $5.20. **13.** $71.25. **14.** $93\frac{1}{3}$ ft. **15.** 2,070 head. **16.** $9,900.

Page 138.—**17.** $562.50. **18.** $533\frac{1}{3}$ bu. **19.** $29\frac{1}{4}$ mi. **20.** $6\frac{2}{3}$ hr. **21.** $1\frac{3}{4}$ bu. **22.** 175 lb. **23.** 42 bales. **24.** 360 panels. **25.** $6\frac{2}{3}$ da. **26.** 220 bu. **27.** $\$\frac{14}{25}$. **28.** 32 ¢. **29.** 864 ties. **30.** $1,167.96. **31.** $4\frac{58}{70}$ a.

Page 139.—**32.** 29,400 bricks. **33.** 1,948,527 people. **34.** 100 bbl. **35.** $47\frac{1}{2}$ bu. **36.** $15\frac{3}{4}$ lb. **37.** $\frac{7}{18}$ of fence. **38.** $5{,}714\frac{2}{7}$ lb. **39.** $8\frac{3}{4}$ gal. **40.** $\$24\frac{1}{5}$. **41.** $\$8\frac{3}{4}$. **42.** 24,000 lb. **43.** 48 yd. **44.** $30. **45.** $6.40. **46.** 1 bu. **47.** 1,900 lb. **48.** 10 min. **49.** $\frac{25}{132}$. **1.** $180.

Page 141.—**2.** $18.

Page 142.—**3.** 2,430 bundles. **4.** $3\frac{3}{4}$ da. **5.** $4,000. **6.** $90. **7.** $54. **8.** 4 horses. **9.** 1,200 ties. **10.** $81. **11.** 6 da. **12.** $5\frac{2}{3}$ da. **13.** $27. **14.** $1.80. **15.** $\$41\frac{2}{3}$. **16.** $10.

Page 143.—**17.** $22\frac{1}{2}$ bu. **18.** $6\frac{3}{10}$ A. **1.** $1\frac{3}{5}$ bu. corn. **2.** $\frac{1}{20}$ T. hay. **3.** $\frac{1}{4}$ melon. **4.** $\frac{4}{7}$ da.

Page 144.—**1.** $312.50; $187.50. **2.** $480; $520. **3.** 285 A.; 75 A. **4.** $\$642\frac{6}{7}$; $\$357\frac{1}{7}$. **5.** $300; $600; $600. **6.** $400; $600; $600. **7.** $400; $600; $800. **8.** $1,500. **9.** $1,600. **10.** $1,200.

Page 145.—**11.** $\$1{,}111\frac{1}{9}$. **12.** $42; $35; $21. **13.** $960; $1,080; $1,200. **14.** $11,200; $5,600; $4,200. **15.** $1,200; $600; $1,050.

Page 146.—**16.** $360; $160; $210. **1.** $\frac{3}{25}$ bu. **2.** 7,040 rails. **3.** $1.60. **4.** 5,445. **5.** 12,400 gr. **6.** $636. **7.** 12. **8.** 350 mi.

9. N. O. 12.52 A.M. 10. 288 mi. 11. $1\frac{1}{8}$. 12. \$140. 13. 9. 14. 6 turkeys.

Page 147.—15. $158\frac{2}{3}$ furrows. 16. 264. 17. \$14. 18. $7\frac{7}{9}$ min. 19. 37.4 lb. 20. 1,920 palings. 21. 420 qt. 22. \$731.50. 23. 35 mi. 24. \$2,000. 25. 36, 45, and 63 trout. 26. 16 axes.

Page 148.—27. \$570. 28. $187\frac{1}{2}$ mi. 29. 7 P.M. 30. \$.56. 31. 10 cents. 32. \$$187.68\frac{3}{4}$. 33. $36\frac{3}{10}$ bu. 34. \$15. 35. \$87.50; \$90; \$87.50. 36. 14 hr., 43 min., 20 sec.

Page 149.—37. 1.45 P.M. 38. \$.07. 39. 459 lb. 40. 1,950 lb. 41. \$52.80. 42. \$120. 43. \$$.66\frac{2}{3}$; \$373.33. 44. $1,166\frac{2}{3}$. 45. 3 da. 46. \$.07.

Page 152.—1. $\frac{5}{100}$. 2. .06. 3. 7%. 4. $\frac{12}{100}$; .12. 5. $\frac{9}{100}$; .09. 6. $\frac{5}{100}$; .05. 7. 75%. 8. 30%. 9. $87\frac{1}{2}$%. 10. $37\frac{1}{2}$%. 1. \$24. 2. 77 A. 3. 24,750 people. 4. 1,215 lb.

Page 153.—5. 5980.05 sq. mi. 6. \$72.25. 7. \$$.24\frac{3}{4}$. 8. 357 acres. 9. \$152. 10. 23,220 gr. 11. \$57.50. 12. $109\frac{2}{10}$ A. 13. \$66. 14. 1,200 qt. 15. 100,000 lb. 16. 90 min. 17. 11 yd.

Page 154.—18. 10,000 cents. 19. 8,000 oz. 20. 1,200 pt. 21. 1,008 sec. 22. 160 rd. 23. 48,400 sq. ft. 24. 2,323,200 sq. yd. 25. 46,656 cu. in. 26. 140 gr. 27. 1,280 far. 28. $43\frac{3}{4}$ bu. 29. $5\frac{1}{4}$ gal. 30. 5 mi. 31. 96,000 sq. rd. 32. 70 da. 33. $4\frac{1}{2}$ min. 34. .01543. 35. .33399. 36. .05005. 37. .2992. 38. 107.87.

Page 155.—1. 16%. 2. $14\frac{2}{7}$%. 3. 20%. 4. 20%. 5. $3\frac{1}{17}$%. 6. 3,000%.

Page 156.—7. 46%. 8. $8\frac{1}{3}$%. 9. $7\frac{9}{13}$%. 10. 10%. 11. 10%. 12. $5\frac{15}{17}$%. 13. $12\frac{1}{2}$%. 14. $4\frac{8}{23}$%. 15. $7\frac{6}{17}$%. 16. $\frac{5}{16}$ of 1%. 17. $3\frac{1}{3}$%. 18. $6\frac{1}{4}$%. 19. $\frac{5}{8}$ of 1%. 20. $8\frac{1}{3}$%. 21. $83\frac{1}{3}$%. 22. $16\frac{2}{3}$%. 23. $\frac{1}{8}$ of 1%.

Page 157.—24. $\frac{1}{8}$ of 1%. 25. $30\frac{1}{4}$%. 26. $\frac{1}{2}$ of 1%. 27. $8\frac{1}{3}$%. 28. $6\frac{1}{4}$%. 1. $208\frac{1}{3}$ A. 2. 200 mi. 3. \$1,500. 4. \$60,000. 5. \$1,125. 6. \$12,000.

Page 158.—7. 100. 8. \$4. 9. \$90. 10. \$100. 11. \$100. 12. \$32. 13. 10 A. 14. 150 gal. 15. 1 sq. mi., 384 A. 16. 40 sq. rd. 17. 1 bu. 18. 384 lb. 19. $266\frac{2}{3}$ bu. 20. \$2,500. 21. \$50. 22. \$3,750. 23. \$$53\frac{1}{3}$.

Page 159.—24. \$7.50. 25. \$112.50 26. 100. 27. 400. 28. 10,000. 29. 200. 30. 300. 1. \$384. 2. \$.84. 3. \$3.91. 4. \$2.30. 5. \$60. 6. 8%.

Page 160.—7. $9\frac{1}{11}$%. 8. $11\frac{1}{9}$%. 9. \$200. 10. \$400. 11. \$320. 12. \$.10 per lb. 13. \$77.50. 14. \$.25. 15. $6\frac{1}{4}$%. 16. $7\frac{59}{163}$%. 17. $5\frac{5}{9}$ %. 18. \$.30. 19. \$5,310. 20. \$25 per head. 21. \$.16. 22. \$750.

Page 161.—23. \$5,000. 24. \$.35. 25. $5\frac{5}{9}$%. 26. $9\frac{1}{11}$%. 27. 40%. 28. \$.08$\frac{5}{8}$. 29. $6\frac{1}{4}$%. 30. \$1.75. 31. $14\frac{2}{7}$%. 32. $16\frac{2}{3}$%. 33. 25¢ per doz. 34. 10¢. 35. 44%. 36. $16\frac{2}{3}$%. 37. 25¢ per doz.

Page 162.—38. $16\frac{2}{3}$%. 39. \$5.50 per bbl. 40. \$1. 41. $16\frac{2}{3}$%. 42. 20%. 43. $2\frac{2}{49}$%. 44. 486 head. 45. \$7 per A. 46. $13\frac{7}{11}$%. 47. $368\frac{3}{4}$%. 48. $16\frac{2}{3}$%. 49. 40%. 50. \$50,000. 51. \$400.

Page 163.—52. \$960. 1. \$320. 2. \$262.50. 3. \$360. 4. \$1.32. 5. \$12.75. 6. 25%. 7. \$396.72. 8. \$242.25. 9. \$307.80. 10. 20% is $\frac{3}{4}$% greater.

Page 164.—11. \$250. 12. \$400. 13. 4%. 14. \$500. 1. \$2,925. 2. \$521.19.

Page 165.—3. \$1,950. 4. \$1,682.26. 5. 1,379 A. 6. \$3,900. 7. \$3,600. 8. \$16.45. 9. \$58$\frac{14}{17}$ or \$58.82+. 10. 2%.

Page 166.—11. 5%. 12. 3,000 bu. 1. \$5,625. 2. \$57.75. 3. \$11,269,208.20.

Page 167.—4. $\frac{1}{2}$. 5. \$3,556,000. 6. \$250. 7. \$137,755.10. 8. \$15,000. 9. \$7,996.80. 10. \$22,750. 11. $7\frac{1}{2}$ mo. 12. \$2,800,000.

Page 168.—1. \$220. 2. \$4.25. 3. \$15.10. 4. $32\frac{68}{151}$%.

Page 169.—5. \$1.75. 6. \$18.92. 7. \$52.25. 8. \$.60$\frac{5}{12}$. 1. \$45.

Page 170.—2. $1\frac{1}{4}$%. 3. \$5,400. 4. \$441.
5. \$4,467.01, nearly. (In such a case policy would be written for \$4,500).
6. \$2,060. 7. $\frac{1}{2}$ of 1%. 8. \$7,500. 9. 1%. 10. \$4,421.25. 11. $2\frac{3}{16}$%. 12. \$6.62$\frac{1}{2}$. 13. \$20.62$\frac{1}{2}$. 14. \$98.75.

Page 172.—1. \$90. 2. \$590. 3. \$72.80. 4. \$172.50. 5. \$520.80. 6. \$211.50. 7. \$45.024. 8. \$928.47—.

Page 173.—9. \$394.98. 10. \$544.80$\frac{1}{4}$. 11. \$1,179.61. 12. \$2,177.95. 13. \$2,997.40. 14. \$162.096. 15. \$1,050.625. 16. \$870.528. 1. \$10. 2. \$140. 3. \$67.50. 4. \$2.25.

Page 174.—5. \$123. 6. \$17.96. 7. \$65.71. 8. \$200.40. 9. \$2,936.08. 10. \$620.54. 11. \$1,486.40. 12. \$935.53. 13. \$1,867.59. 14. \$165. 15. \$1,698.31. 16. \$1,110.56. 17. \$1,033.76. 18. \$350.87. 19. \$849.49.

Page 175.—20. \$195. 21. \$2,644.45. 22. \$3,903.75. 1. \$1.40. 2. \$2.20. 3. \$7.60. 4. \$2.43. 5. \$2.40. 6. \$1,002.33.

Page 176—7. $301.38. 8. $502.53. 9. $381.64. 10. $1,439.06. 11. $977.27. 12. $126.15. 13. $1,708.17. 14. $1,101.29. 15. $1,210.30. 16. $.08+. 17. $40.50. 18. $600. 19. $150.36. 20. $245. 21. $403.67. 22. $3,666.59.

Page 177.—23. $515.33. 24. $83.125. 25. $362.83. 26. $486. 27. ——. 28. The latter; $5.

Page 178.—1. $15. 2. $180. 3. $1,125. 4. $199.50. 5. $52.50. 6. $60. 7. $108. 8. $135. 9. $75.60. 10. $43.20. 11. $235.20. 12. $6.51. 13. $8.44. 14. $168.75. 15. $735.94.

Page 179.—1. $100. 2. $57. 3. $127. 4. $109.44. 5. $45.94. 6. $358.92. 7. $91.88. 8. $212.05. 9. $17.01. 10. $921.38. 11. $1,136.65. 12. $1,356.30.

Page 180.—1. $.144. 2. $.218$\frac{1}{8}$. 3. $.2895. 4. $.235$\frac{5}{8}$. 5. $109.20. 6. $118.25. 7. $194.93. 8. $110.69. 9. $200.70. 10. $137.48. 11. $114.28. 12. $42.17. 13. $880.51.

Page 181.—14. $48.31. 15. $1,129.69. 16. $358.18. 1. $47.15. 2. $124.833. 3. $106.75. 4. $200.52.

Page 182.—5. $132.77. 6. $267.01. 7. $946.17. 8. $358.21. 9. $92.31. 10. $60.88. 1. $167.52. 2. $197.31. 3. $119.56. 4. $75.10. 5. $131.35. 6. $160.65.

Page 183.—7. $1,511.70. 8. $890.92. 9. $131.75. 1. $60.61. 2. $68.87. 3. $31.73. 4. $60.45. 5. $1.15.

Page 185.—1. $288.75. 2. $359.10.
3. (*a*) $1,771.93. (*b*) $1,807.81. 4. $1,239.46.

Page 186.—5. $741.06. 6. $1,179.53. 7. $1,067.91. 8. $903.97. 9. $792.04. 10. $1,121.85. 1. 8%. 2. 9%. 3. 4%. 4. 6%. 5. 5%.

Page 187.—6. 8%. 7. 7%. 8. 12%. 9. 6%. 10. 10%. 11. 8%.

Page 188.—1. $600. 2. $625. 3. $675. 4. $800. 5. $1,200. 6. $213. 7. $840. 8. $1,080. 9. $1,350.

Page 189.—10. $1,588.24 nearly. 11. $1,080. 12. $450. 1. 5 yr 2. 4 yr. 3. 5 yr. 4. 9 mo. 5. 10 yr. 6. 1 yr., 9 mo. 7. 2 yr., 6 mo. 8. 11 mo. 9. 26 da.

Page 190.—10. Aug. 13, 1892. 11. 2 yr., 7 mo. 1. 8%. 2. 2 yr., 5 mo., 14 da. 3. $673.62. 4. $750. 5. 3 yr., 2 mo., 12 da. 6. 12%. 7. $890.

Page 191.—8. $626.75. 9. 7%. 10. 1 yr., 10 mo., 20 da. 11. Jan. 10, 1893. 12. $800. 13. The first; $95. 14. A, 6%; B, 5$\frac{1}{2}$%.

15. 12%. 16. 1 yr., 10 mo., 15 da. 17. Each, 7 mo. 18. $420. 19. 6%. 20. 41⅔%.

Page 192. — 1. $165.60. 2. $44.87. 3. $7.16. 4. $16.26. 5. $30.80.

Page 193. — 6. $807.94. 7. $265.09. 8. $181.85. 9. $209.20. 10. $272.82. 11. $433.31. 12. $433.77. 13. $232.86. 14. $861.65. 15. $1,240.69. 16. $1,108.76.

Page 195. — 1. $363; $63. 2. $1,752.

Page 196. — 3. $4.92. 4. $1,087.21.

Page 197. — 5. $3,660. 6. $9,112.50. 7. $1,849.89. 8. $1,602.

Page 198. — 1. $349.80. 2. $586.56. 3. $611.89. 4. $410.55. 5. $632.50.

Page 199. — 6. $328.27. 7. $517.48.

Page 200. — 8. $147.84. 9. $917.14.

Page 201. — 1. $230.50. 2. $56.39. 3. $44.19.

Page 202. — 1. $591.09.

Page 203. — 2. $202.56. 3. $4,094.05. 4. $145.90.

Page 204. — 1. $5.25 bank discount; $294.75 proceeds. 2. $6.20 bank discount; $193.80 proceeds. 3. $11 bank discount; $1,989 proceeds. 4. April 6 to 9, 1893, maturity; $60.02 bank discount; $2,843.98 proceeds.

Page 205. — 5. July 9 to 12, 1892, maturity; $9.58 bank discount; $1,035.42 proceeds. 6. Sept. 3 to 6, 1892, maturity; $13.79 bank discount; $861.71 proceeds. 7. April 17 to 20, 1893, maturity; $13.44 bank discount; $1,246.71 proceeds. 8. Sept. 27 to 30, 1891, maturity; $33.49 bank discount; $1,046.96 proceeds. 9. May 29 to June 1, 1893, maturity; $20.62 bank discount; $907.32 proceeds. 10. $11.35. 11. Feb. 12 to 15, 1891, maturity; $12.33 bank discount; $1,285.47 proceeds. 12. April 13 to 16, 1893, maturity; $8.61 bank discount; $340.31 proceeds. 13. Dec. 31, 1892, to Jan. 2, 1893, maturity; $14.46 bank discount; $1,498.59 proceeds. 14. June 23 to 26, 1893, maturity; $73.55 bank discount; $2,484.85 proceeds. 15. April 2 to 5, 1893, maturity; $1.50 bank discount; $118.69 proceeds.

Page 206. — 1. $1,000. 2. $735.29. 3. $384.94. 4. $2,028.40. 5. $1,263.48. 6. $3,562.34.

Page 207. — 1. $200 present worth; $28 true discount. 2. $100 present worth; $5 true discount. 3. $192.59 present worth; $15.40 true

discount. 4. $283.48 present worth; $25.52 true discount. 5. $711.38 present worth; $163.62 true discount. 6. $41.77+. 7. $1,092.06 present worth; $142.94 true discount. 8. $981.82. 9. $453.70. 10. $227.27.

Page 208.—11. $0.943. 12. ——. 13. $31.41.

Page 209.—1. $501.25. 2. $887.69. 3. $400. 4. $479.40. 5. $752.81.

Page 210.—6. $538.65. 7. $2,480.

Page 211.—8. $2,000. 9. $1,972 proceeds; $5 exchange. 10. $963.08. 11. Proceeds, $948.15; exchange, $2.40. 12. $\frac{1}{4}$% premium. 13. $1,548.86. 14. $976.66. 15. $1,519.10. 16. $3,498.62.

Page 213.—1. $200. 2. Semi-annual income, $108. 3. $160. 4. $500. 5. 150 shares.

Page 214.—6. $1,875. 7. $2,931.25. 8. 7%. 9. $1,631.25. 10. $2,055. 11. 40 bonds. 12. 23 bonds. 13. $56.25. 14. 200 shares. 15. $78. 16. $31.87.

Page 215.—17. $86.50. 18. $122\frac{5}{8}$, or $1,226.25 for each bond. 19. $63.75 per share. 20. $7,500. 21. 8%. 22. 4%. 23. 9%. 24. 309 bonds. 25. $41,208.75. 27. $10\frac{77}{100}$%.

Page 216.—28. $62\frac{1}{4}$. 29. 223 bonds; $9\frac{3}{8}$%. 30. $$149\frac{7}{8}$. 31. $80. 32. $8\frac{2}{31}$%. 33. $77,250. 34. $6\frac{14}{23}$%. 35. $5\frac{5}{8}$%. 36. To loan money. 37. $2,560. 38. $7\frac{1}{3}$%. 39. Income would be increased $700.

Page 217.—1. 1 mo., 28 da.

Page 218.—2. $2\frac{5}{8}$ months. 3. $1\frac{10}{11}$ months. 4. March 2, 1893. 5. 1 yr., 3 mo.; April 15, 1891. 6. In 9 months.

Page 219.—7. June 14. 8. July 31.

Page 220.—11. $3\frac{2}{13}$ months. 12. 70 days. 13. May 1, 1893. 1. Nov. 27.

Page 221.—2. $551.25. 3. $551.25. 4. Oct. 13, 1892. 5. $410. 6. $400.

Page 222.—1. 2,000. 2. 444.44 acres. 3. $7\frac{1}{2}$%. 4. March 9, 3 P.M. 5. $1,758.85. 6. $2.25. 7. 16 rd., 4 yd., 1 ft., $2\frac{2}{3}$ in. 8. $2.67. 9. 676 acres. 10. Second term, $$.05\frac{5}{19}$; third term, 2,736 bu. of corn. 12. 25%.

Page 223.—13. $7\frac{1}{10}$%. 14. $156\frac{1}{4}$ days. 15. $605.71. 16. $11\frac{1}{4}$ degrees. 17. $750. 18. 20°. 19. $15.66. 20. $64.27. 21. $12. 22. $804.99. 23. $120. 24. 2 yr., 6 mo.

Page 224.—25. $2\frac{1}{2}\%$. 26. $1\frac{1}{2}\%$. 27. $12\frac{52}{73}\%$. 28. \$10,204.08+ 29. \$7,500. 30. \$200. 31. B, \$1,260; C, \$1,320; D, \$1,350. 32. Aug. 18. 33. \$2. 34. $12\frac{4}{33}\%$. 35. \$30.94.

Page 225.—1. 9. 2. 432. 3. 1,024. 4. 625. 5. 4,096. 6. 729. 7. 4,096. 8. 6,561. 9. $3\frac{3}{8}$. 10. $20\frac{51}{64}$. 11. $915\frac{1}{16}$. 12. 5.0625. 13. 7.5625. 14. 259.21. 15. $\frac{8}{27}$. 16. .00275625. 17. $\frac{64}{125}$. 18. 4,741.632. 19. $4{,}629\frac{17}{27}$. 20. $1{,}977{,}539\frac{1}{16}$. 21. .027. 22. $.0277\frac{7}{9}$. 23. .0196. 24. 3.375. 25. .003375. 26. 3,375.

Page 226.—27. 99. 28. 9,900. 29. 990,000. 30. 1; 100; 10,000; 1,000,000. 31. 999. 32. 999,000. 33. 999,000,000.

Page 227.—1. 15. 2. 28.

Page 228.—3. 396. 4. 21; 63; 42. 5. 108; 285; 273.

Page 229.—1. 625. 2. 164,025. 3. 9,604. 4. 206,116. 5. 130,321. 6. 186,624. 7. 361. 8. 30,625. 9. 46,225.

Page 230.—1. 25. 2. 24. 3. 32. 4. 26. 5. 35. 6. 41. 7. 45. 8. 51. 9. 120.

Page 232.—10. 37. 11. 140. 12. 64. 13. .25. 14. .31. 15. 38. 16. 19. 17. 17. 18. 73. 19. 86. 20. 9.2. 21. 97. 22. 2,030.

Page 233.—23. $\frac{10}{13}$. 24. 7.07+. 25. 10.049. 26. .935+. 27. 14.142+; .6324+; .6546+. 28. 252. 29. 256. 30. 1,352. 31. 1,776. 32. 40 rd. 33. 240 rd.; 120 rd.

Page 234.—34. 10 in. 35. 10 in. 36. 5 in.

Page 235.—37. 40 ft. 38. 56.56+ rd. 39. 92.19+ yd. 40. $7\frac{1}{2}$ acres. 41. 37 ft. 1. 5. 2. 12. 3. 36. 4. 45. 5. 25. 6. 35. 7. 24. 8. 33.

Page 240.—1. 23; 24; 26. 2. 27; 31; 33. 3. 42; 45; 63. 4. 123. 5. 1,020. 6. $\frac{2}{3}$; $\frac{5}{6}$; $\frac{4}{7}$. 7. 2.22.

Page 241.—8. 6.5. 9. 1.61+. 10. 1.44+. 11. 3.68+. 12. $\frac{1}{4}$. 13. $\frac{1}{10}$. 14. .01. 15. 3.23+. 16. 729 sq. in 17. 4 ft. by 4 ft. by 8 ft. 18. 12 ft. by 4 ft. by 4 ft. 19. Length, breadth, and thickness, ea. 12.907 in. 20. Length, breadth, and thickness, ea. 6.135+ in. 21. Length, breadth, and thickness, ea. 9 ft., 6.52+ in. 22. 10 ft., 10.32+ in.

Page 243.—1. 11 yr. 2. \$124. 3. \$120. 4. 70.

Page 244.—5. 5. 6. $20\frac{1}{2}$. 7. 150. 1. 8 ft. 2. 3. 3. $\frac{1}{12}$.

Page 245.—4. .001. 5. $\frac{1}{2}$ rd. 6. 2 lb. 7. \$1,500. 8. $2\frac{3}{16}$. 1. 13. 2. 30. 3. 9. 4. 71. 5. 1,490.

Page 246.—6. 3. 1. 50. 2. 336.

Page 247.—3. 5,050. 4. 16,244. 5. $\frac{1}{2}$; 60. 6. 495,550. 1. 486.

Page 248.—2. 4. 3. $\frac{3}{32}$. 4. $133.82255776. 5. 137,781. 1. 10.

Page 249.—2. 10. 3. 2. 1. 4.

Page 250.—2. 5. 3. 7. 1. 363. 2. 7,775. 3. 11,110. 4. 279,935.

Page 251.—1. $7\frac{1}{7}$%. 2. 87 bu., 6 qt. 3. 1,755 pieces. 4. $628.
5. 540 trees. 6. $13\frac{8}{9}$ bu. 7. 11.
8. 400 greater number; 150 less number. 9. 52 rows. 10. 135 girls.
11. $3\frac{1}{3}$%. 12. 1,600. 13. 2,057.

Page 252.—14. $3,000. 15. 2,500. 16. $26,852.85. 17. 200 da.
18. $449.68. 19. $\frac{1}{4}$ sq. in. 20. $144.93. 21. $1\frac{1}{8}$ bu. 22. $24.27.
23. 111.71 qt. 24. $29,406.25. 25. $5\frac{1}{25}$ hr. 26. $33.50.
27. 1 mi., 240 rd., 3 yd., 2 ft., $8\frac{20}{113}$ in. 28. $39.82.

Page 253.—29. $.002. 30. $380.09. 31. 40 yd. 32. $3.48.
33. $20\frac{10}{27}$%. 34. $56. 35. $96.
36. Ætna, $3,000; Hartford, $2,250; Globe, $1,875. 37. $2\frac{7}{8}$ cords.
38. $472.52. 39. 229 rd., 3 yd. 40. $9\frac{727}{997}$%. 41. 1,000 nails.

Page 254.—42. $48\frac{2}{5}$ bu. 43. 40 da. 44. $13\frac{13}{30}$.
45. 103 A., 8 sq. rd., 22 sq. yd., 6 sq. ft., 27 sq. in. 46. $11\frac{1}{7}$.
47. 31.30+ miles. 48. $6\frac{12}{73}$%. 49. $11\frac{1}{9}$ min. 50. $1.88.
51. 7 mi., 160 rd. 52. $6\frac{2}{3}$%. 53. $6\frac{2}{3}$ min.

Page 255.—54. A's, $1,000; B's, $1,200; C's $800.
55. 516 lb.; 430 lb. 56. 8 in. long, 4 in. wide, and 2 in. thick.
57. 240 rd. 58. Chair, $5; lamp, $3; book, $2. 59. $500. 60. $33\frac{1}{3}$%.
61. 18 ft.; 16 ft. 62. $2,460\frac{3}{8}$. 63. $15; $20; $25. 64. 20 mi.
65. 40 rd.; $53\frac{1}{3}$ rd. 66. $11\frac{1}{9}$%.

Page 256.—67. 12.20 P.M. 68. 60 acres. 69. 150 rd.
70. 4,800 mi. 71. 150 mi. 72. $\frac{5}{9}$ of 1%. 73. $\frac{9}{25}$. 74. 800.
75. 36 horses. 76. 450. 77. $\frac{1}{3}$ of 1%.

Page 257.—78. 17%. 79. $1\frac{103}{107}$ hr. 80. 500 yd. 81. 12 plows.
82. 1,000 yd. 83. 1,200 ft. 84. 54 in. 85. 14 mi. 86. 14,934 plants.
87. 1,320 posts. 88. $16\frac{2}{3}$%. 89. 330 ft.

Page 258.—90. 1 mi., 132 rd., 3 yd. 91. 20%.
92. Peach, 162; apple, 144; pear, 128. 93. 9 T., 16 cwt.
94. 320 bu.; 360 bu. 95. greater, 112; less, 56. 96. 20¢
97. 1,312 cu. in. 98. $16\frac{1}{2}$ mi. 99. 5 bu. 100. $6\frac{5}{8}$%.

Page 260.—1. 288 sq. ft. 2. 64 yd. 3. 9 ft. 4. 60 ft.
5. 40 sq. in. 6. 54,000 sq. ft. 7. 720 sq. ft.

Page 261.—8. $7\frac{1}{2}$ sq. ft. 9. 190 sq. ft. 10. $56\frac{1}{4}$ sq. ft.

Page 262.—1. 12 ft., .513+ in. 2. 7,957+ miles. 3. 254+ sq. ft. 4. 804+ sq. ft. 5. 4 ft., 3.5+ in. 6. 113+ sq. yds.
1. 198,943,846 sq. mi. 2. 804+ sq. in. 3. 412 sq. ft. 4. 4,188+ cu. in. 5. 14+ cu. in. 6. 12 sq. ft. 7. 27 sq. ft. 8. 160 sq. ft. 9. 65.97+ sq. ft. 10. 36 cu. ft. 11. 63.617+ cu. ft.

Page 264.—12. 144 sq. in. 13. 932,080 sq. ft. 14. 35,200 cu. ft. 15. 324 cu. ft. 16. 1,200 sq. in.

Page 265.—17. 94.2477 sq. in. 18. 201.062+ cu. ft. 19. 343+ cu. in. 20. 2,416 cu. ft. 21. 202.6332 cu. ft. 1. $49\frac{7}{9}$ yards. 2. $22.40.

Page 266.—3. $94. 4. $94.22. 5. $93.60. 6. 8,280 shingles. 7. $180. 8. $83.16. 9. $12. 10. 81.4+ bu.

Page 267.—1. 3,443.8+ bushels. 2. 51 gallons. 3. 4,793.8+ bu. ear corn.

Page 271.—1. 13,000,000 Ml.

Page 272.—2. .32,975 Km. 3. 7,820 M.
4. 23.48 H.M.; 2,348,000 Mm. 5. 50 da. 6. 34.02 m.
7. 2,982.72 miles. 8. $80.41. 9. 77.14 steres. 10. 2,780 Kg.
11. $218,750. 12. $25\frac{551}{720}$%. 13. $328,125. 14. 600 Hl.
15. 1,493 G.; 907+ G. 16. 4%. 17. 7.31+ Hl.; 2.38+ Hl.
18. 6 Km. 19. $266\frac{2}{3}$ cl.; $\frac{1}{4500}$ cg. 20. $1,443.98.

Page 274.—1. 36 Mex. in.—$33\frac{1}{3}$ Am. in.—$33\frac{1}{3}$. 2. $11\frac{1}{9}$ in.
3. 5.4 varas; 7.56 varas; 21.6 varas. 4. $\frac{1}{2}$ vara. 5. 5.94; 475.2 varas.
6. 8 rd., 1 ft., 4 in. 7. 28,512 varas. 8. 11 miles. 9. 950.4 varas.
10. 157.14 varas.

Page 275.—11. 3,613,040.64 sq. varas; 1,806,520.32 sq. varas; 903,260.16 sq. varas. 12. 265.7+ A. 13. 5,000 varas.
14. 2 mi., 201 rd., 4 yd., $4\frac{2}{3}$ in. 15. 177.13 A. 16. 491,147.712 sq. varas.
17. 141,134,400 sq. varas. 18. 40.29+ A.

APPENDIX.

Page 276. — ILLINOIS. 1. $2,756. 2. 20.787$\frac{1}{8}$. 3 A. $32; B, $40; C, $108. 4. 3$\frac{22}{51}$%. 5. 1; Upon the principle given in Art. 59, p. 30, and repeated in Art. 102 (6) p. 57; A fraction is a ratio, but a ratio may be a whole number. TENNESSEE. 1. .0069265625; for principle involved, see Art. 142, p. 91. 2. $1,600; $1,664; $2,112; 3. 714. 4. 9 dozen. 5. (*a*) 897.64\frac{1}{2}$; (*b*) $150.

Page 277. — GEORGIA. 1. 72$\frac{3}{4}$ yd. 2. First process, see p. 77; second process, see p. 78; third process, reduce to decimal fractions before dividing; most satisfactory analysis for young pupils, p. 77; for advanced pupils, p. 78; for this problem analysis of the decimal process is complicated because $\frac{2}{3}$ cannot be reduced to an exact decimal.
3. The latter is worth per annum $18 the more. 4. $10.
5. Plastering, $23.07+; carpeting, $46.66+. MICHIGAN. 1. Farm, $1,600; horse, $100. *Analysis:* Value of $\frac{1}{4}$ of farm = 4 times the value of horse; value of $\frac{1}{16}$ of farm = value of horse; value of $\frac{16}{16}$ of farm + $\frac{1}{16}$ of farm = value of both; continue by Art. 192. 2. $84. 3. 57.64 + stat. miles. 4. Explanation and answer: In 1 da. A can mow $\frac{1}{7}$ of $\frac{2}{3}$ of field = $\frac{2}{21}$ of field; in 1 da. B can mow $\frac{1}{8}$ of $\frac{2}{3}$ of field = $\frac{1}{12}$ of field; in 1 da. A and B can mow $\frac{2}{21} + \frac{1}{12}$ of field = $\frac{15}{84}$ of field. If A and B can mow $\frac{15}{84}$ of field in 1 da., they can mow $\frac{84}{84}$ of field in as many times 1 da. as $\frac{84}{84}$ of field contains $\frac{15}{84}$ of field. $\frac{84}{84}$ field ÷ $\frac{15}{84}$ field = 84 ÷ 15 = 5$\frac{3}{5}$ times. Then 5$\frac{3}{5}$ times 1 da. = 5$\frac{3}{5}$ da. *Answer*. Or, by reciprocal, $\frac{15}{84}$ of all work = 1 day's work. Therefore $\frac{84}{15}$ days' work = all work = 5$\frac{3}{5}$ days' work. See Comparisons in Probs. 8 and 11, p. 158. 5. 144 yd.

Page 278. — INDIANA. 1. 11$\frac{1}{4}$ oz. 2. $\frac{1}{8}$ in. 3. Daily, A, $2.25; B, $2. 4. 25 cts. per keg greater. 5. $150.85.
WISCONSIN. 1. 12 oxen. 2. 270.5 + rd. 3. 4$\frac{97}{150}$ da.
4. 9.40 A.M.; 109$\frac{2}{3}$ mi. 5. See S. & K. Lower Book, p. 161.

Page 279. — SOUTH CAROLINA. 1. $45. 2. 42$\frac{2}{3}$ da. 3. 40.
4. 156$\frac{3}{3}\frac{2}{8}\frac{1}{8}\frac{1}{8}$ cu. yd. 5. $200. NEBRASKA. 1. 384. 2. There are 5

fractional units in the reciprocal of $\frac{4}{5}$. *Analysis:* (1) To find the reciprocal of $\frac{7}{16}$. The reciprocal of $\frac{1}{16} = 16$. The reciprocal of $\frac{7}{16} = \frac{1}{7}$ of $16 = \frac{16}{7}$. (2) To reduce $\frac{16}{7}$ to a mixed number. See Art. 105, p. 59. **3.** $1\frac{7}{8}$ lb. tea $= \frac{15}{8}$ lb. tea ; cost of $\frac{15}{8}$ lb. tea $=$ \$1.05 ; cost of $\frac{1}{8}$ lb. tea $= \frac{1}{15}$ of \$1.05 $=$ \$.07 ; cost of $\frac{8}{8}$ lb. tea $= 8$ times \$.07 $=$ \$.56; cost of 16 lb. tea $= 16$ times \$.56 $=$ \$8.96. **4.** $1\frac{1}{2}$. **5.** $23\frac{127}{226}$. **6.** $1328\frac{1}{8}$. **7.** \$5,000. **8.** 240 rd. ; 120 rd. **9.** $79\frac{3}{4}$.

Page 280.— NEW YORK. **1.** (*a*) 4 mo., 15 da., 13 hr., 7 min., 58.08 sec. (*b*) $\frac{11}{96}$ bu. **2.** 1,400 handbills. **3.** $401\frac{1}{2}$ lb. ; 99 lb. ; 33 lb. ; 66 lb. ; $16\frac{1}{2}$ lb. **4.** \$270.93. **5.** $66\frac{2}{3}$%. MISSISSIPPI. **1.** \$120. **2.** $\$249.56\frac{1}{4}$. **3.** 4.479 + A. **4.** A, $\$1,714\frac{2}{7}$; B, $\$2,285\frac{5}{7}$. **5.** See pp. 46, 47.

Page 281. — NEW JERSEY. **1.** \$1,980. **2.** A, $\$4,398\frac{78}{89}$; B, $\$3,337\frac{7}{89}$; C, $\$5,764\frac{4}{89}$. **3.** 148 yd. **4.** \$47,000 ; $7\frac{7}{9}$%. **5.** $4^2 : 8^2 = 1 : 4$. COLORADO. **1.** 27 bu. **2.** \$368.25. **3.** \$137.35. **4.** .07889 +. **5.** 24.73 + ft. $(\sqrt{16^2 + 16^2 + 10^2})$.

Page 282. — ARKANSAS. **1.** 3 hr., 13 min., 40 sec. P.M. **2.** \$2.35 +. **3.** 10.6 + sq. rd. **4.** 73.8 + ft. **5.** \$1,800 ; \$1,200 ; \$900. IOWA. **1.** 5,544 planks. **2.** $13\frac{11}{25}$ A. **3.** \$2,500. **4.** $\$142.91\frac{2}{3}$. **5.** See Art. 77; Art. 78 ; Art. 73 ; Art. 74 ; Art. 65 ; Art. 107, respectively.

Page 283. — KENTUCKY. **1.** 9 lb., 4 oz., 2 pwt., $4\frac{1}{2}$ gr. **2.** \$2.80 +. **3.** $\$9.77\frac{1}{2}$. **4.** 30 ; $33\frac{3}{4}$; $26\frac{1}{4}$. **5.** \$77.94 ; 66 + %. MINNESOTA. **1.** $20\frac{96}{100}$% (dimensions of hall are superfluous). **2.** $67\frac{1}{3}$ sq. ft. **3.** $37\frac{44}{100}$%. **4.** 75.15 ; 83.5 ; 300.6. **5.** 15,840 ft. $=$ 3 mi. **6.** See S. & K. Lower Book, p. 161, probs. 1, 2, 4 ; also S. & K. Higher Book, Art. 123. **7.** (*a*) $\frac{144}{1500}$; (*b*) $\frac{36}{375}$.

Page 284. — ONTARIO, PUBLIC. **1.** \$700. **2.** \$.968 + per bu. (includes premium as part of cost). **3.** \$730.18 (with 3 da. grace). **4.** March 3, 1894. **5.** 22.25 + cu. ft. See Art. 393. ONTARIO, H. S. **1.** A, \$439.68; B, \$293.12; C, \$183.20. **2.** \$119.15 (with 3 da. grace). **3.** $16\frac{2}{3}$, see Art. 352. **4.** 1.5033 + C. **5.** 20 in.

Page 285. — ONTARIO UNIV. **1.** 40,170, pop. **2.** (*a*) \$29.18 +; (*b*) \$544.64. **3.** Tea, \$2,401.96; total com. \$98.04. **4.** Gains \$15.60 instead of \$12. **5.** $9\frac{23}{33}$%. **6.** $30\frac{10}{11}$%. **7.** \$364.94 (with 3 da. grace).

Page 286. — **8.** 2.952%. **9.** 3 hr. 38 min. $10\frac{10}{11}$ sec. **10.** 128,679,526,400 Acres.

TEXAS. **1.** (*a*) Price of 5 melons $=$ 75 cents ; price of 1 melon $= \frac{1}{5}$ of 75 cents ; $\frac{1}{5}$ of 75 cents $=$ 15 cents ; price of 1 melon $=$ 15 cents. (*b*) 5 cents $=$ price of 1 melon ; 75 cents $=$ price of as many times 1

melon as 75 cents contains 5 cents; 75 cents ÷ 5 cents = 15 times; 15 times 1 melon = 15 melons. **2.** $13,846. **3.** 35,325. **4.** $62\frac{25}{336}$. **5.** $\frac{9}{32}$. **6.** 8%. **7.** 50. **8.** $119.42; $16.33; for analysis, see p. 207. **9.** $11,187.50. **10.** $\frac{2}{3}$; $\frac{1}{3}$; $\frac{3}{8}$.

Page 287.—**1.** 432. **2.** 24; 12; 2. **3.** $166\frac{2}{3}$, see p. 72. **4.** XCVIII. **5.** See p. 40. **6.** $\frac{7}{20} \times \frac{5}{5} = \frac{35}{100} = .35$; .225; .136. **7.** $288.75. **8.** 2:3. **9.** $109\frac{1}{11}$; $145\frac{5}{11}$; $163\frac{7}{11}$; $181\frac{9}{11}$. **10.** 226.2 + rd. **11.** .004132 + A. **12.** 1.66\frac{2}{3}$ per yd. **13.** $500. **14.** $600; $8\frac{1}{3}$%. **15.** 9,408.

Page 288.—**16.** A, $1,000; B, $1,400. **17.** $.36 per yd. **18.** 82 bu. **19.** $504 \times 12 = 6{,}048$. **20.** $780. **21.** $573.13+. **22.** $6,909.97+. **23.** $71\frac{3}{7}$. **24.** 42 (see p. 228); 15 (see p. 235). **25.** 219 + mi. **26.** 42 (see pp. 235, 240). **27.** $129.60. **28.** 8.33\frac{1}{3}$. **29.** 27; 18; 9; 33 + ft. **30.** 4,800 bbl.

Page 289.—**31.** $480. **32.** Nearly 20 bunches. **33.** $50.17. **34.** $266.25. **35.** 21. **36.** 72 in.; 36 in.; 36 in. **37.** 1,854.63 +. **38.** $99. **39.** $19\frac{5}{9}$ loaves. **40.** 4 ft. $9\frac{3}{5}$ in. **41.** A, $364; B, $308; C, $198. **42.** 2 yr. 6 mo. **43.** Lost 166\frac{2}{3}$. **44.** $16\frac{2}{3}$ sq.ft.